EngrD 2190 – Lecture 12

Concept: Process Analysis by Mathematical Modeling

Context: Energy Balance Examples, cont’d
Liquid-Gas separator with a heat exchanger.

Defining Question: What are the two chief differences between mass balances and energy balances?

Read Chapter 3 pp. 132-139 (energy balances with chemical reaction)

Summary of Energy Balances to date

- Cannot calculate the energy of a mass stream;

  cannot state $q_1 = 100$ kJ/min

  Must calculate the *change* in energy of a mass stream;

  must calculate $\Delta q_{1\rightarrow 2} = q_2 - q_1$

- Cannot measure Δq; must measure ΔT *or* Δ*phase*

  and use thermodynamics to convert to energy.

- Simple mass units can be complex energy units.

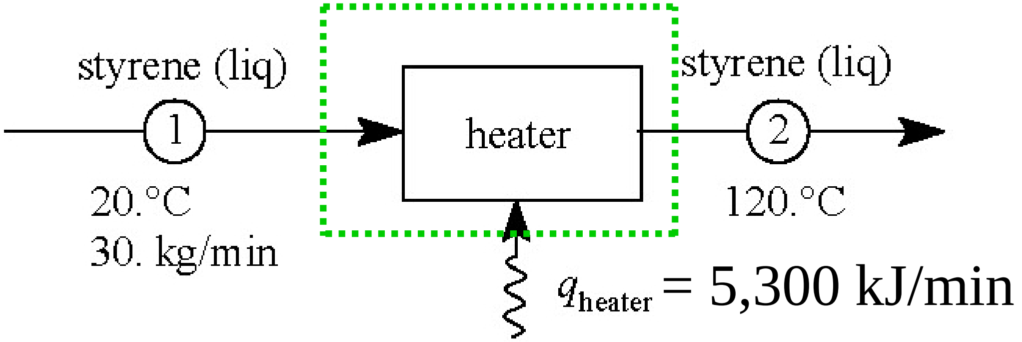
  Create an equivalent unit of elementary energy units.

An elementary energy unit has

- one mass stream in, one mass stream out
- one energy change; ΔT *or* Δ*phase*
- ***all*** mass passing through the unit is heated or changes phase.

Summary from Lecture 11

Example 1. **Calculate** q_{heater}.



styrene (liq)

1

20.°C

30. kg/min

heater

styrene (liq)

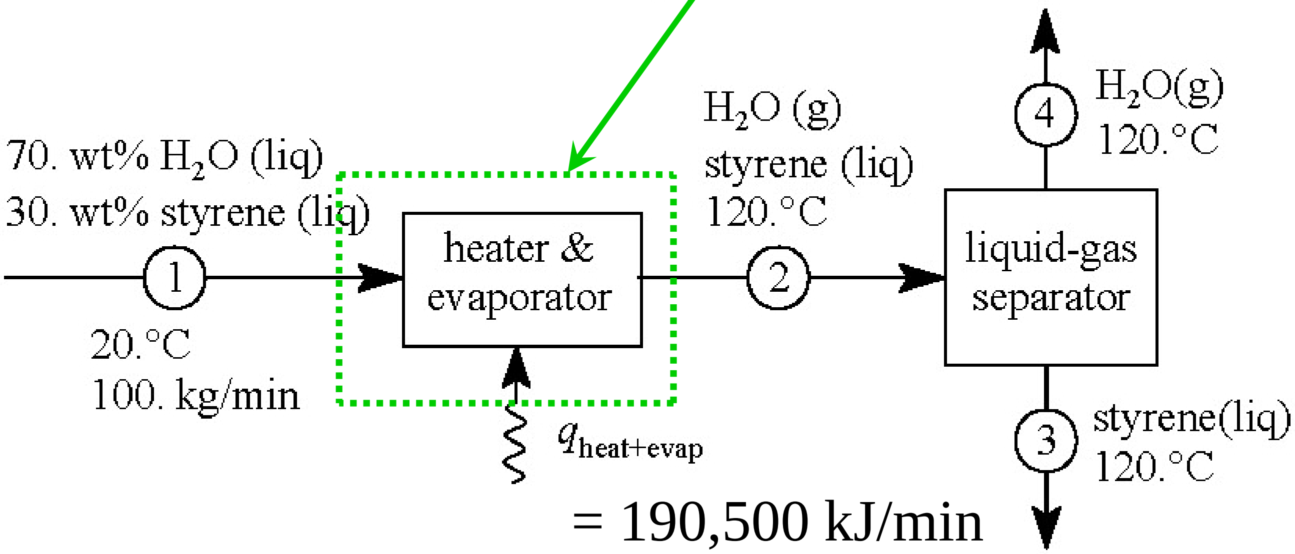
2

120.°C

q_{heater} = 5,300 kJ/min

Example 2: Exercise 3.63(A). **Calculate** $q_{\text{heat+evap}}$.

analyzed an equivalent unit



70. wt% H_2O (liq)

30. wt% styrene (liq)

1

20.°C

100. kg/min

heater & evaporator

$q_{\text{heat+evap}}$

= 190,500 kJ/min

H_2O (g)

styrene (liq)

120.°C

2

liquid-gas separator

4

H_2O(g)

120.°C

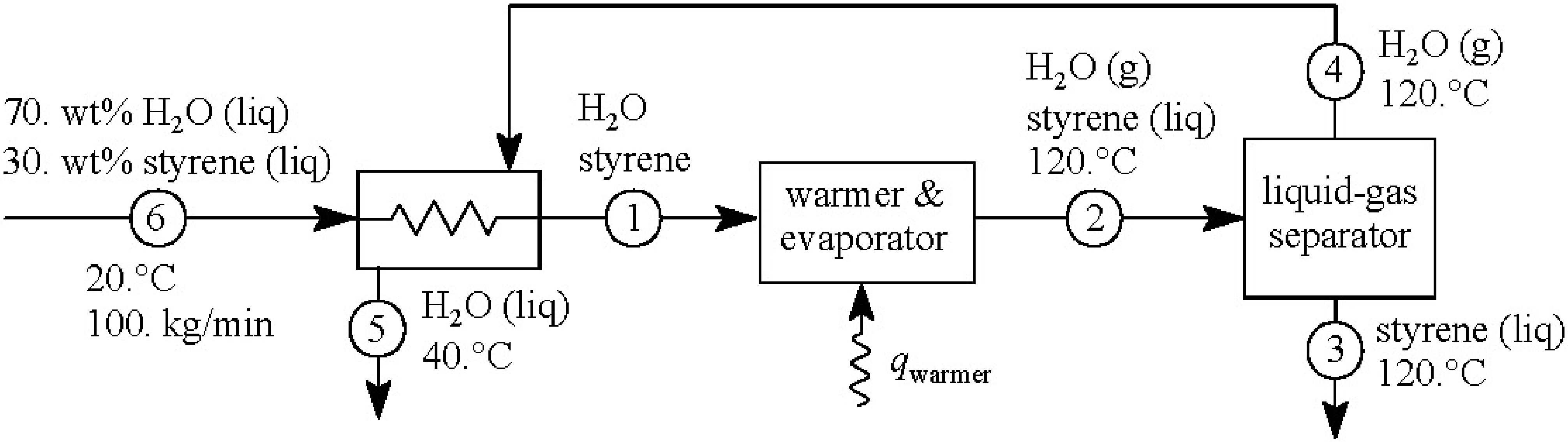
3

styrene(liq)

