EngrD 2190 – Lecture 14

Concept: Process Analysis by Mathematical Modeling

Context: Energy Balance Examples

Exercise 3.65: Equivalent units of elementary energy changes.

Exercise 3.78: Using energy balances to calculate mass flow rates.

Defining Question: When is an energy balance necessary to calculate a mass flow rate?

Read Chapter 3, pp. 145-151. Process Economics

Lecture 15 will follow the textbook.

EngrD 2190 – Heat Exchanger Module

Sign-up for a 50-minute lab session. Two homework teams per session.

Lecture Monday 9/29 is cancelled to accommodate the lab session.

Although your lab session is unlikely to be Monday 9/29 9:05-9:55 a.m.

The cancelled lecture allows for the lab session
without increasing the net contact hours.

My office hours Monday 9/29 are cancelled.

Professional Development – LinkedIn Pages

If you have not already done so, enroll in the Cornell Career Services “Career Development Toolkit” available via https://scl.cornell.edu/news-events/news/new-career-development-toolkit

Complete the LinkedIn module located within the “Networking” tab within the Toolkit.

Based on the guidance provided within the LinkedIn education module, create a LinkedIn account/page for yourself that is comprehensive, engaging, and effective.

Using the guidance provided in the education module, along with your own experience in creating your account, create a LinkedIn grading rubric and use it to assess the LinkedIn page of two of your class peers. Refer to the grading rubrics for homework as an example.

By Wednesday Oct 1, 5pm, email:

Last name beginning A-J Email to Professor Woltornist (AW499)

Last name beginning K-Z Email to Professor Bauer (btb42)

- A link to your LinkedIn page
- Completed grading rubrics for the LinkedIn pages of two peers.

Homework 5

Homework 5 due Friday 10/3

Formal Energy Balances: exercises **3.59, 3.72, and 3.77.**

See Thermodynamic Data on p. 200.

Thermodynamics of Chemical Reactions: exercise **3.80(A)&(B) only** (Lecture 13)

Append a list of 'take-aways' to each exercise.

Requirements for Formal Energy Balances

- Define nomenclature.
- Show system borders and state assumptions.
- State source of equations. ***Every** equation must have an explicit source.*
  examples: "apply conservation of energy" or "thermodynamic relation" or "reactor specification."
- Describe derivation. "Substitute equations (1) and (2) into equation (3)"
- Box your answers. Numbers must have proper significant figures and include units.

See Examples of Formal Energy Balances: See posted solutions for exercises 3.63, 3.64, 3.65, 3.66, 3.67, 3.68, and 3.78.

Flow sheets for mass & energy balance exercises are posted on-line.
Select the "Textbook" item at the EngrD 2190 homepage.

Textbook Errata

Page 200, $C_{P,air} = \cancel{42}\ \text{J/(°C} \cdot \text{mol)}$

$29\ \text{J/(°C} \cdot \text{mol)}$

Update your textbook with the corrections posted at the EngrD 2190 homepage:

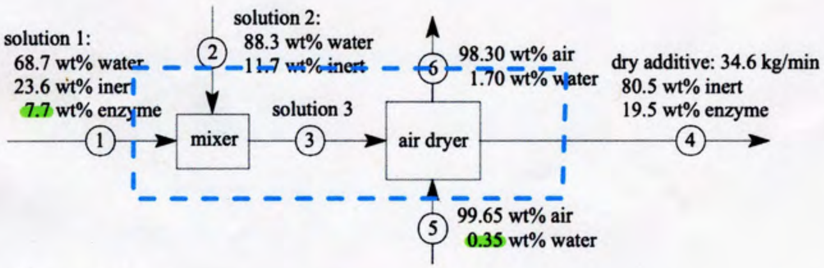
EngrD 2190 homepage ⇒ Textbook ⇒ Textbook Errata, Graphs, and Figures ⇒ Errata

Homework 3 Excellence – Exercise 3.12 - Team 18

Amber Belk, Dolly Hritz, Parth Vaidyanath (Team Coordinator) Team 18

Homework 3

3.12 A process to produce a detergent additive.



2 significant figures

Nomenclature:

- $T \equiv$ total
- $W \equiv$ water
- $E \equiv$ enzyme
- $I \equiv$ inert
- $A \equiv$ air
- $F_{i,j}$ where F is the flow rate, i is th...

Assumptions:
- Steady-state
- no reaction
- Conservation of mass

$\therefore$ rate of mass in = rate of mass out

A) System: — Want: $F_{T,1}$ and $F_{T,2}$ Parameters: W, I, E

Rate In = Rate Out

(5 streams) × (4 parameters/stream) = 20 parameters

Balances:

$F_{W,1} + F_{W,2} + F_{W,5} = F_{W,4} + F_{W,6}$ (1) Conservation of Mass

$F_{I,1} + F_{I,2} + F_{I,5} = F_{I,4} + F_{I,6}$ (2) Conservation of Mass

$F_{E,1} + F_{E,2} + F_{E,5} = F_{E,4} + F_{E,6}$ (3) Conservation of Mass

$F_{I,2} = 0.117 F_{T,2}$ (4)

Amber Belk, Dolly Hritz, Parth Vaidyanath (Team Coordinator) Team 18

3.12 A (Cont.)

Specific:

$F_{W,6} = 0$ (5)

$F_{I,5} = F_{I,6} = 0$ (6), (7)

$F_{E,2} = F_{E,5} = F_{E,6} = 0$ (8), (9), (10)

$F_{T,4} = 34.6$ kg/min (11)

$F_{I,4} = 0.805 F_{T,4}$ (12)

$F_{E,4} = 0.195 F_{T,4}$ (13)

$F_{E,1} = 0.077 F_{T,1}$ (14)

$F_{I,1} = 0.236 F_{T,1}$ (15)

$F_{W,1} = 0.687 F_{T,1}$ (16)

$F_{\ldots} = 0.883 F_{\ldots}$ (17)

$F_{W,4} = 0.0170 F_{T,4}$ (18)

$F_{W,5} = 0.0035 F_{T,5}$ (19)

$F_{T,1} + F_{T,2} + F_{T,5} = F_{T,4} + F_{T,6}$ (20)

Honorable Mention to Teams 1, 3, 4, 13, 22, and 24

...13) into (3), (Enzyme Balance)

...$5 F_{T,4}$ (21)

$0.077 F_{T,1} = 0.195 (34.6 \text{ kg/min})$

$0.077 F_{T,1} = 6.747$ kg/min

$\boxed{F_{T,1} = 88 \text{ kg/min}}$ (22)

Substitute (4), (6), (7), (12), (15) into (2) (Inert Balance)

$0.236 F_{T,1} + 0.117 F_{T,2} + 0 = 0 + 0.805 F_{T,4}$ (23)

Substitute (22), (11) into (23)

$0.236 (88 \text{ kg/min}) + 0.117 F_{T,2} = 0.805 (34.6 \text{ kg/min})$

$20.77 \text{ kg/min} + 0.117 F_{T,2} = 27.85 \text{ kg/min}$

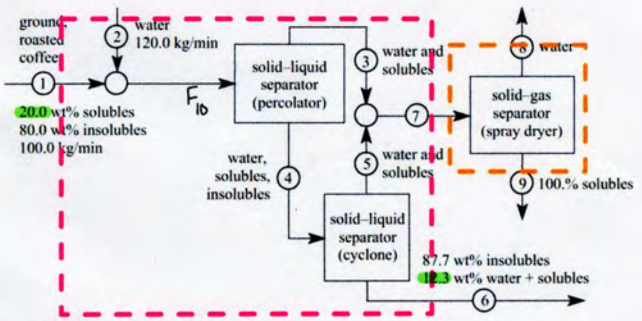
$\boxed{F_{T,2} = 61 \text{ kg/min}}$ (24)

Homework 3 Excellence – Exercise 3.28 - Team 18

Amber Belk, Dolly Hritz, Parth Vaidyanath (Team Coordinator) Team 18

Homework 3

3.28 Instant coffee (the soluble portion of ground roasted coffee) is produced by the process in the simplified flowsheet below. Calculate the rate of production of dried soluble coffee (stream 9). Note that the ratio of water to soluble components is the same in streams 3, 4, 5, 6, and 7.



System 1 ✓

System 2 ✓

Nomenclature:

3 significant figures ✓

- T ≡ total
- W ≡ water
- S ≡ soluble ✓
- I ≡ insolubles
- $F_{i,j}$ where F is the flow rate, i is the component, and j ...

Assumptions:
- Steady-state ✓
- no reaction
- Conservation of mass

∴ rate of mass in = rate of mass out   Rate In = Rate out ✓

(4 streams)×(3 parameters/stream) = 12 parameters

Balances:

$F_{T,1} + F_{T,2} = F_{T,6} + F_{T,7}$ ✓ (1) Conservation of Mass

$F_{I,1} + F_{I,2} = F_{I,6} + F_{I,7}$ ✓ (2) Conservation of Mass

$F_{T,1} = 100.0$ kg/min (3)

$F_{T,2} = 120.0$ kg/min (4)

$F_{w,1} = 0$ (5)

$F_{S,1} = 0.20\, F_{T,1}$ ✓ (6)

$F_{I,1} = 0.80\, F_{T,1}$ (7)

$F_{I,2} = F_{S,2} = 0$ ✓ (8), (9)

$F_{I,6} = 0.877\, F_{T,6}$ ✓ (10)

$F_{I,7} = 0$ (11)

$F_{S,6} + F_{w,6} = 0.123\, F_{T,6}$ (12)

Amber Belk, Dolly Hritz, Parth Vaidyanath (Team Coordinator) Team 18

3.28 (cont)

Substitute (7)(8)(10)(11) into (2) (Insolubles Balance)

$0.80\, F_{T,1} + 0 = 0.877\, F_{T,6} + 0$ (13)

Substitute (3) into (13)

$0.80(100.0 \text{ kg/min}) = 0.877\, F_{T,6}$

$80.0 \text{ kg/min} = 0.877\, F_{T,6}$

$\boxed{F_{T,6} = 91.2 \text{ kg/min}}$ (14)

Substitute (14), (3), (4) into (1) (total balance)

$100.0 \text{ kg/min} + 120.0 \text{ kg/min} = 91.2 \text{ kg/min} + F_{T,7}$

$\boxed{F_{T,7} = 129 \text{ kg/min}}$ (15)

Honorable Mention to Teams 5, 6, 9, and 19.

Rate out ✓

...re and equations, let $F_{T,10}$ represent $F_{T,1} + F_{T,2}$ after the combiner.

$F_{w,9} = F_{S,8} = 0$ (22), (23)

$F_{w,7} = F_{w,8} + F_{w,9}$ (20)

$F_{w,7} = \frac{6}{7} F_{T,7}$ (24)

w:s ratio $\frac{F_{w,2}}{F_{S,1}} = \frac{120.\text{ kg/min}}{20.0\text{ kg/min}} = 6$; $F_{w,7} = 6F_{S,7}$ (21)

Substitute (24), (22) into (20)

$\frac{6}{7} F_{T,7} = 0 + F_{w,8}$ (25)

Substitute (15) into (25)

$\frac{6}{7}(129 \text{ kg/min}) = F_{w,8}$

$\boxed{F_{w,8} = F_{T,8} = 111 \text{ kg/min}}$ (26)

Substitute (26), (15) into (19)

$129 \text{ kg/min} = 111 \text{ kg/min} + F_{T,9} \longrightarrow \boxed{F_{T,9} = 18.0 \text{ kg/min}}$

Try to avoid intermediate rounding → 18.4

Overall Takeaways:

- Using information from the problem statement that is not directly included in the prices diagram is important in solving for flow rates; for instance, the ratio between water and soluble compositions.
- Make sure to always define the system, state assumptions, choose parameters, and state nomenclature.
- Some systems are too complicated and need at least two models/system boundaries to solve.