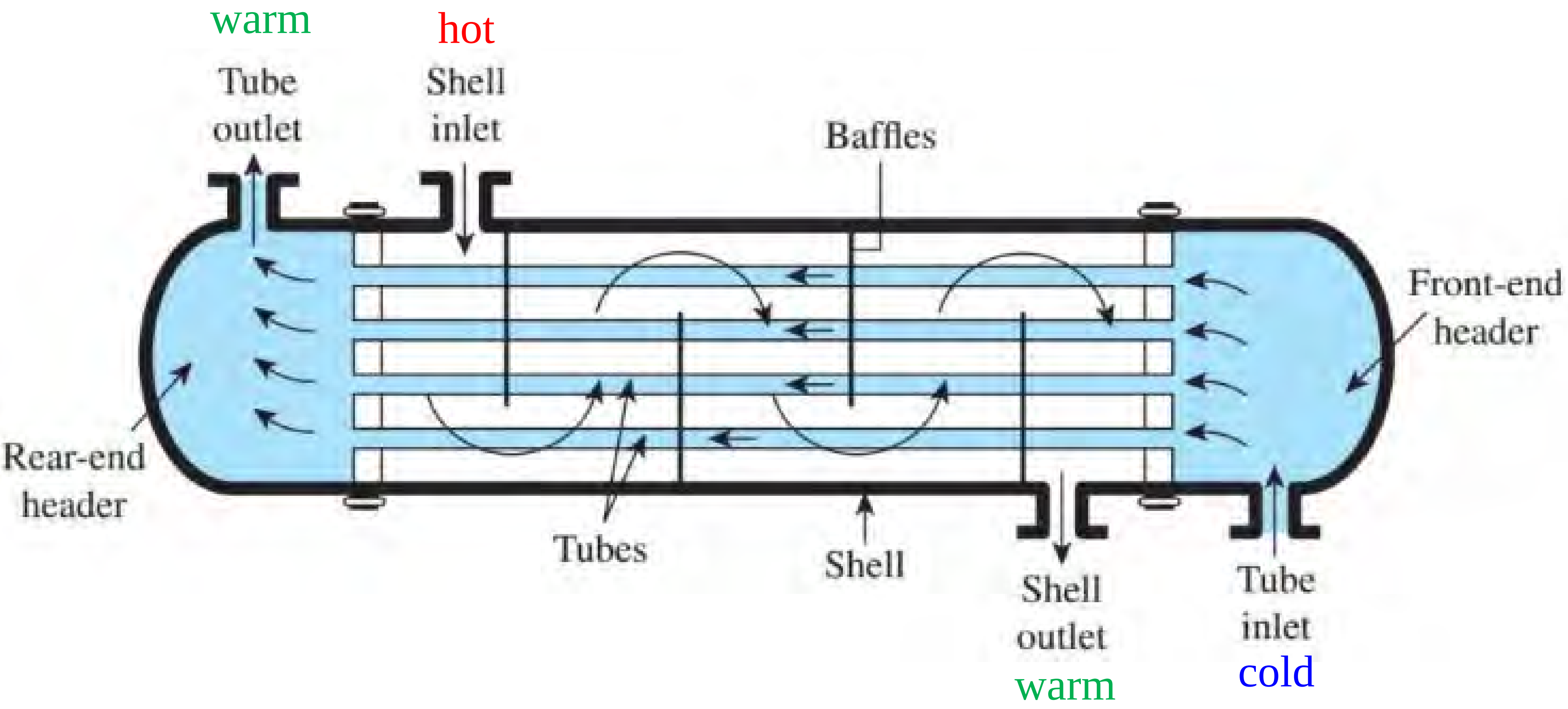
120.°C

Energy Example 3

The energy demand in the heater-evaporator in example 2 increased substantially. The chief energy demand was the evaporation of water. Add a heat exchanger to reclaim this energy by condensing the steam.



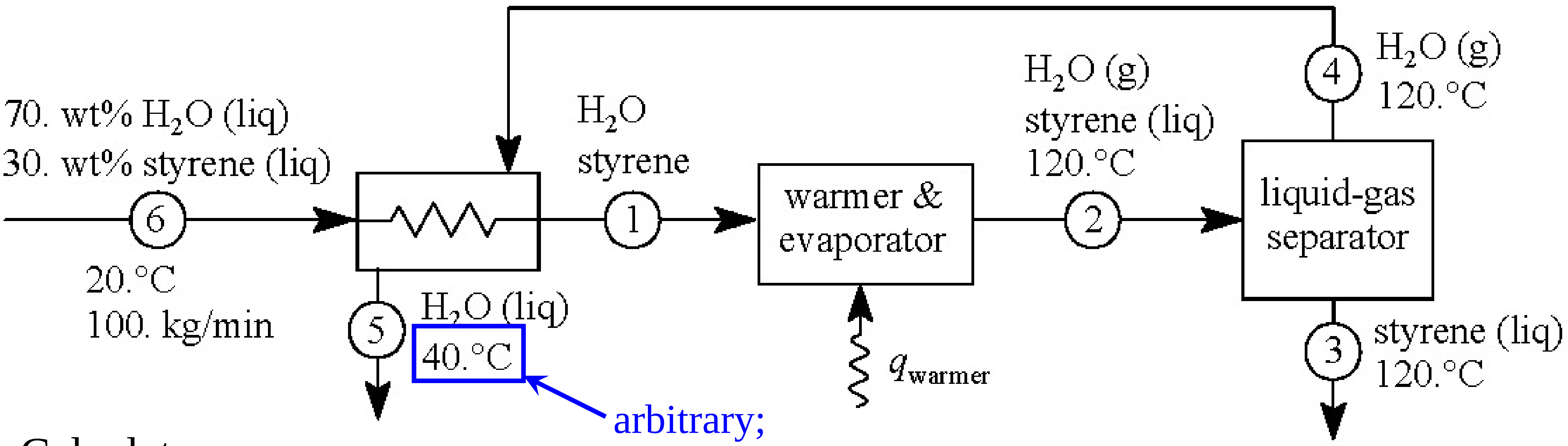
A Heat Exchanger



Heat Exchangers

Energy Example 3

The energy demand in the heater-evaporator in example 2 increased substantially. The chief energy demand was the evaporation of water. Add a heat exchanger to reclaim this energy by condensing the steam.



arbitrary;
must be >20° C and <120° C

Calculate q_{warmer}.

Method 1.

If we knew the state of stream 1 (temperature and phase(s)),

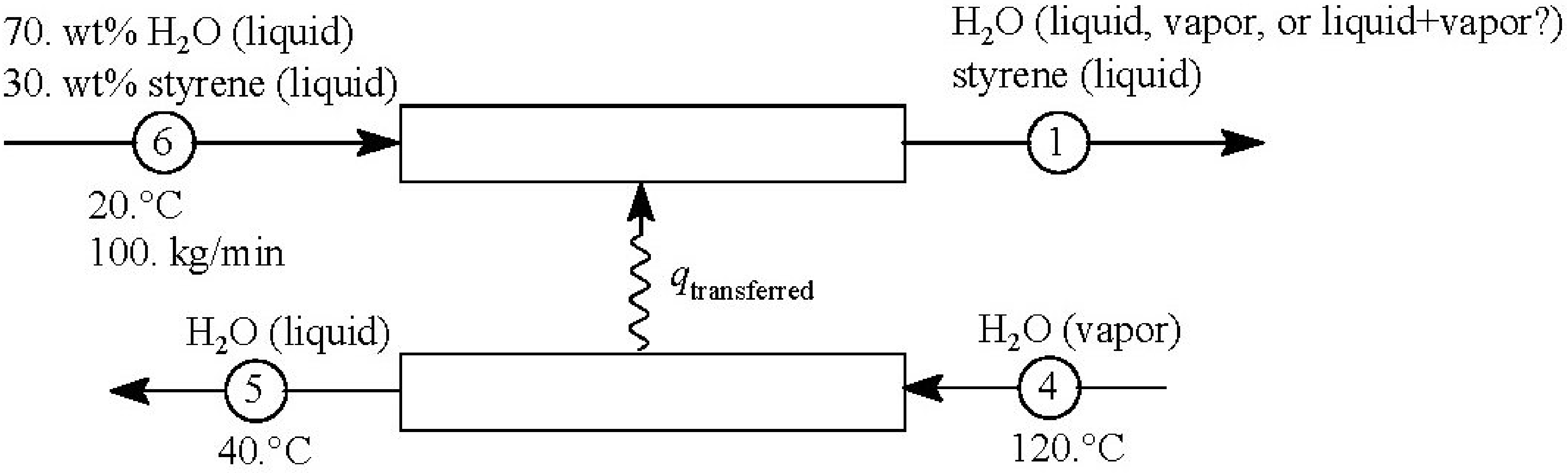
we could calculate $\Delta q_{1\to 2} = q_{warmer}$

Because streams 4 and 5 are completely specified (temperature and phase(s)),

we can calculate $\Delta q_{4\to 5}$

From an energy balance on the heat exchanger, $\Delta q_{4\to 5} = \Delta q_{6\to 1}$

Heat Exchanger Mathematical Model



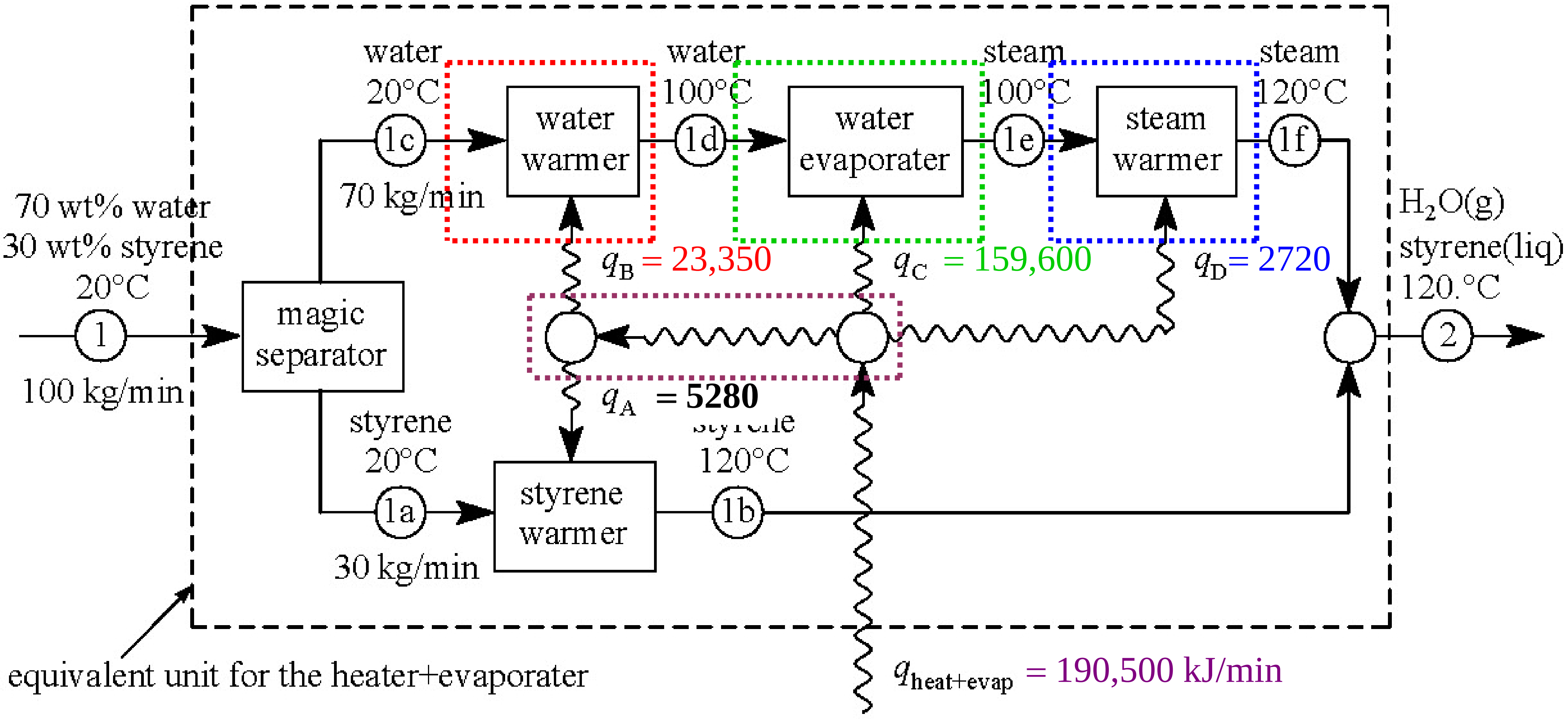
Devise equivalent units for the heat exchanger. Must guess the phase(s) of stream 1.

1. $T < 100^{\circ}$ C and H_2O is entirely liquid.
2. $\boxed{T = 100^{\circ} \text{ C and } H_2O \text{ is a mix of liquid and vapor.}}$ 1st guess
3. $T > 100^{\circ}$ C and H_2O is entirely vapor.