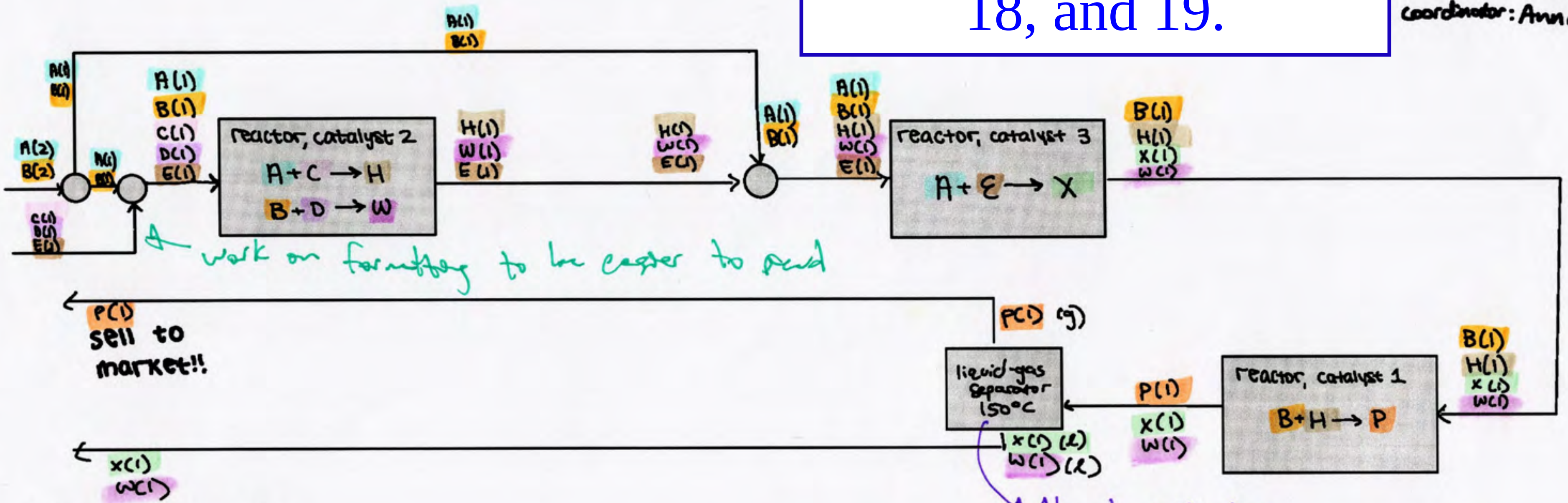
Homework 3 Excellence – Exercise 3.115 - Team 11

Honorable Mention to Teams 8, 10, 13, 15, 18, and 19.

3.115

Beautiful colors! Great job!

TEAM 11
Isabelle Bennie
Steffanie Jones
coordinator: Anna Voronova



Takeaways:

you don't do this?

1. Using separators earlier in the process can help reduce reactor sizes later, resulting in a cheaper overall process in many cases
2. In some cases, unwanted byproducts can only be minimized, not avoided completely. ✓
3. Always check & prioritize design goals to keep the overarching aim in mind while designing

A little generic

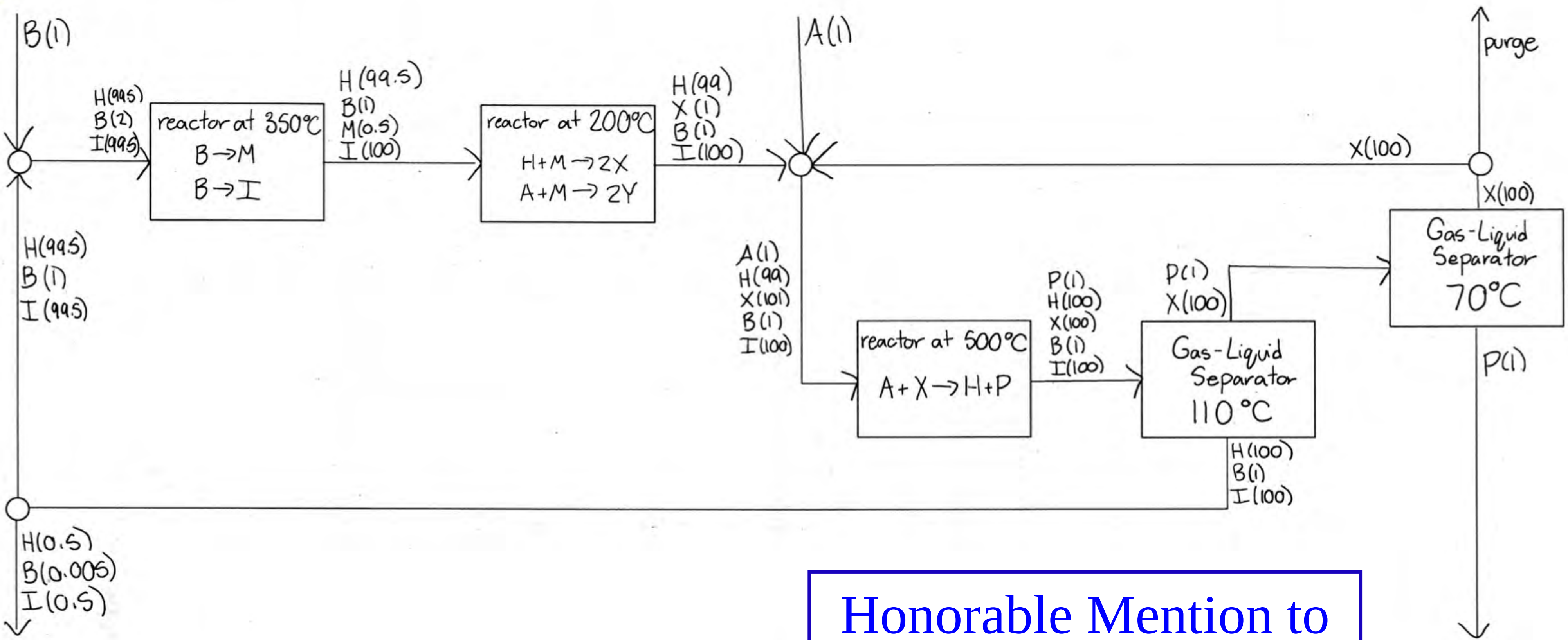
Homework 3 Excellence – Exercise 3.125 - Team 11

Simeon Hur, Grace Reid, Sofia Ziegler
coordinator

Team 21

3.125 $A + B \rightarrow \frac{1}{2}H + \frac{1}{2}I + P$

* flow rates in $\frac{mol}{min}$



Take aways:

Calculate the overall reaction before starting.

Keep design priorities in mind.

Honorable Mention to Teams 8, 10, 13, 15, 18, and 19.

Prelim 1

- Prelim 1: Tuesday 10/7, 7:30-9:30 p.m. 128 and 245 Olin Hall

  Covers Chapter 2 and mass balances (formal and informal).

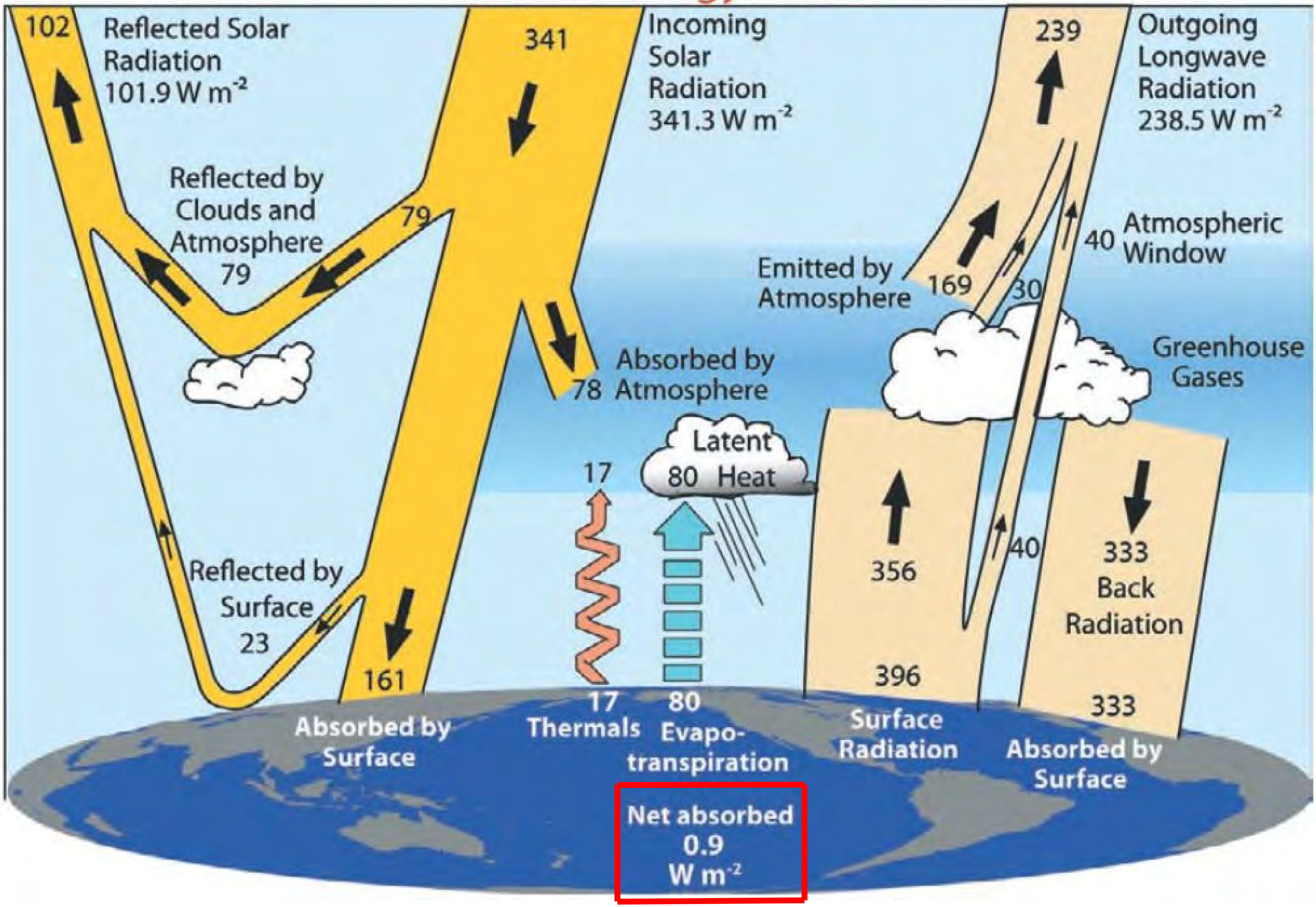
  Covers through Lecture 10, Homework 4, Calculation Session 5.