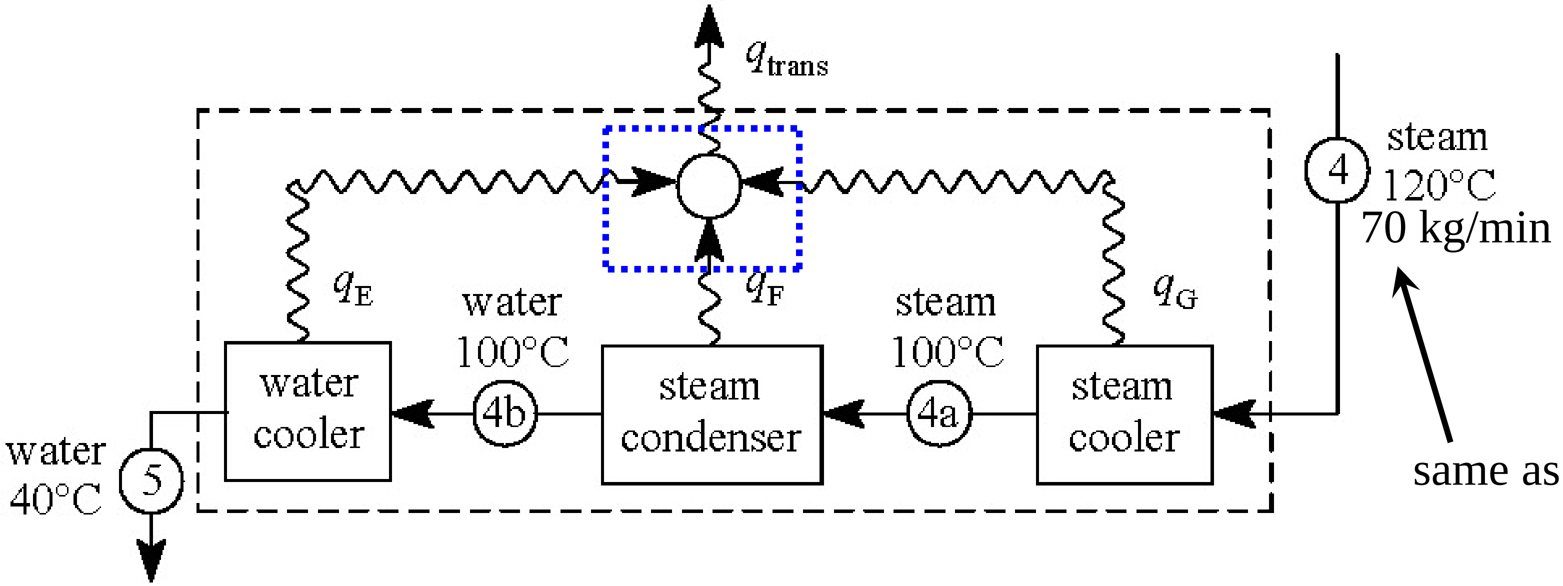
Note: We need not guess the *temperature* of stream 1.

Recap from Lecture 11

Example 2: Exercise 3.63(A). Equivalent unit for heater+evaporator:



Example 3: Exercise 3.63(B) Equivalent unit for heat exchanger: Stream 4 to stream 5.



same as example 2

Informal energy balances

From example 2, $q_G = 2720$ kJ/min

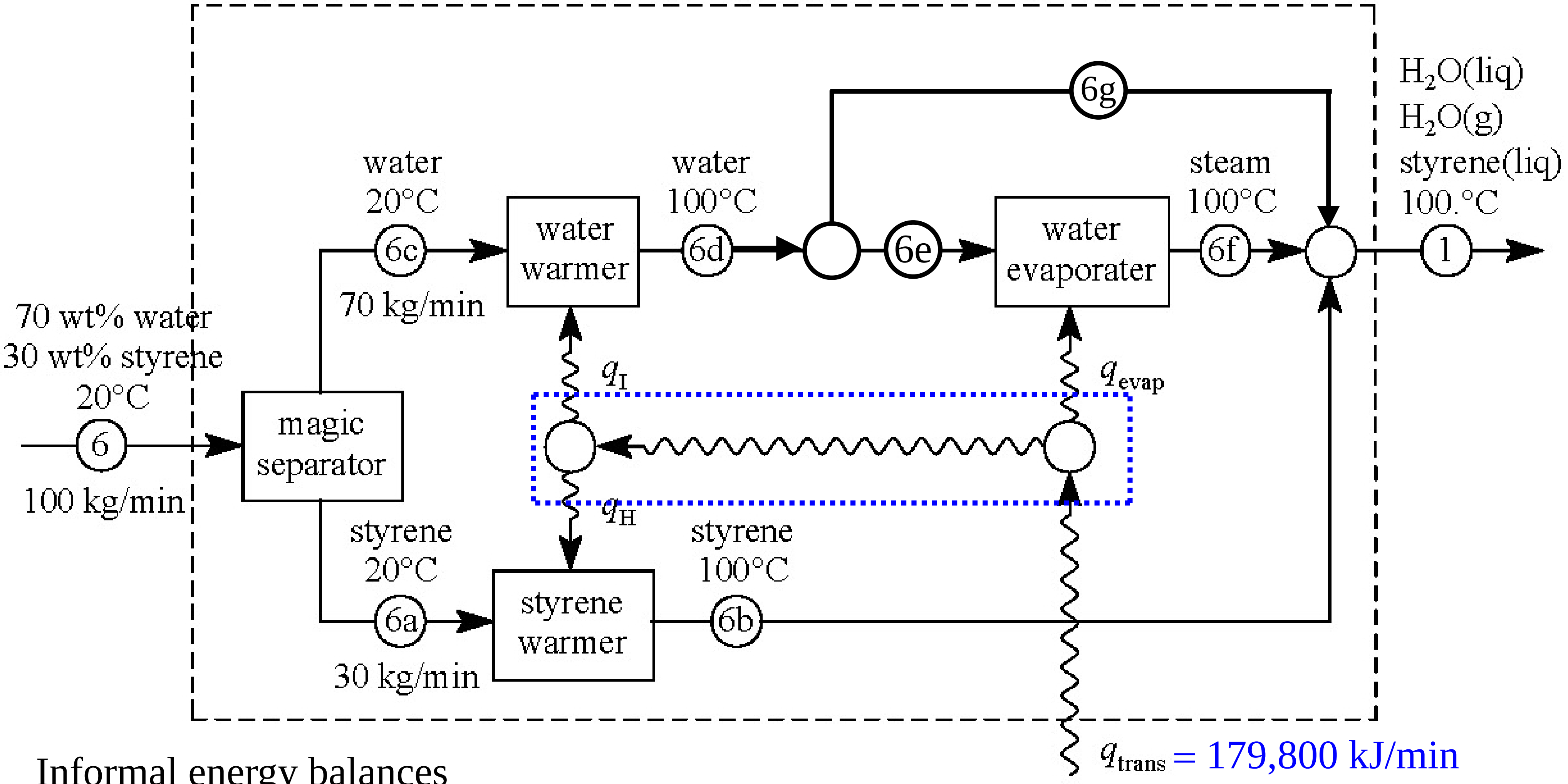
From example 2, $q_F = 159{,}600$ kJ/min

$$\Delta q_{4b\to 5} = \Delta q_{100°C\to 40°C} = \left(\frac{100-40}{100-20}\right)\times \Delta q_{100°C\to 20°C} = \left(\frac{100-40}{100-20}\right)\times 23{,}350 = 17{,}500 \text{ kJ/min}$$

Energy balance on fictitious energy combiner.

$$q_{\text{transferred}} = q_E + q_F + q_G = 17{,}500 + 159{,}600 + 2{,}720 = 179{,}800 \text{ kJ/min}$$

Example 3: Exercise 3.63(B) Equivalent unit for heat exchanger: Stream 6 to stream 1.