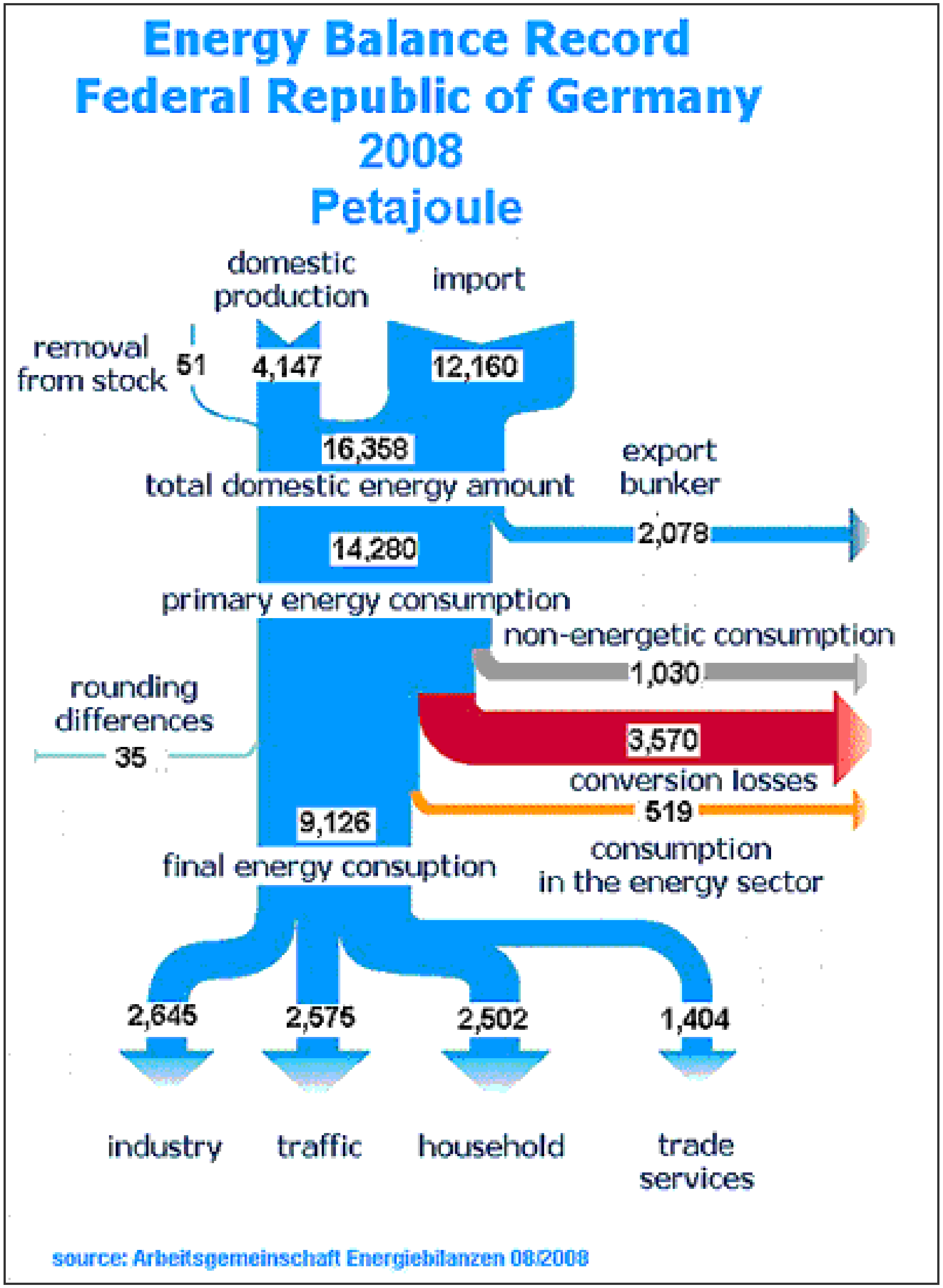
  Open paper notes, open exercise solutions.

  Bring a calculator. Graphing calculators are allowed.

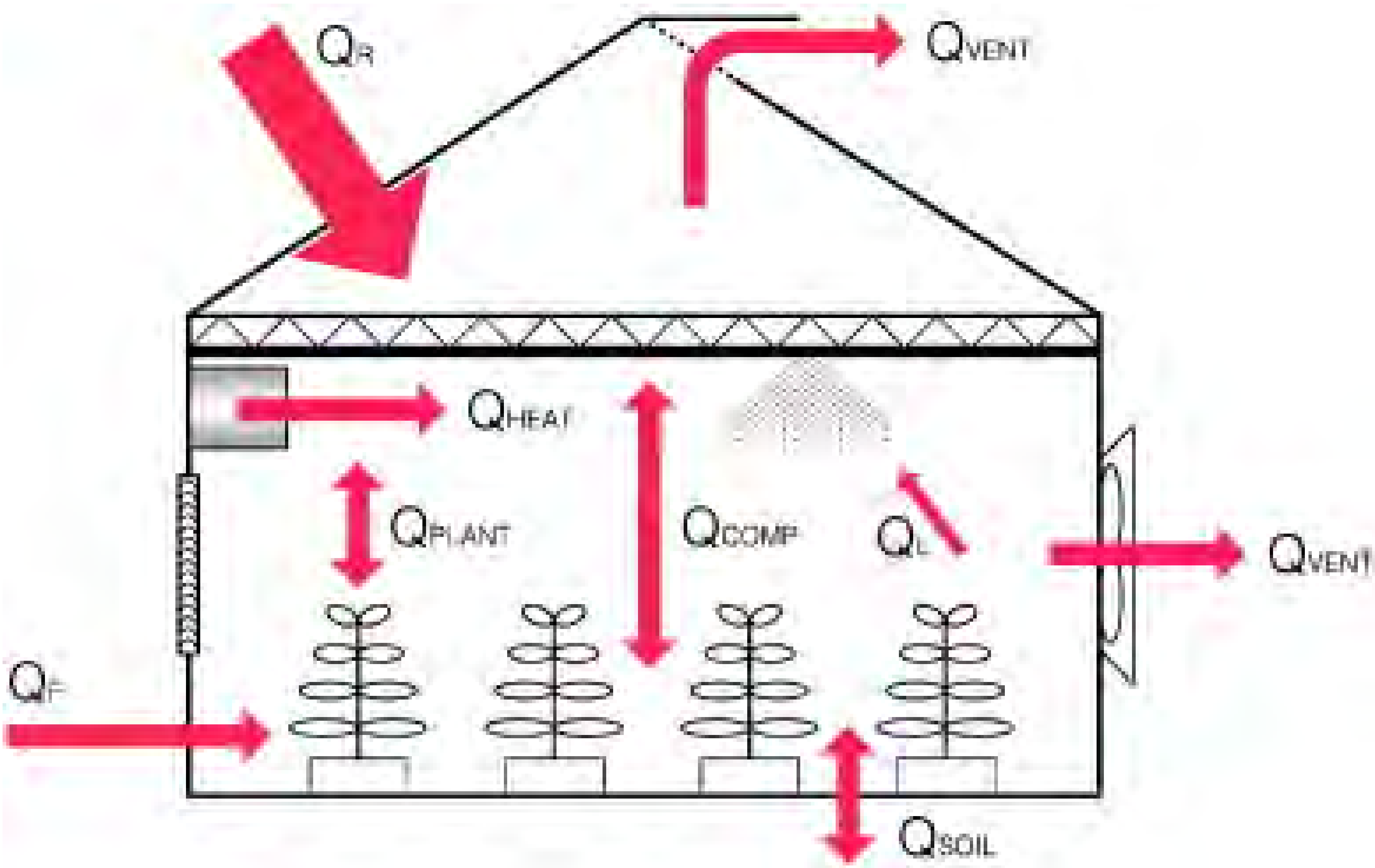
  No laptops and no iPads.

Global Energy Flows W m^{-2}




Energy Balance Record
Federal Republic of Germany
2008
Petajoule
domestic production
import
removal from stock
51
4,147
12,160
16,358
total domestic energy amount
export bunker
2,078
14,280
primary energy consumption
non-energetic consumption
1,030
rounding differences
35
3,570
conversion losses
519
9,126
final energy consuption
consumption in the energy sector
2,645
2,575
2,502
1,404
industry
traffic
household
trade services
source: Arbeitsgemeinschaft Energiebilanzen 08/2008

Energy Balance on a Greenhouse



Energy Balance – The London Underground

The London Tube ‘Feels Like Hell.’ Efforts to Cool It Just Make It Hotter.

Wall Street Journal, *July 16, 2025*.

“Summer temperatures on the Tube now regularly exceed 30 degrees Celsius, or 86 degrees Fahrenheit, the legal limit for transporting cattle, pigs and sheep in the U.K.”

“Industrial fans were installed at stations. Surveys indicated the fast-flowing air made people feel more comfortable.”

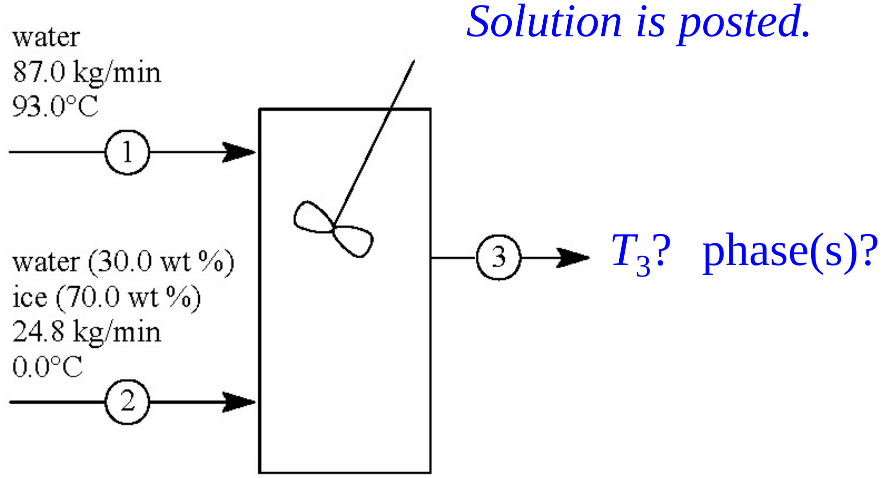
But the Tunnel for London (TfL) operators discovered the fans actually increased the air temperature.   Surprising?


ENERGY
BALANCE
PS4

Exercise 3.65

3.65 Calculate the temperature of the stream leaving the mixer below.

Solution is posted.

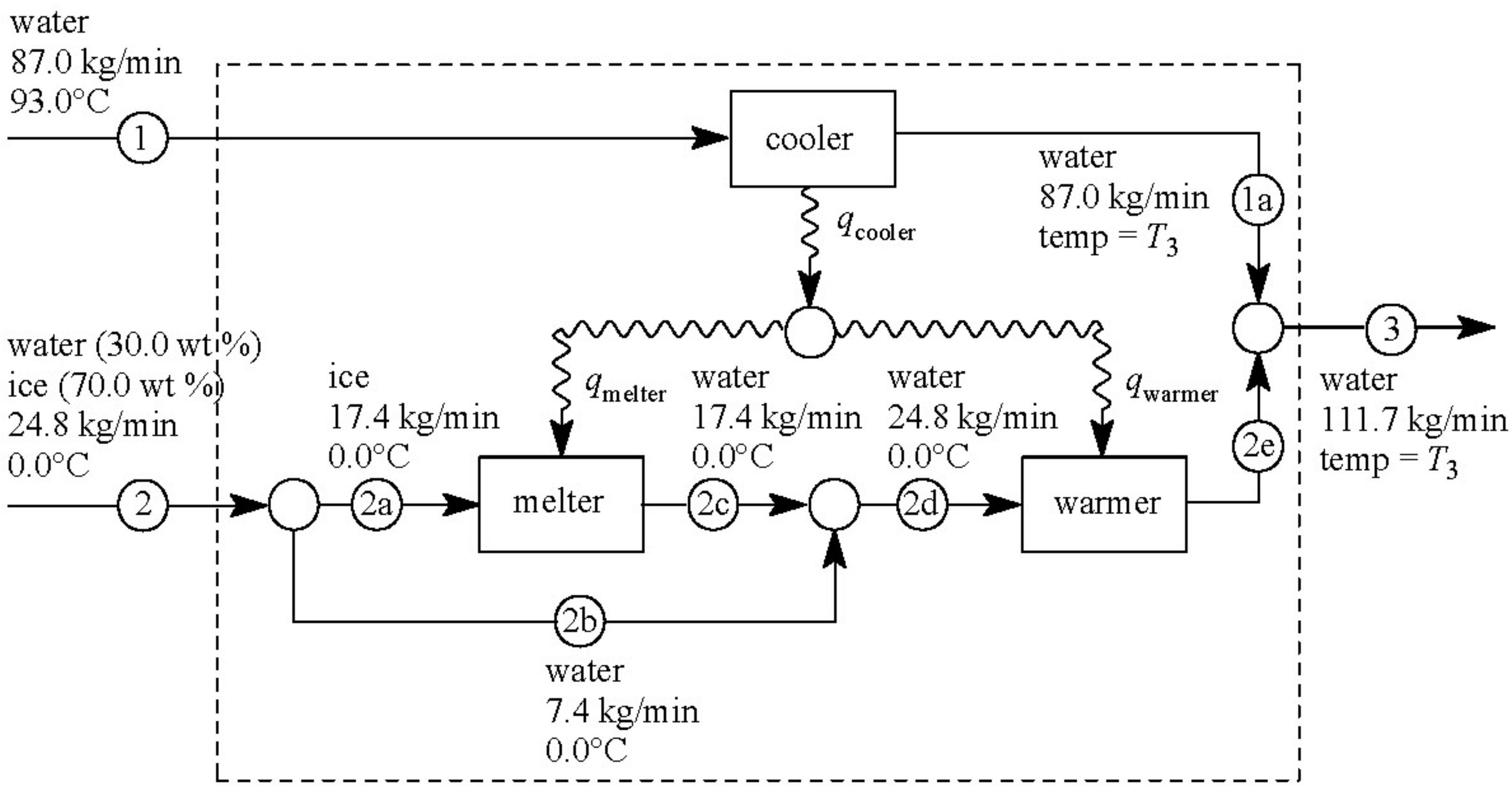


Units with simple mass flows may have complex energy flows.

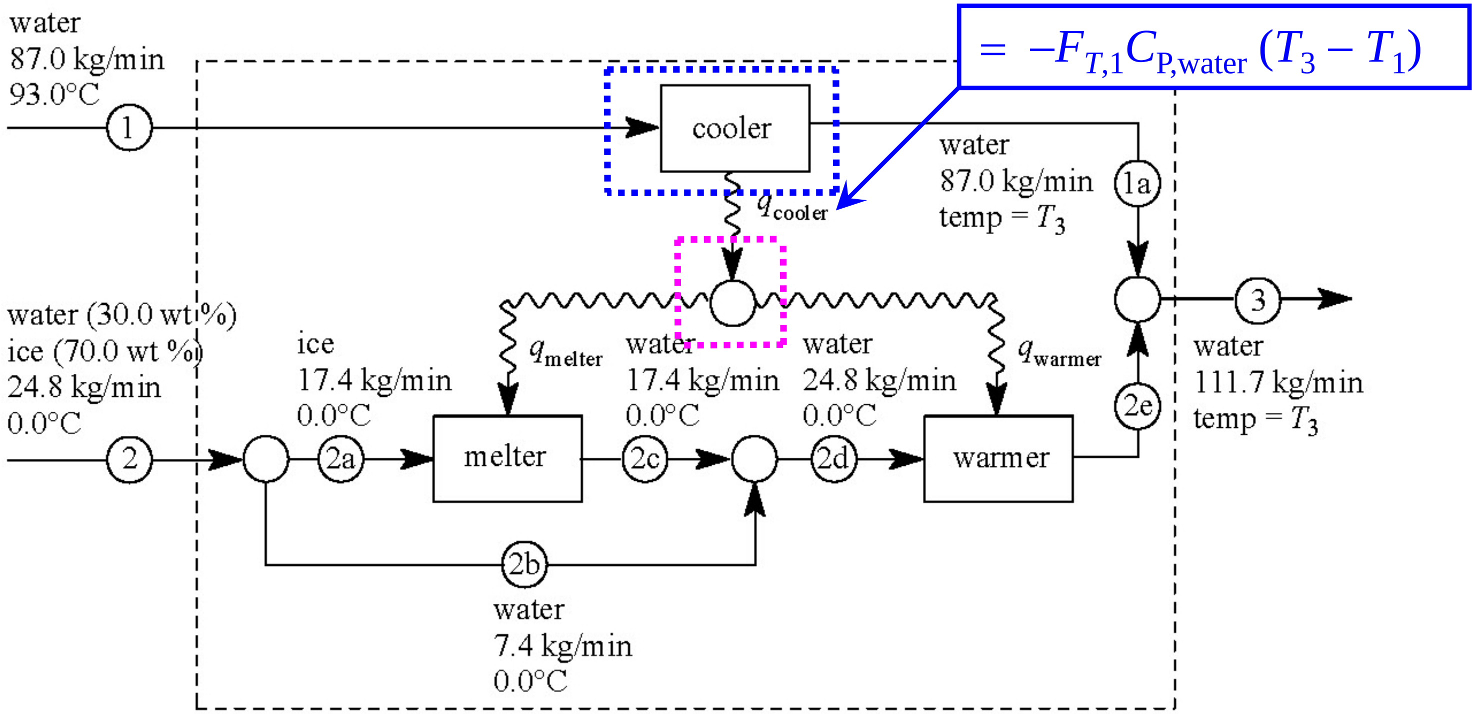
Guess: Stream 3 is liquid water.

⇒ 1. The ice in stream 2 melts;
the melted ice and water warm to T_3.

⇒ 2. All the water in stream 1 cools to T_3.



Exercise 3.65: Equivalent Unit for Water+(Water+Ice) Mixer



Energy balance on fictitious energy splitter.

rate of energy in = rate of energy out

$$q_{cooler} = q_{melter} + q_{warmer}$$

Energy balance on fictitious cooler.

rate of energy in = rate of energy out

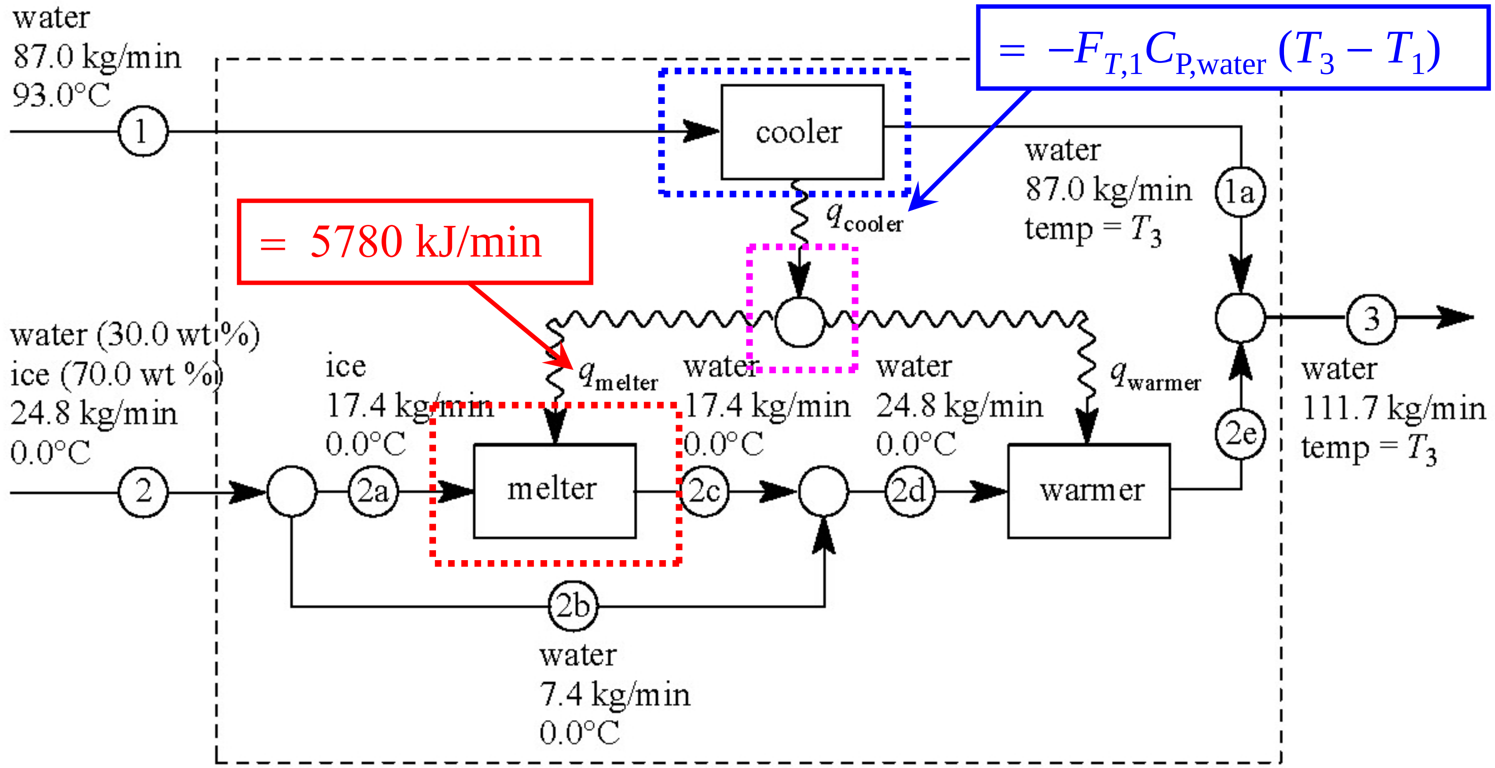
$$q_1 = q_{1a} + q_{cooler}$$

$$q_{cooler} = -(q_{1a} - q_1) = -\Delta q_{1\rightarrow 1a} = -F_{T,1}C_{P,water}(T_3 - T_1)$$

out

in

Exercise 3.65: Equivalent Unit for Water+(Water+Ice) Mixer



Energy balance on fictitious melter.

rate of energy in = rate of energy out

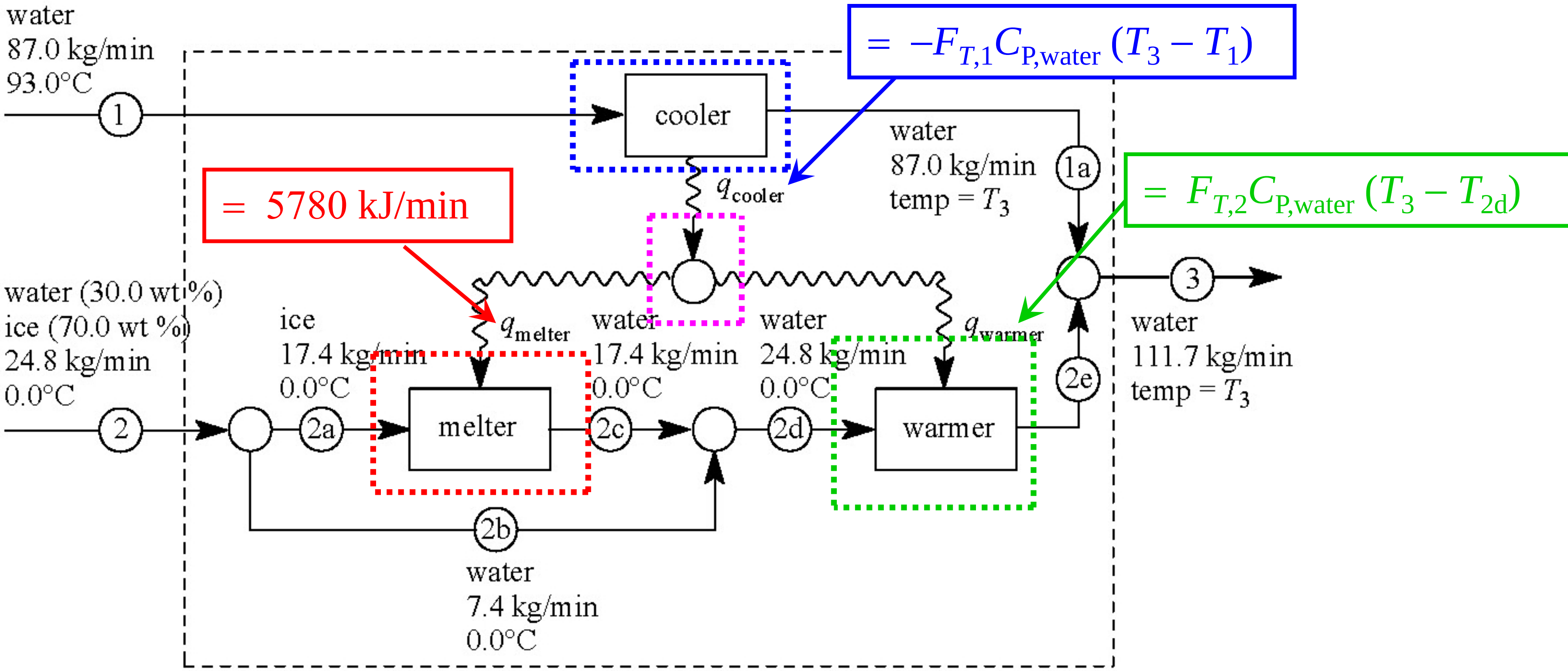
$$q_{2a} + q_{\text{melter}} = q_{2c}$$

$$q_{\text{melter}} = q_{2c} - q_{2a} = \Delta q_{2a\to 2c} = 0.70 F_{T,2}\Delta H_{\text{melt, water}}$$

out (→ q_{2c}) in (→ q_{2a})

$$q_{\text{melter}} = 0.70\left(\frac{24.8\text{ kg water}}{\text{min}}\right)\left(\frac{6.0\times 10^3\text{ J}}{\text{mol water}}\right)\left(\frac{1000\text{ mol water}}{18\text{ kg water}}\right) = 5780\text{ kJ/min}$$