Informal energy balances

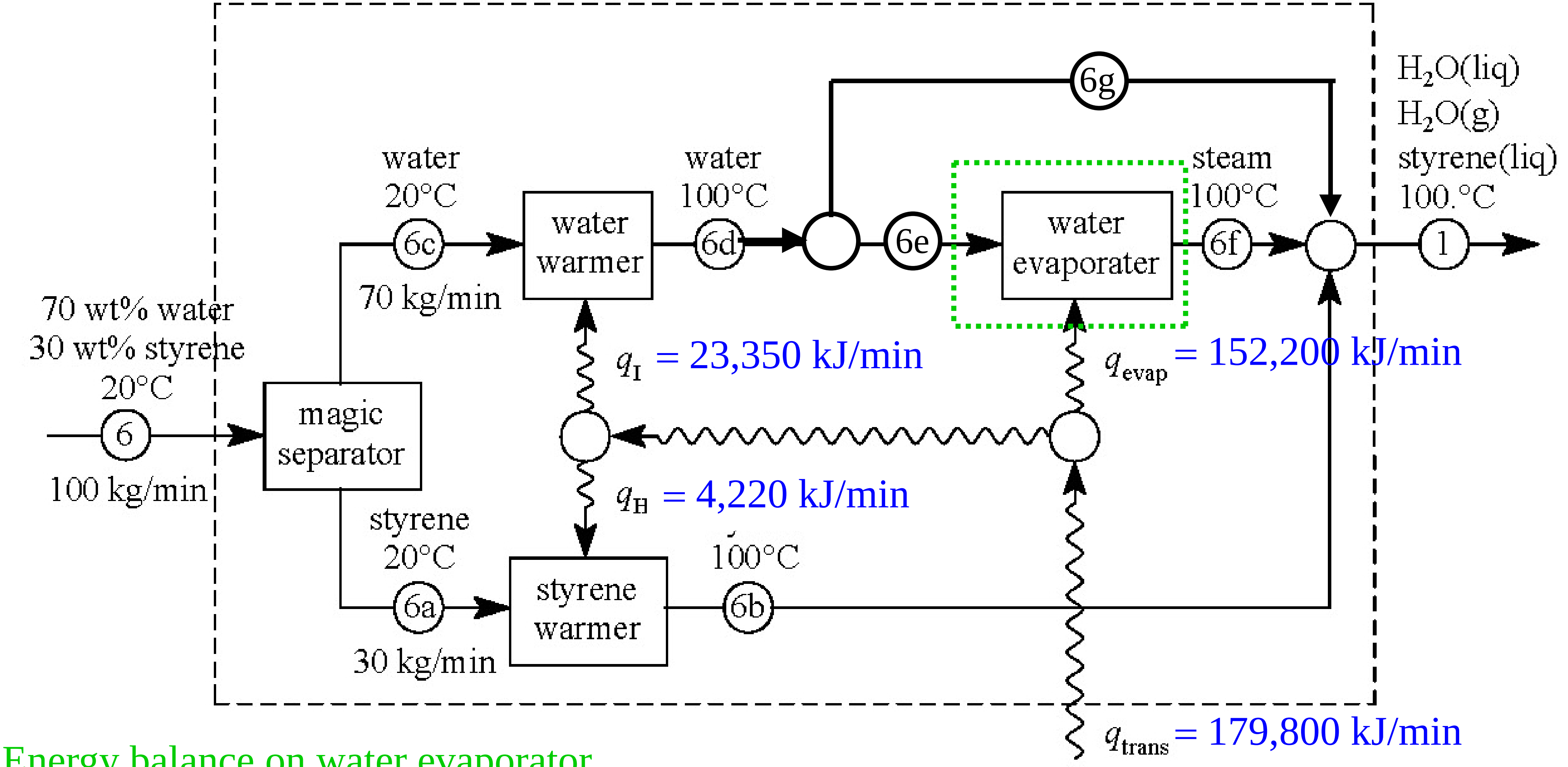
$$q_H = \Delta q_{6a\to 6b} = \Delta q_{20\to 100°C} = \left(\frac{100-20}{120-20}\right)\times \Delta q_{20°C\to 120°C} = \left(\frac{100-20}{120-20}\right)\times 5280 = 4220 \text{ kJ/min}$$

From example 2, $q_I = 23{,}350$ kJ/min

Energy balance on fictitious energy combiner.

$$q_{evaporator} = q_{transferred} - q_H - q_I = 179{,}800 - 4220 - 23{,}350 = 152{,}200 \text{ kJ/min}$$

Example 3: Exercise 3.63(B) Equivalent unit for heat exchanger: Stream 6 to stream 1.

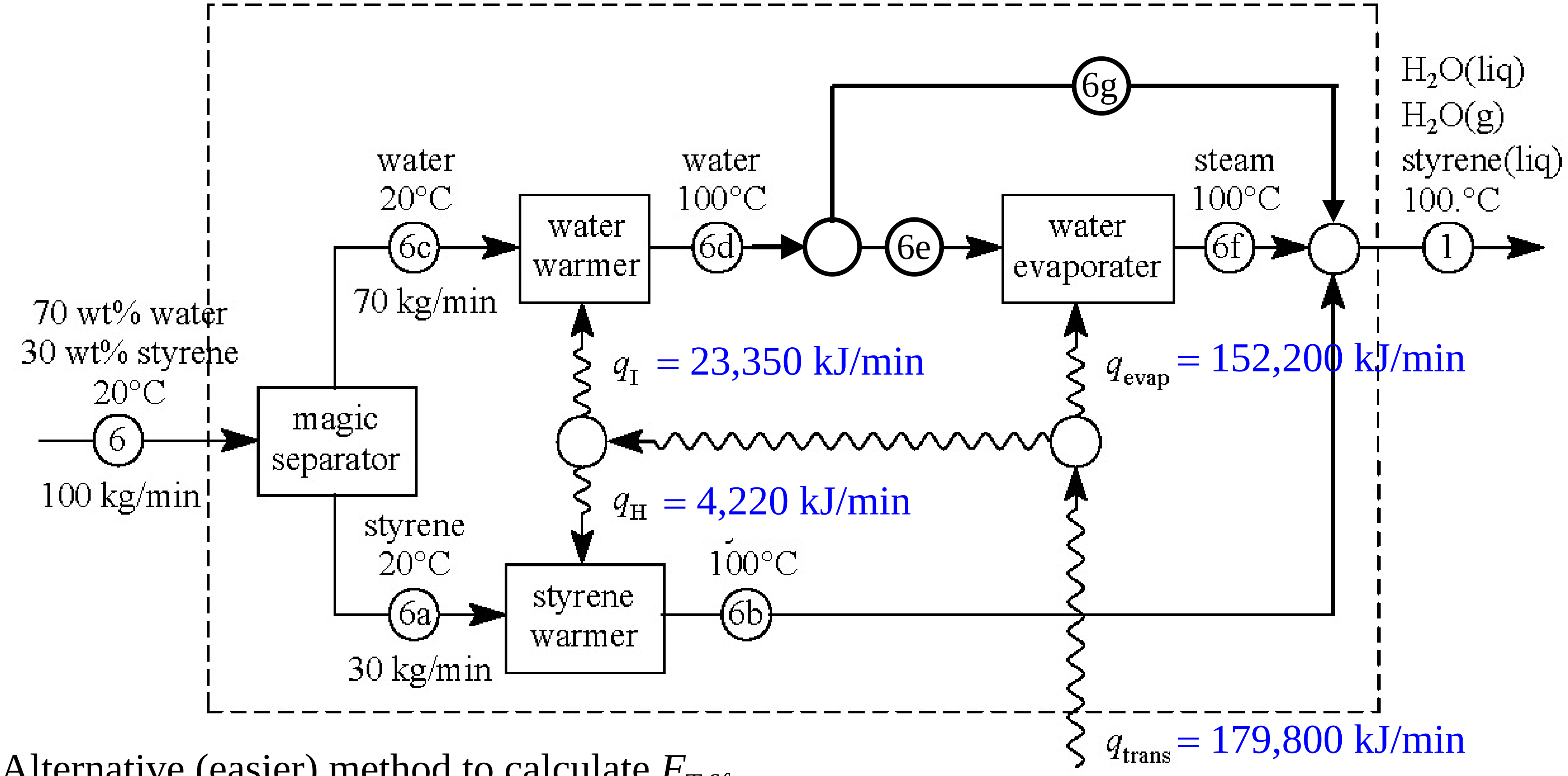


Energy balance on water evaporator

$$q_{\text{evaporator}} = q_{6\text{f}} - q_{6\text{e}} = F_{\text{T,6f}}\,\Delta H_{\text{vap}}$$

$$F_{\text{T, 6f}} = \frac{q_{\text{evap}}}{\Delta \bar{H}_{\text{vap}}} = \frac{152{,}200\ \text{kJ/min}}{\left(\dfrac{4.1\times10^{4}\ \text{J}}{\text{mol H}_2\text{O}}\right)\left(\dfrac{1000\ \text{mol H}_2\text{O}}{18\ \text{kg H}_2\text{O}}\right)\left(\dfrac{1\ \text{kJ}}{1000\ \text{J}}\right)} = 66.8\ \text{kg H}_2\text{O/min}$$

Example 3: Exercise 3.63(B) Equivalent unit for heat exchanger: Stream 6 to stream 1.



Alternative (easier) method to calculate $F_{T,6f}$.

From example 2, 159,600 kJ/min evaporates 70. kg H_2O/min.

$$(70.\,\text{kg H}_2\text{O})\times\left(\frac{152{,}200\text{ kJ/min}}{159{,}600\text{ kJ/min}}\right) = 66.8\text{ kg H}_2\text{O/min}$$

Check: What if calculation yielded q_{evap} > 159,600 kJ/min?

Calculate $q_{\text{warmer}} = \Delta q_{1\to 2}$

Stream 1 is 66.8 kg steam/min, 3.2 kg water/min and 30. kg styrene/min at 100º C.

Must convert to 70 kg steam/min and 30. kg styrene/min at 120º C.

1. Evaporate 3.2 kg/min of water:

$$\Delta q \;=\; F_{\text{water}}(\Delta H_{\text{vap}}) \;=\; \frac{3.2\text{ kg H}_2\text{O}}{\text{min}}\left(\frac{2280\text{ kJ}}{\text{kg H}_2\text{O}}\right) \;=\; 7{,}300\text{ kJ / min}$$

2. Warm 70 kg/min of steam to 120ºC from 100ºC:

$$\Delta q \;=\; F_{\text{steam}}C_{P,\,\text{steam}}(T_2 - T_1) \;=\; \frac{70\text{ kg steam}}{\text{min}}\left(\frac{1.94\text{ kJ}}{(\text{kg steam})(°\text{C})}\right)(120-100) \;=\; 2{,}720\text{ kJ / min}$$

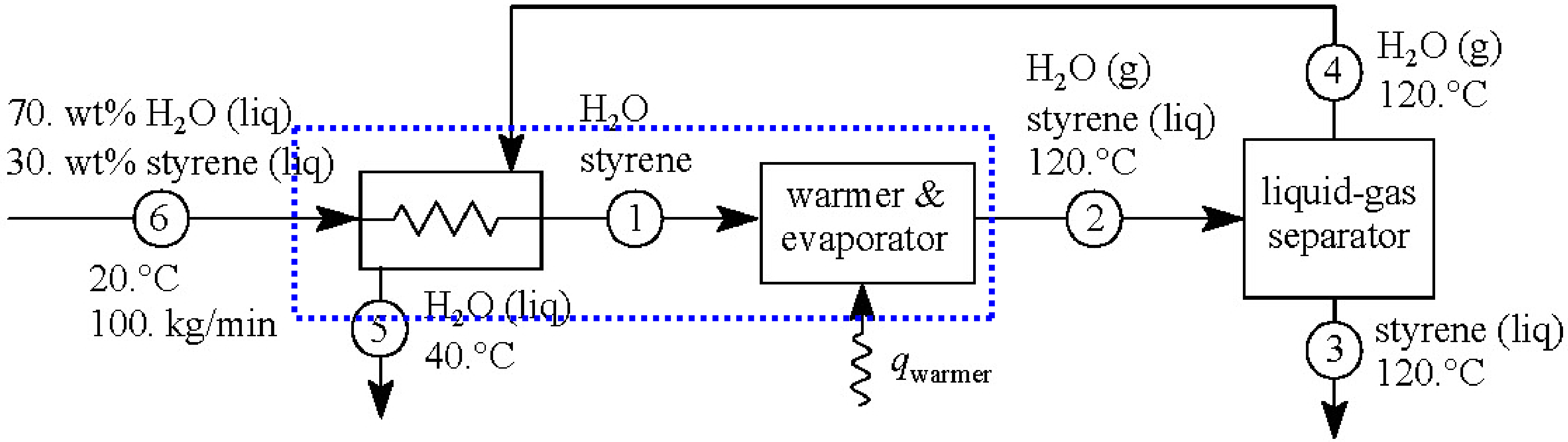
3. Warm 30 kg/min of styrene to 120ºC from 100ºC:

$$\Delta q \;=\; F_{\text{styrene}}\,C_{P,\,\text{styrene}}(T_2 - T_1) \;=\; \frac{30\text{ kg styrene}}{\text{min}}\left(\frac{1.76\text{ kJ}}{(\text{kg styrene})(°\text{C})}\right)(120-100) \;=\; 1060\text{ kJ / min}$$

The total energy rate into the warmer+evaporator
is therefore 7300 + 2720 + 1060 = 11,080 kJ/min.

The energy rate is only 6% of that in Example 2.