Exercise 3.65: Equivalent Unit for Water+(Water+Ice) Mixer



Energy balance on fictitious warmer.

rate of energy in = rate of energy out

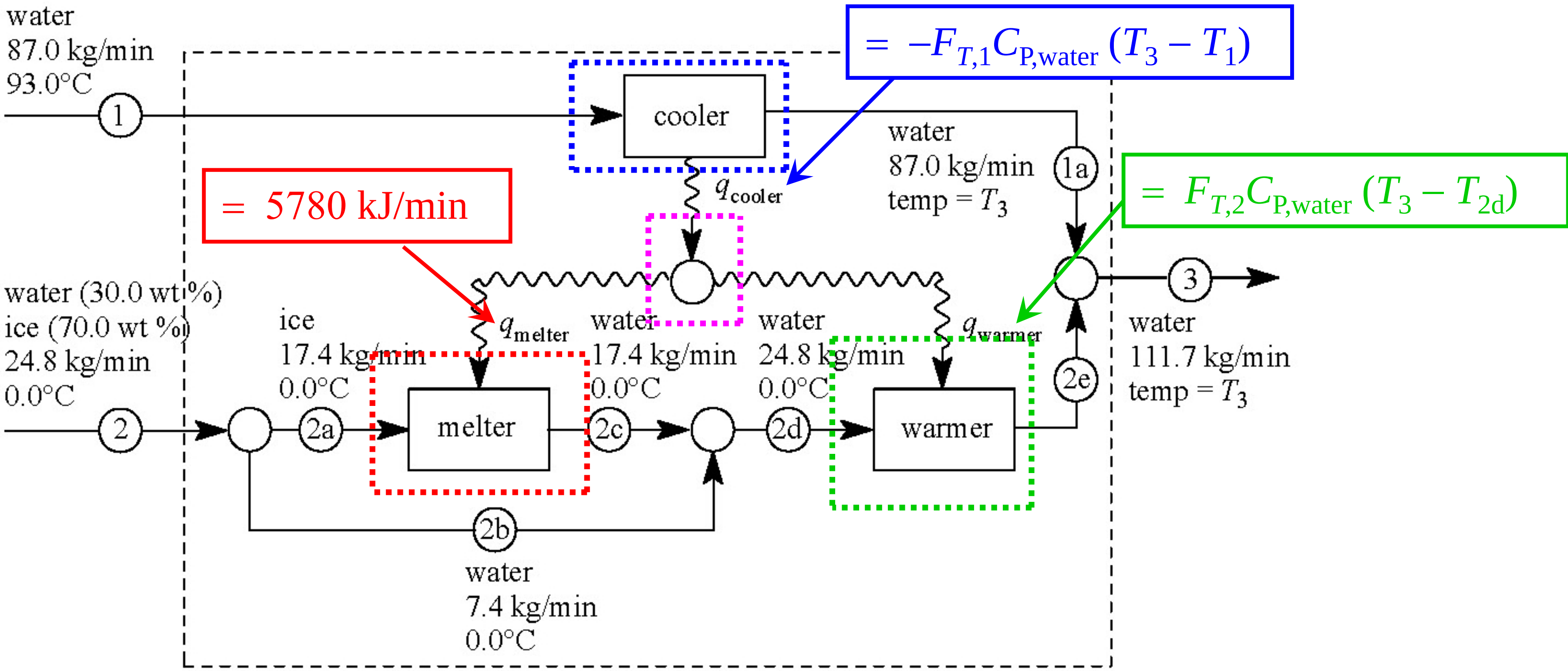
$$q_{2d} + q_{\text{warmer}} = q_{2e}$$

$$q_{\text{warmer}} = q_{2e} - q_{2d} = \Delta q_{2d \to 2e} = F_{T,2}C_{P,\text{water}}(T_3 - T_{2d})$$

out

in

Exercise 3.65: Equivalent Unit for Water+(Water+Ice) Mixer



Recall equation from energy balance on fictitious energy splitter:

$$q_{cooler} = q_{melter} + q_{warmer}$$

Substitute equations for q_{cooler}, q_{melter}, and q_{warmer}:

$$-F_{T,1}C_{P,water}(T_3 - T_1) = 5780 + F_{T,2}C_{P,water}(T_3 - T_{2d})$$

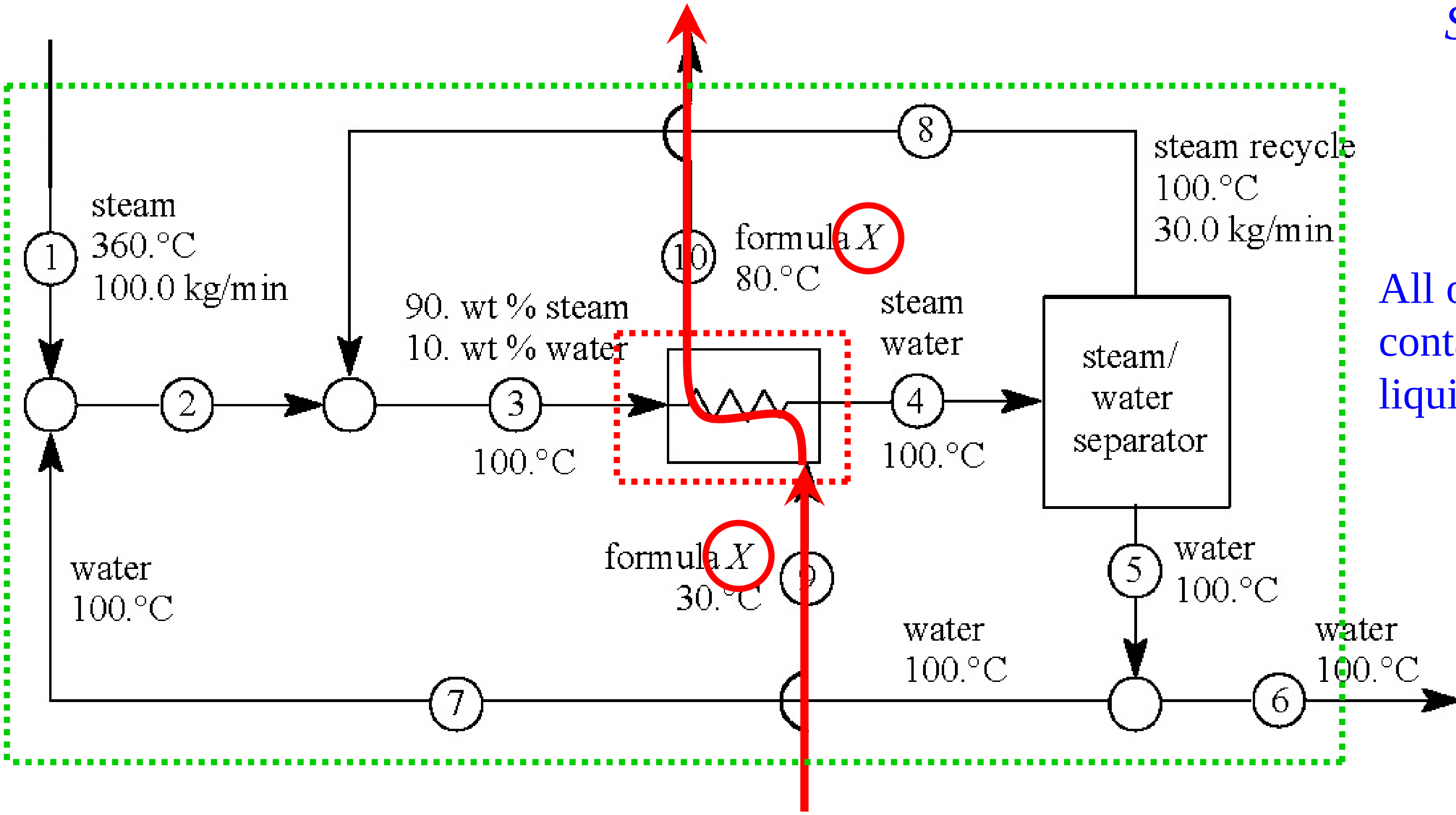
Solve for T_3:

$$T_3 = \frac{\overline{C}_{P,water}(F_{T,2}T_{2d} + F_{T,1}T_1) - 5780}{\overline{C}_{P,water}(F_{T,2} + F_{T,1})}$$

Substitute numerical values: $T_3 = 60.0°C$ Consistent with guess that stream 3 is liquid water.

Exercise 3.78

Solution is posted.



All other streams contain H_2O: liquid water or steam.

(A) Flow rate of water out in stream 6? mass balance on H_2O. 100.0 kg/min

(B) Flow rate of formula *X* in stream 10? mass balance on formula *X* ? No.

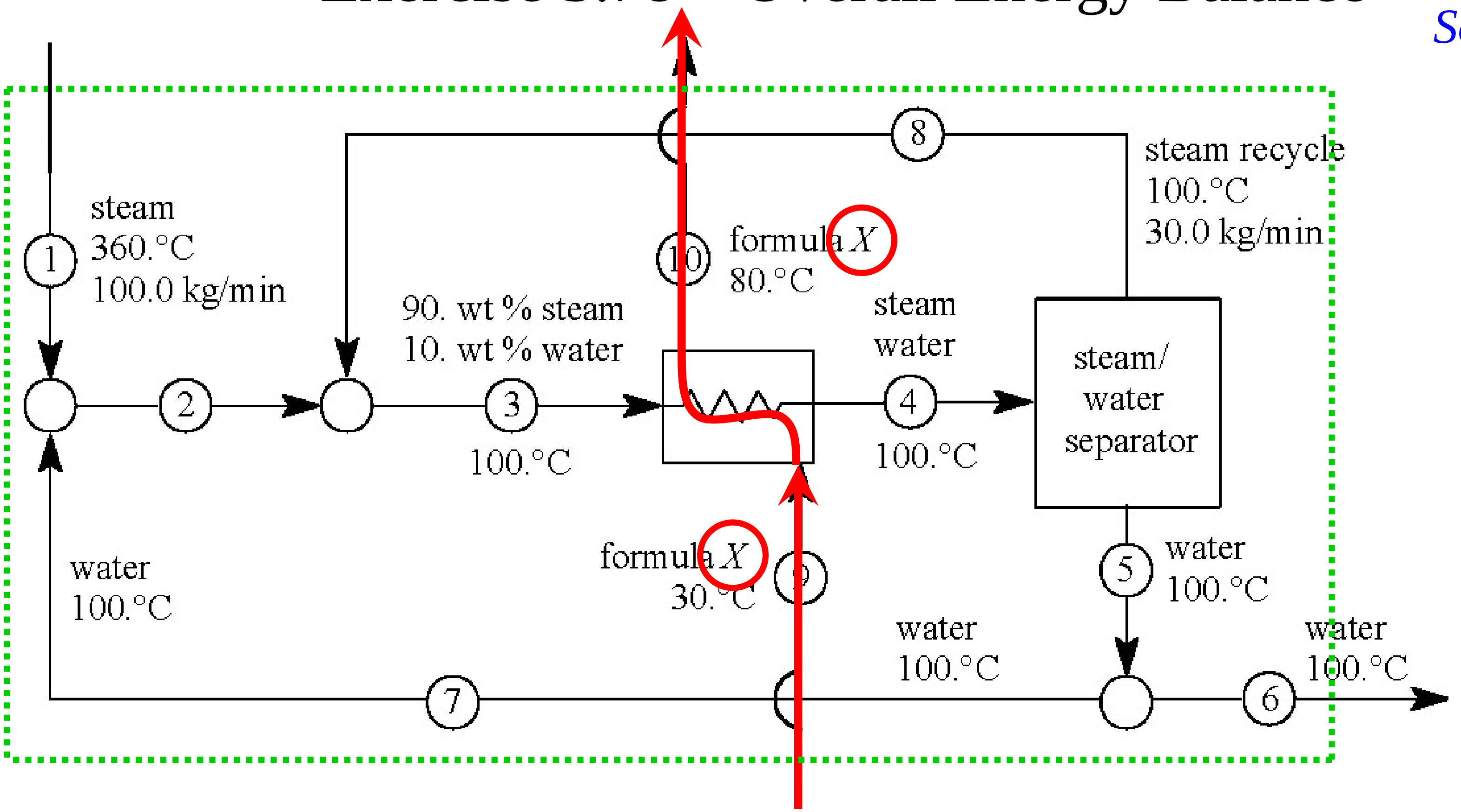
mass balance on formula *X* ? No.

overall energy balance? Yes!