Second Method: Draw strategic system borders.



Avoid calculating the specifics of stream 1 by drawing system borders that enclose stream 1. Draw borders around the heat exchanger and the warmer+evaporator.

rate of energy in = rate of energy out

$$q_{\text{warmer}} + q_6 + q_4 = q_2 + q_5$$

$$q_{\text{warmer}} = (q_2 - q_6) + (q_5 - q_4)$$

$$q_{\text{warmer}} = (\Delta q \text{ for streams } 6 \rightarrow 2) + (\Delta q \text{ for streams } 4 \rightarrow 5)$$

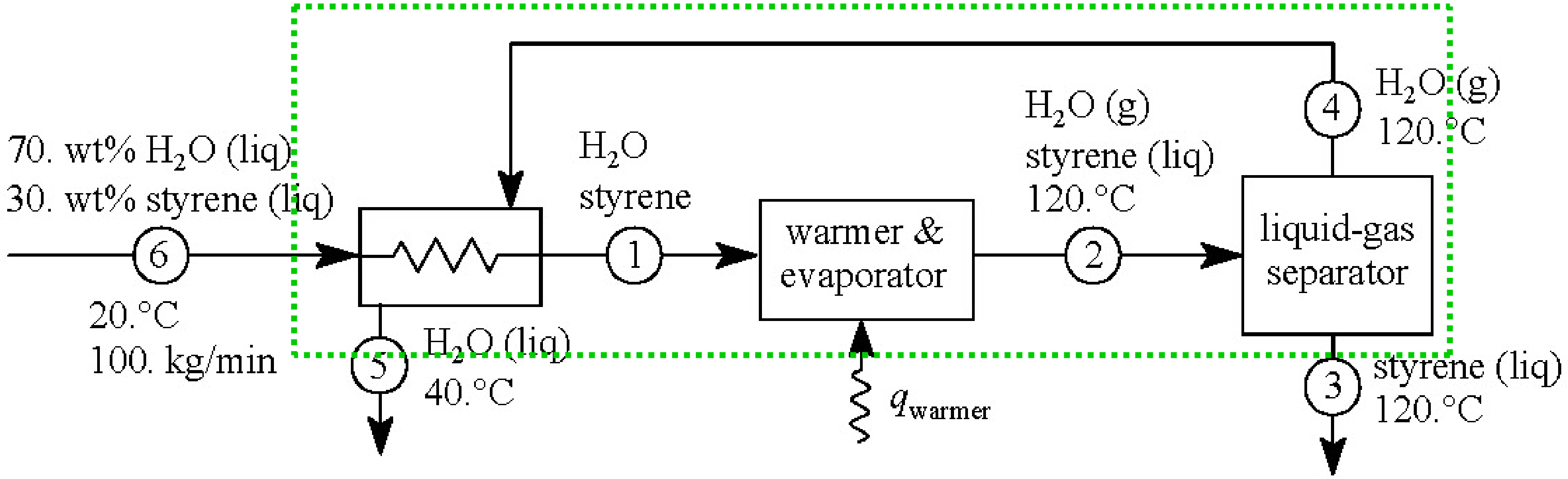
From example 2, Δq for streams 6 → 2 = 190,950 kJ/min.

Δq for streams 4 → 5 = Δq provided by the steam (120°C) to water (40°C) in the heat exchanger

= –179,830 kJ/min (from first method).

$$q_{\text{warmer}} = 190{,}950 + (-179{,}830) = 11{,}120 \text{ kJ/min.}$$

Third Method: Draw strategic system borders.



Avoid calculating the specifics of stream 1 by drawing system borders that enclose stream 1. Draw borders around the entire process.

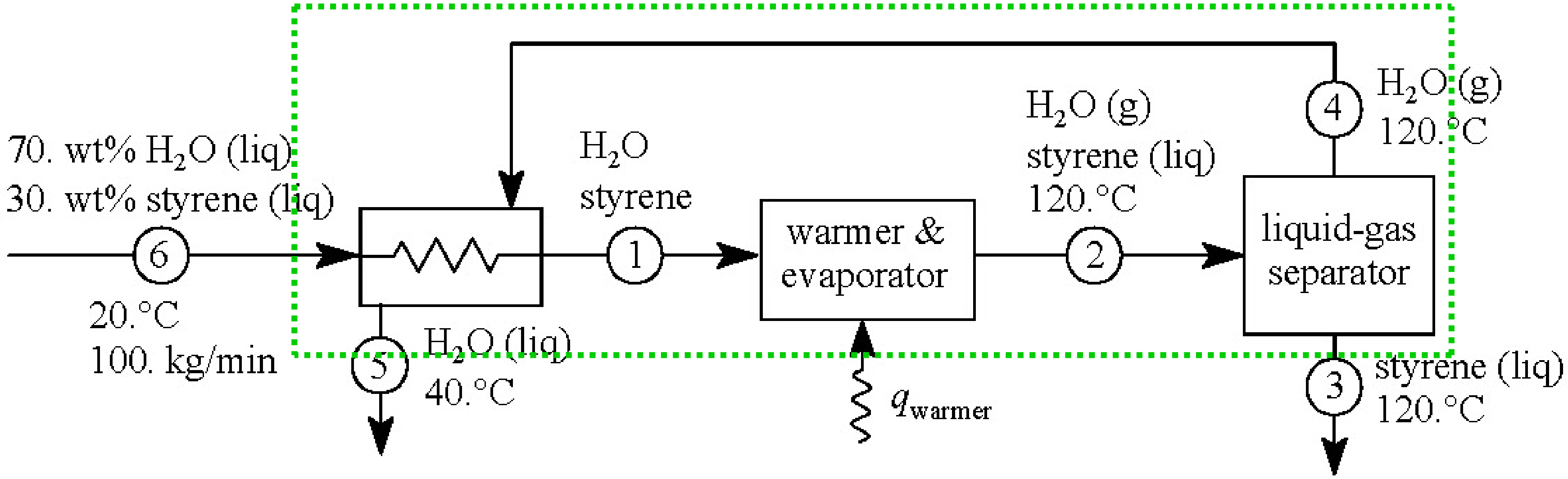
rate of energy in = rate of energy out

$$q_{\text{warmer}} + q_6 = q_3 + q_5$$

$$q_{\text{warmer}} = (q_3 - q_{6,\text{styrene}}) + (q_5 - q_{6,\text{H2O}})$$

$$q_{\text{warmer}} = (\Delta q \text{ for styrene in streams } 6 \rightarrow 3) + (\Delta q \text{ for } H_2O \text{ in streams } 6 \rightarrow 5)$$

Third Method: Draw strategic system borders.



$$q_{\text{warmer}} = (\Delta q \text{ for styrene in streams } 6 \to 3) + (\Delta q \text{ for } H_2O \text{ in streams } 6 \to 5)$$

1. Warm 30. kg/min of styrene to 120ºC from 20ºC:

$$\Delta q = F_{\text{styrene}} C_{P,\text{styrene}} (T_3 - T_6) = \frac{30 \text{ kg styrene}}{\text{min}} \left(\frac{1.76 \text{ kJ}}{(\text{kg styrene})(°\text{C})} \right) (120 - 20) = 5{,}280 \text{ kJ/min}$$