Energy from condensing 100 kg/min steam at 360°C to water at 100°C warms formula *X* from 30°C to 80°C.

Exercise 3.78 – Overall Energy Balance

Solution is posted.



Overall energy balance. rate of energy in = rate of energy out

$$q_1 + q_9 = q_6 + q_{10}$$

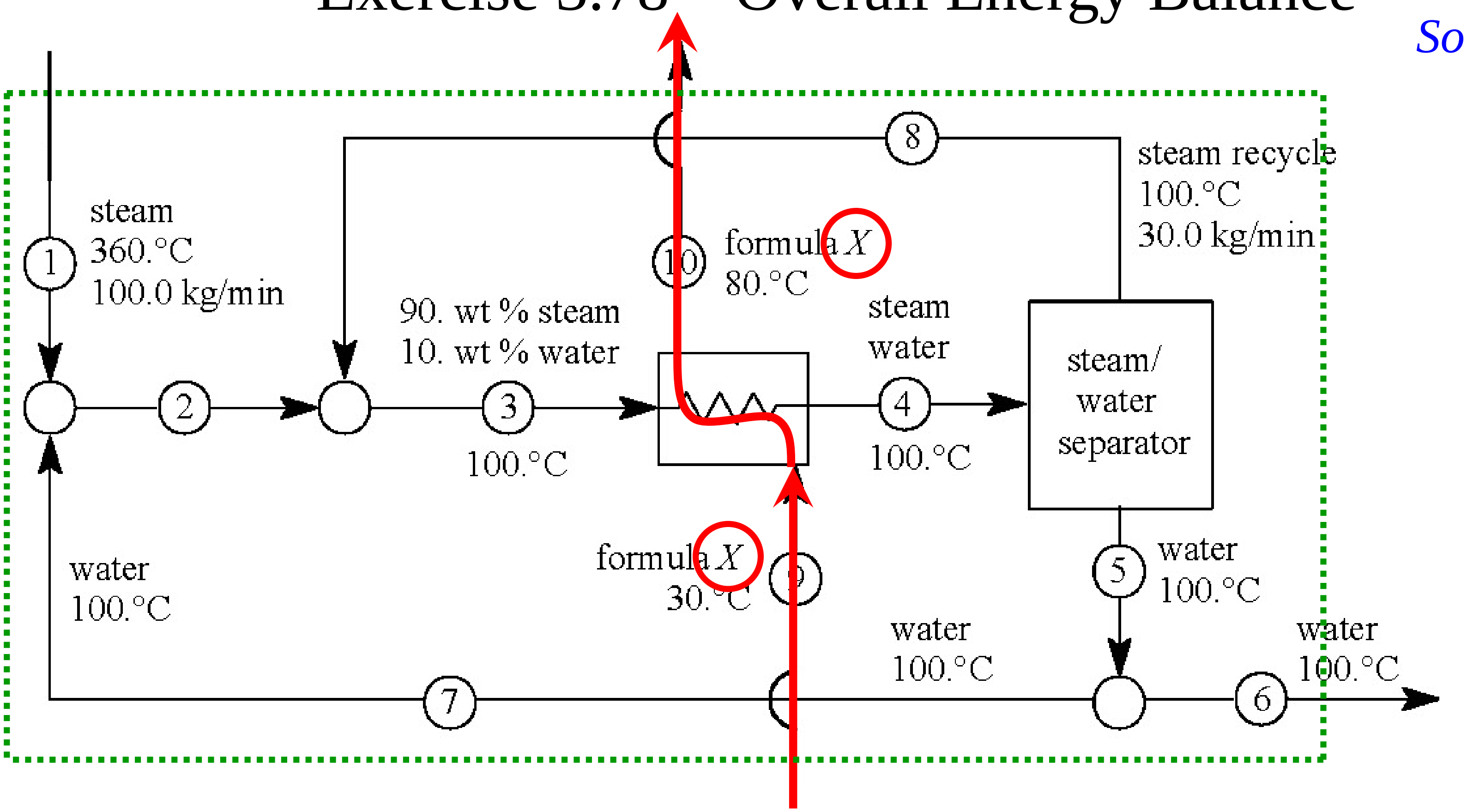
Group q's into "out – in" differences. $-(q_{10} - q_9) = q_6 - q_1$

Substitute thermodynamic relations. $-F_{T,9}C_{P,\,X\,\text{liquid}}\,(T_{10} - T_9) = F_{T,1}(C_{P,\text{steam}}\,(100 - T_1) - \Delta H_{\text{vap}})$

$$F_{T,9} = \frac{F_{T,1}\left(\overline{C}_{P,\text{steam}}(100 - T_1) - \Delta\overline{H}_{\text{vap}}\right)}{-\overline{C}_{P,\,X\,\text{liquid}}(T_{10} - T_9)}$$

Exercise 3.78 – Overall Energy Balance

Solution is posted.



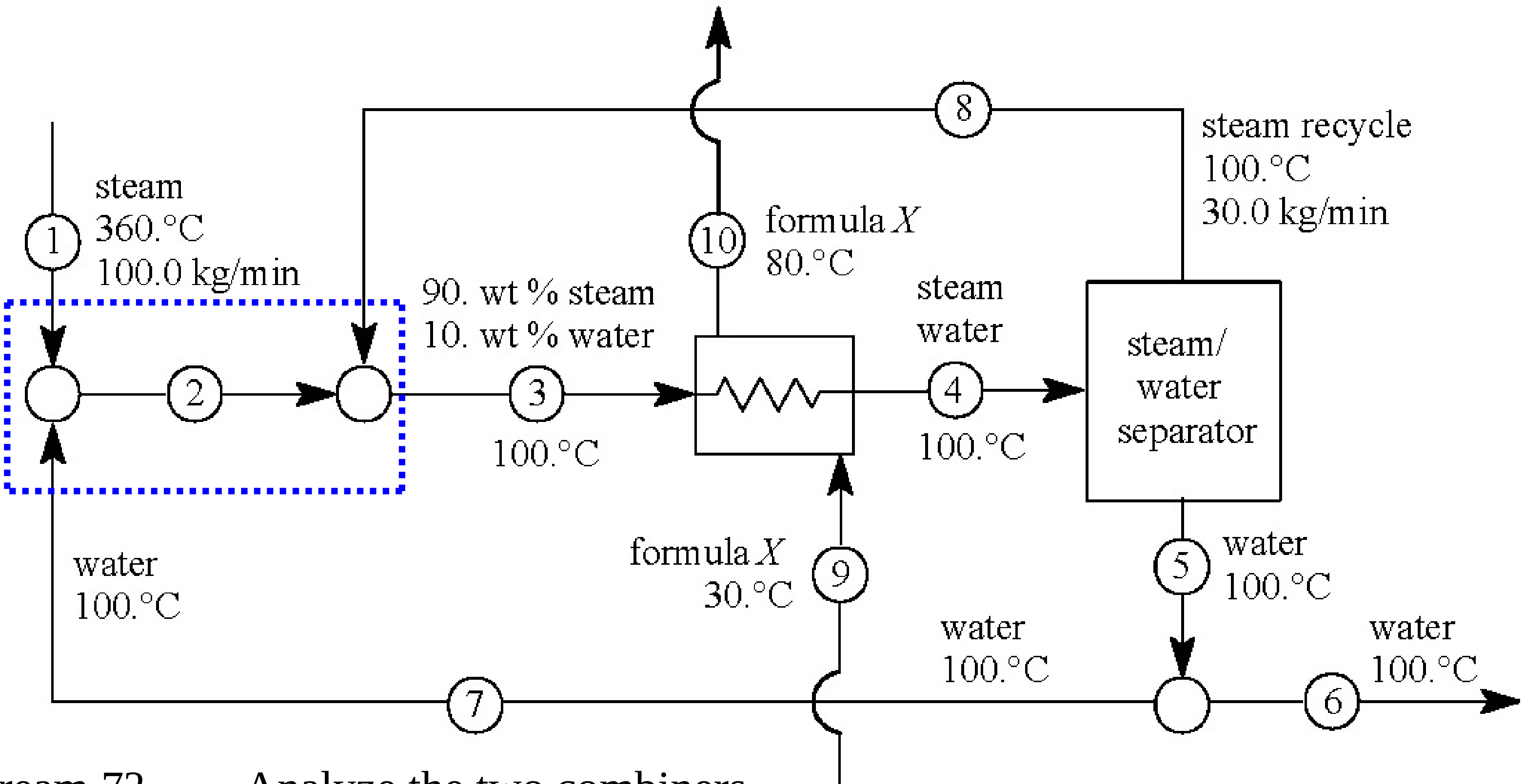
$$F_{T,9} = \frac{F_{T,1}\left(\overline{C}_{\text{P,steam}}(100 - T_1) - \Delta\overline{H}_{\text{vap}}\right)}{-\overline{C}_{\text{P},\,X\text{ liquid}}(T_{10} - T_9)}$$

Substitute numerical values.

$$F_{T,9} = \frac{\left(\frac{100.\text{ kg H}_2\text{O}}{\text{min}}\right)\left(\frac{1000.\text{ mol water}}{18\text{ kg water}}\right)\left[\left(\frac{35\text{ J}}{(\text{mol H}_2\text{O})(^\circ\text{C})}\right)(100 - 360^\circ\text{C}) - \frac{4.1\times10^4\text{ J}}{\text{mol H}_2\text{O}}\right]}{-\left(\frac{451\text{ J}}{(\text{mol }X)(^\circ\text{C})}\right)(80 - 30^\circ\text{C})\left(\frac{1000.\text{ mol }X}{103\text{ kg }X}\right)}$$

$$F_{T,9} = 1270\text{ kg }X/\text{min}$$

Exercise 3.78 – Analysis of Recycle Stream 7



Stream 7? Analyze the two combiners.

Consider options for stream 2.

Stream 2 is water < 100°C? Not possible. Mixing steam at 360°C with water at 100°C must produce $T \geq 100$°C.

Stream 2 is steam > 100°C? Not possible. Stream 2 must provide some water for stream 3.