2. Warm 70. kg/min of water to 40ºC from 20ºC:

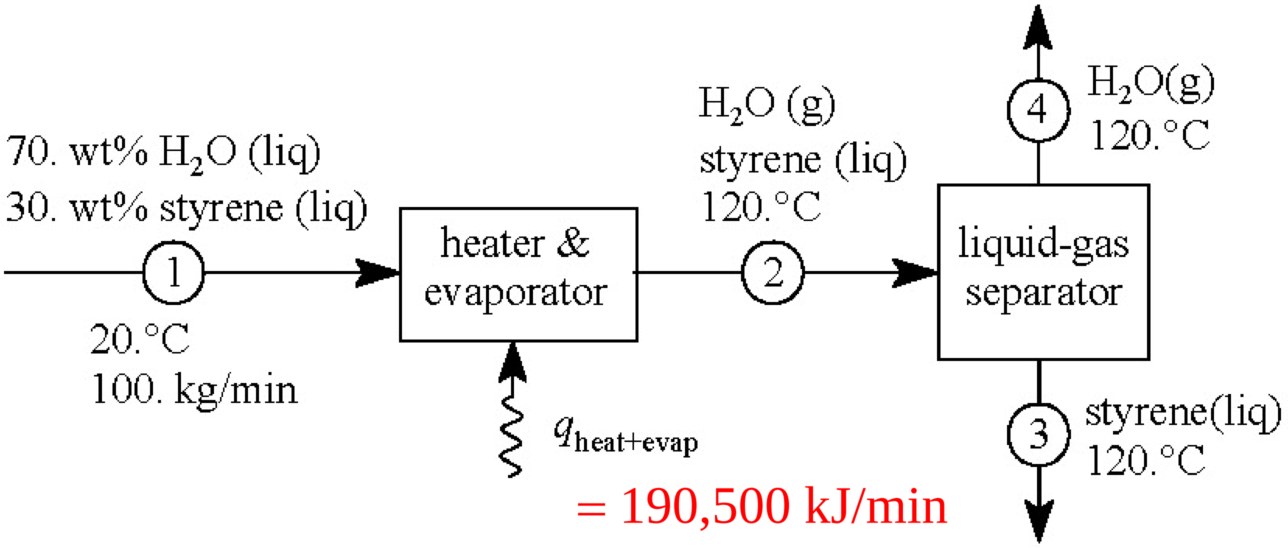
$$\Delta q = F_{\text{water}} C_{P,\text{water}} (T_5 - T_6) = \frac{70 \text{ kg water}}{\text{min}} \left(\frac{4.17 \text{ kJ}}{(\text{kg water})(°\text{C})} \right) (40 - 20) = 5{,}840 \text{ kJ/min}$$

$$q_{\text{warmer}} = 5{,}840 + 5{,}280 = 11{,}120 \text{ kJ / min}$$

Much easier!

Summary of Lectures 11 and 12

Example 2: Exercise 3.63(A). Calculate $q_{\text{heat+evap}}$.



70. wt% H_2O (liq)
30. wt% styrene (liq)
20.°C
100. kg/min

heater & evaporator

$q_{\text{heat+evap}}$ = 190,500 kJ/min

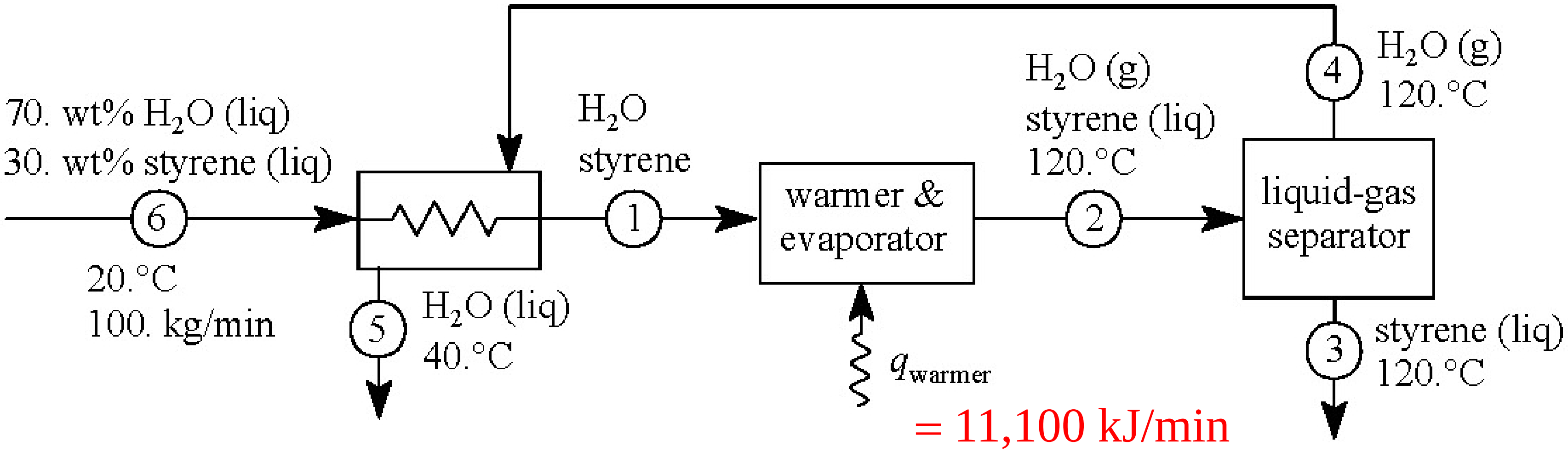
H_2O (g)
styrene (liq)
120.°C

liquid-gas separator

H_2O(g)
120.°C

styrene(liq)
120.°C

Example 3: Exercise 3.63(B). Calculate q_{warmer}.



70. wt% H_2O (liq)
30. wt% styrene (liq)
20.°C
100. kg/min

H_2O (liq)
40.°C

H_2O
styrene

warmer & evaporator

q_{warmer} = 11,100 kJ/min

H_2O (g)
styrene (liq)
120.°C

liquid-gas separator

H_2O (g)
120.°C

styrene (liq)
120.°C

Energy consumption for Example 3 is 17× less than Example 2!

Summary of Energy Balances to date

- Cannot calculate the energy of a mass stream;

  cannot state $q_1 = 100$ kJ/min

  Must calculate the *change* in energy of a mass stream;

  must calculate $\Delta q_{1\rightarrow 2} = q_2 - q_1$

- Cannot measure Δq; must measure ΔT *or* Δ*phase*

  and use thermodynamics to convert to energy.

- Simple mass units can be complex energy units.

  Create an equivalent unit of elementary energy units.

  An elementary energy unit has

  one mass stream in, one mass stream out

  one energy change; ΔT *or* Δ*phase*

  all mass passing through the unit is heated or changes phase.

- Calculating a mass flow may be impossible with mass balances.

  Calculating a mass flow may require an *energy* balance.