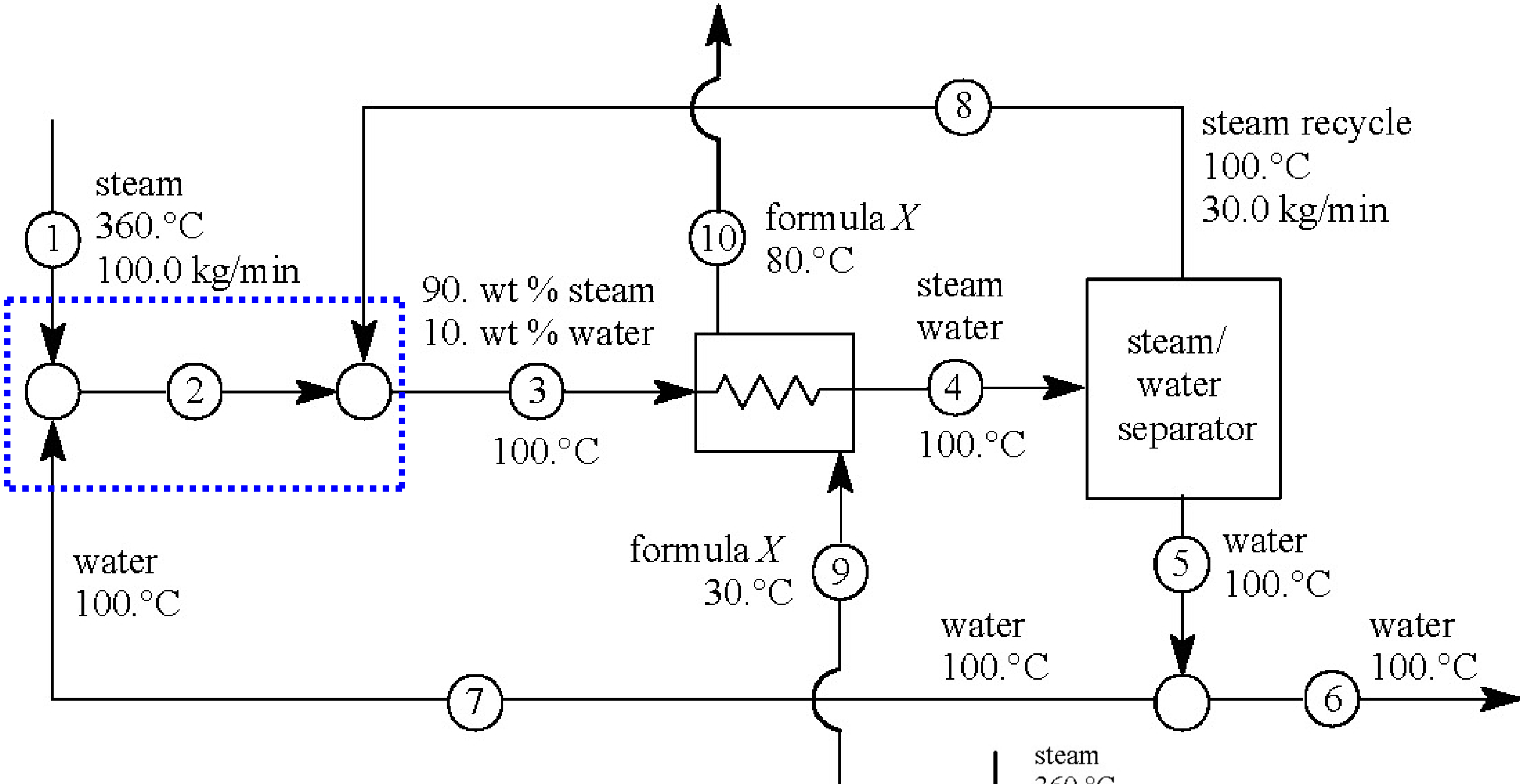
Stream 2 must be mix of water + steam at 100°C.

No steam in stream 8 condenses when mixed with stream 2.

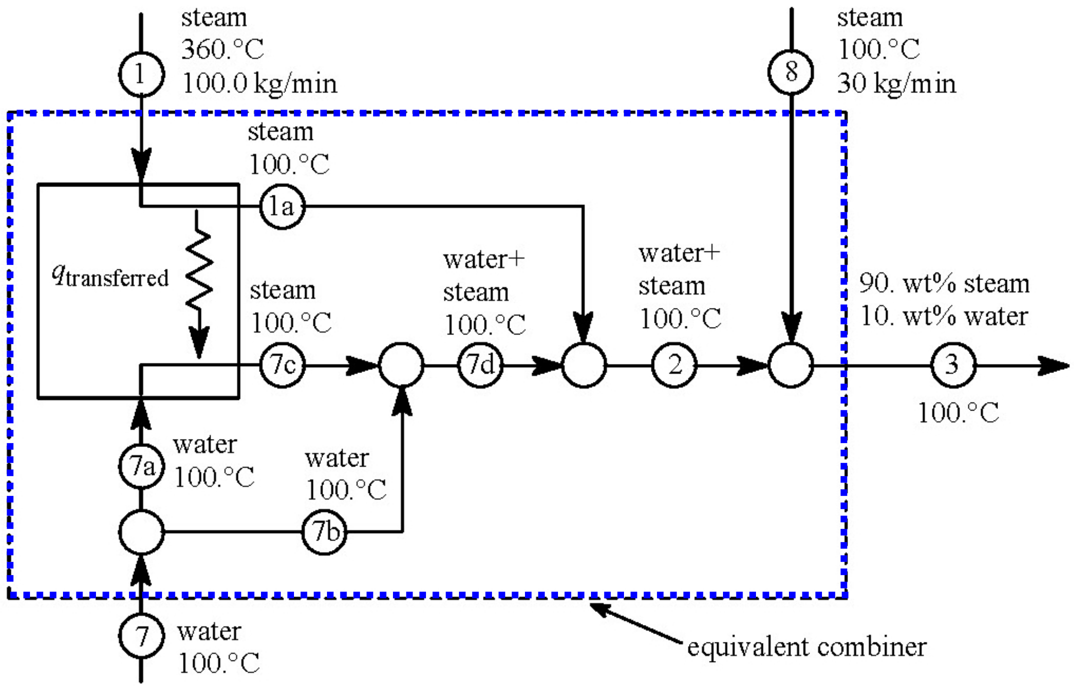
Combiner for streams 1 + 7: *All* steam in stream 1 cools 360°C → 100°C. *None* condenses.

Some water in stream 7 evaporates.

Exercise 3.78 – Analysis of the Two Combiners



No steam in stream 8 condenses when mixed with stream 2.

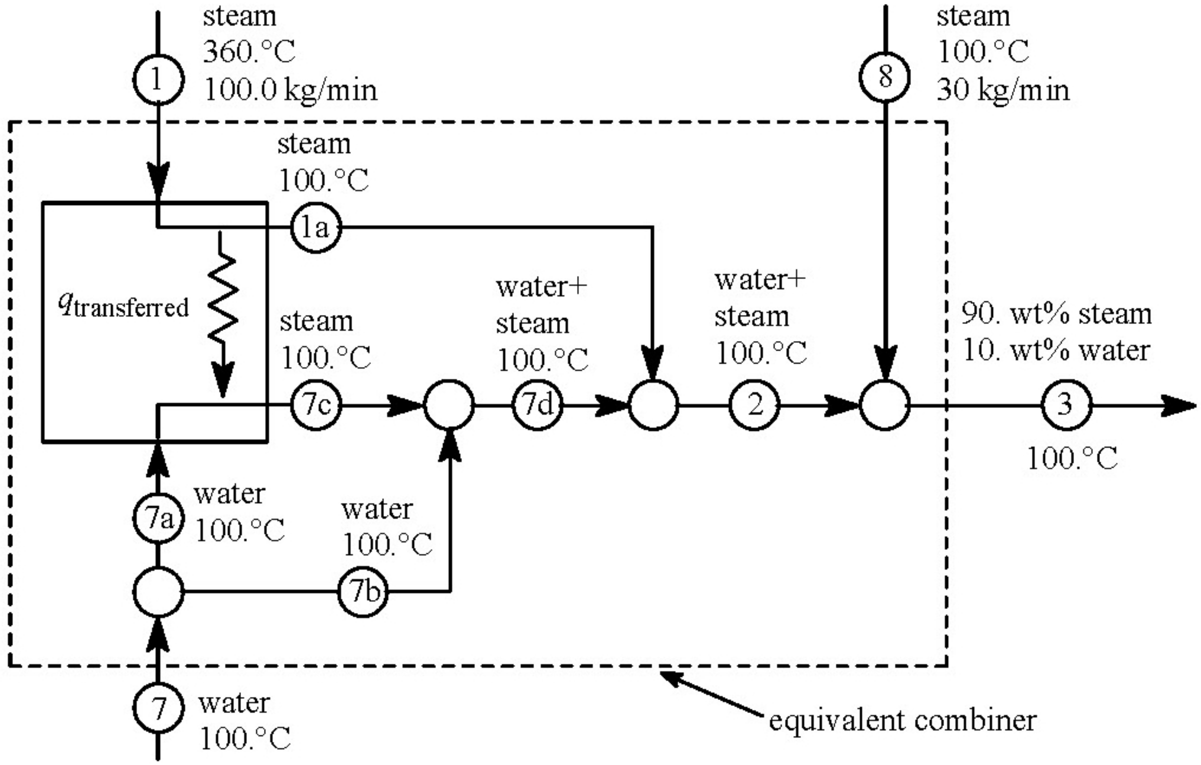


Combiner for streams 1 + 7:

All steam in stream 1 cools 360°C → 100°C. *None* condenses.

Some water in stream 7 evaporates.

Equivalent Unit for the Two Combiners



$$q_{transferred} = \Delta q_{1\rightarrow 1a} = F_{T,1} C_{P,steam} (T_1 - 100) = 50{,}560 \text{ kJ/min}$$

$$q_{transferred} = \Delta q_{7a\rightarrow 7c} = F_{T,7c} \Delta H_{vap,\, water} \quad \Rightarrow F_{T,7c} = q_{transferred} / \Delta H_{vap,\, water} = 22.2 \text{ kg steam/min}$$

Steam in stream 2 = 100 kg/min (from stream 1) + 22.2 kg/min (from stream 7c) = 122.2 kg steam/min

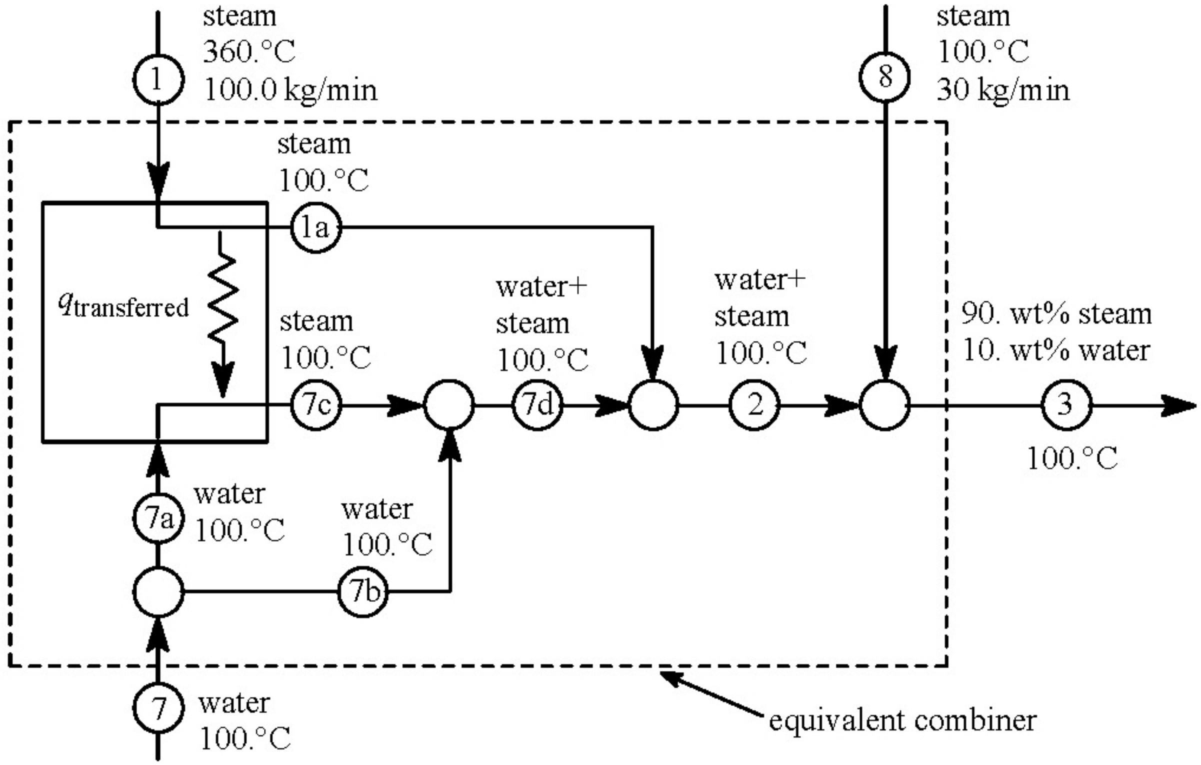
Steam in stream 3 = 122.2 kg/min (from stream 2) + 30 kg/min (from stream 8) = 152.2 kg steam/min

Because stream 3 is 90% steam, $F_{T,3} = 152.2/0.90 = 169$ kg/min.

Because stream 3 is 10% water: $F_{water,3} = 169 \times 0.10 = 16.9$ kg water/min. (= water in stream 7b)

$\Rightarrow$ Stream 7 is 22.2 kg/min (stream 7a) + 16.9 kg/min (stream 7b) = 39 kg/min.

Equivalent Unit for the Two Combiners



Check: Overall mass balance on Equivalent Unit.

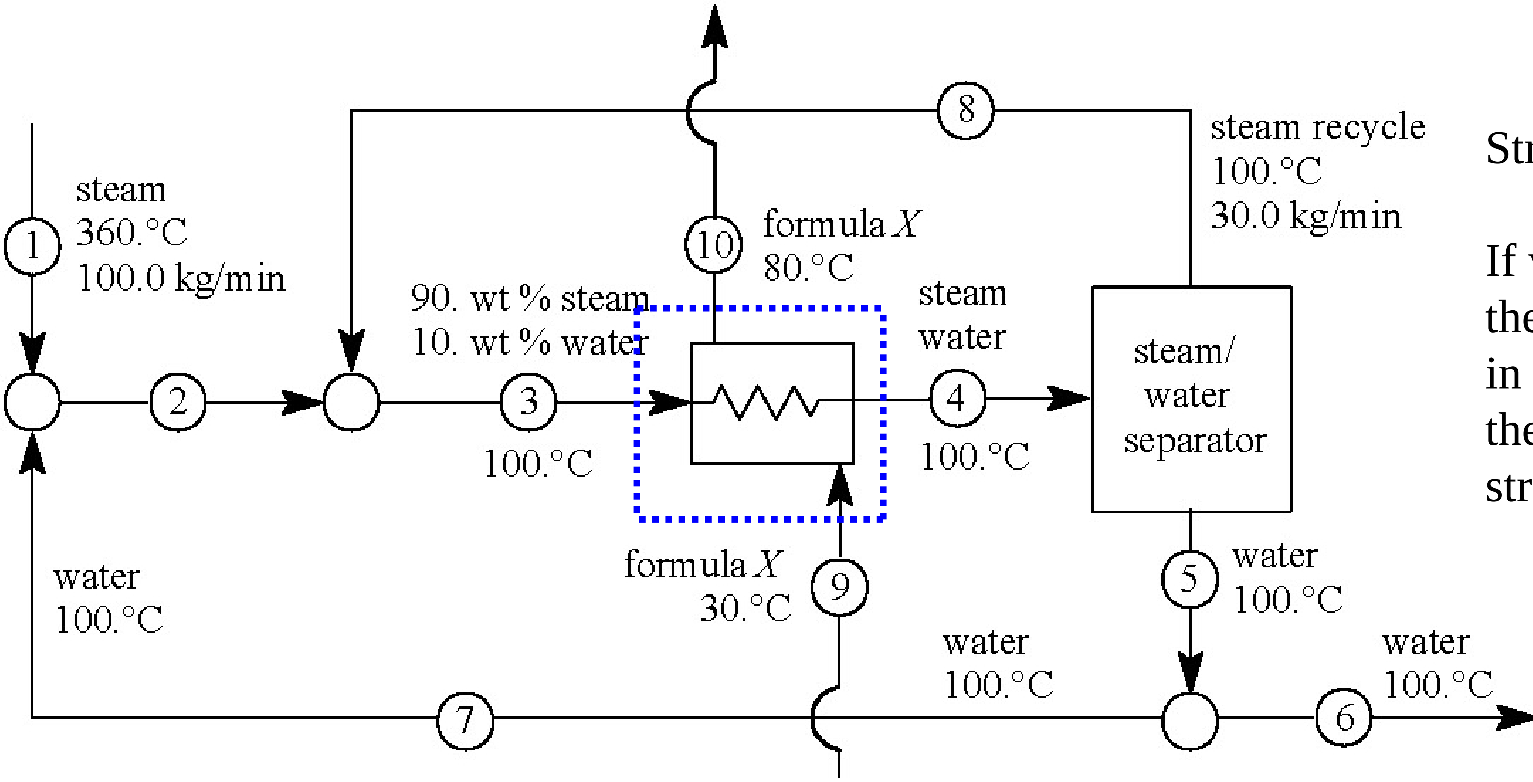
$$\text{rate in} = \text{rate out}$$

$$F_{T,1} + F_{T,7} + F_{T,8} = F_{T,3}$$

$$F_{T,7} = F_{T,3} - F_{T,1} - F_{T,8}$$

$$F_{T,7} = 169 - 100 - 30 = 39 \text{ kg/min.}$$

Exercise 3.78 – Energy Balance on Heat Exchanger



Stream 7?

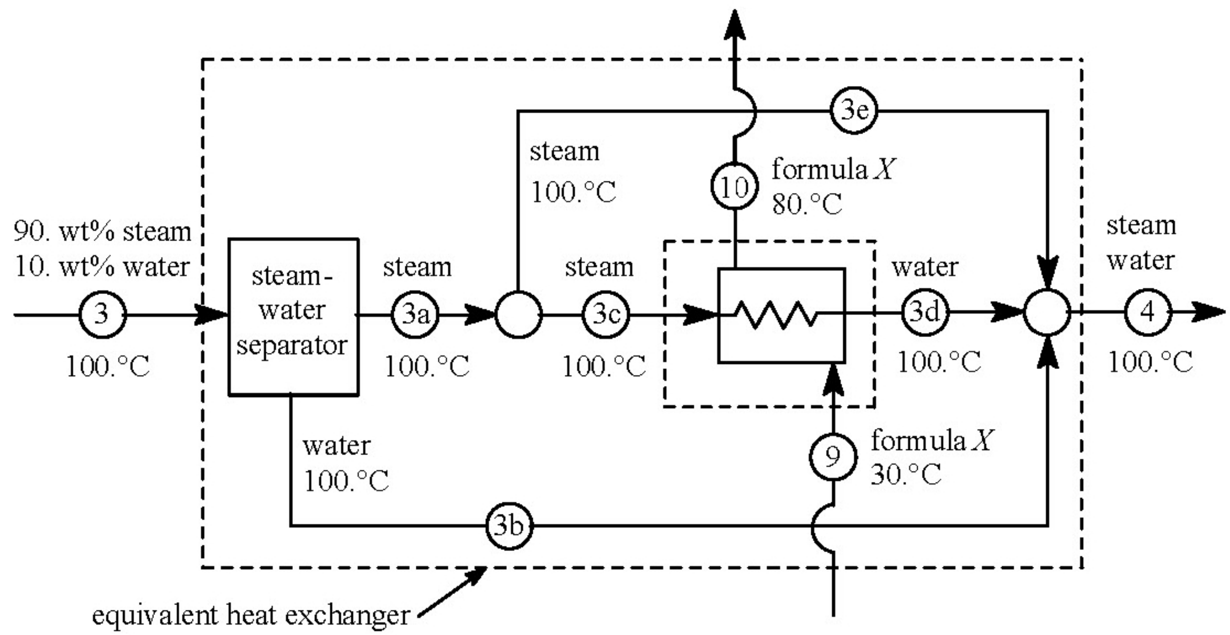
If we can calculate the flow rate of water in stream 4, we have the flow rates of streams 5 and 7.

Consider stream 3 -
Analyze the heat exchanger.

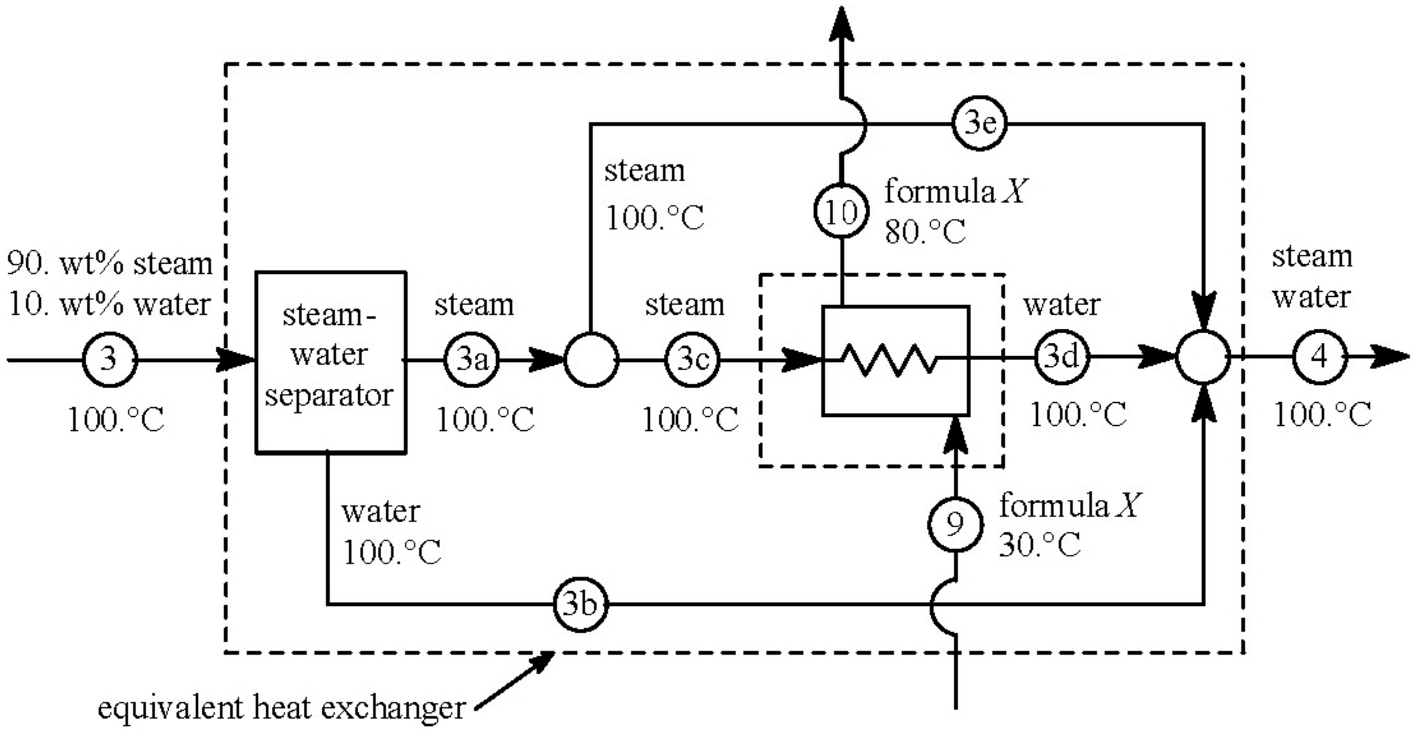
All the water in stream 3 passes through the heat exchanger unchanged.

Some steam condenses to warm formula *X*.

Some steam (30 kg/min) passes through the heat exchanger unchanged.



Equivalent Unit for the Heat Exchanger



Calculate the rate steam condenses to warm formula X.
Write and energy balance on the fictitious heat exchanger. $-\Delta q_{3c\to 3d} = \Delta q_{9\to 10}$

Substitute thermodynamics. $-F_{T,3c}\Delta H_{\text{vap water}} = F_{T,9}C_{\text{P, formula }X}(T_{10} - T_9)$

Solve for $F_{T,3c}$. $F_{T,3c} = \dfrac{F_{T,9}\overline{C}_{\text{P,formula }X}(T_{10}-T_9)}{-\Delta\overline{\overline{H}}_{\text{vap water}}} = 122.2 \text{ kg steam/min}$

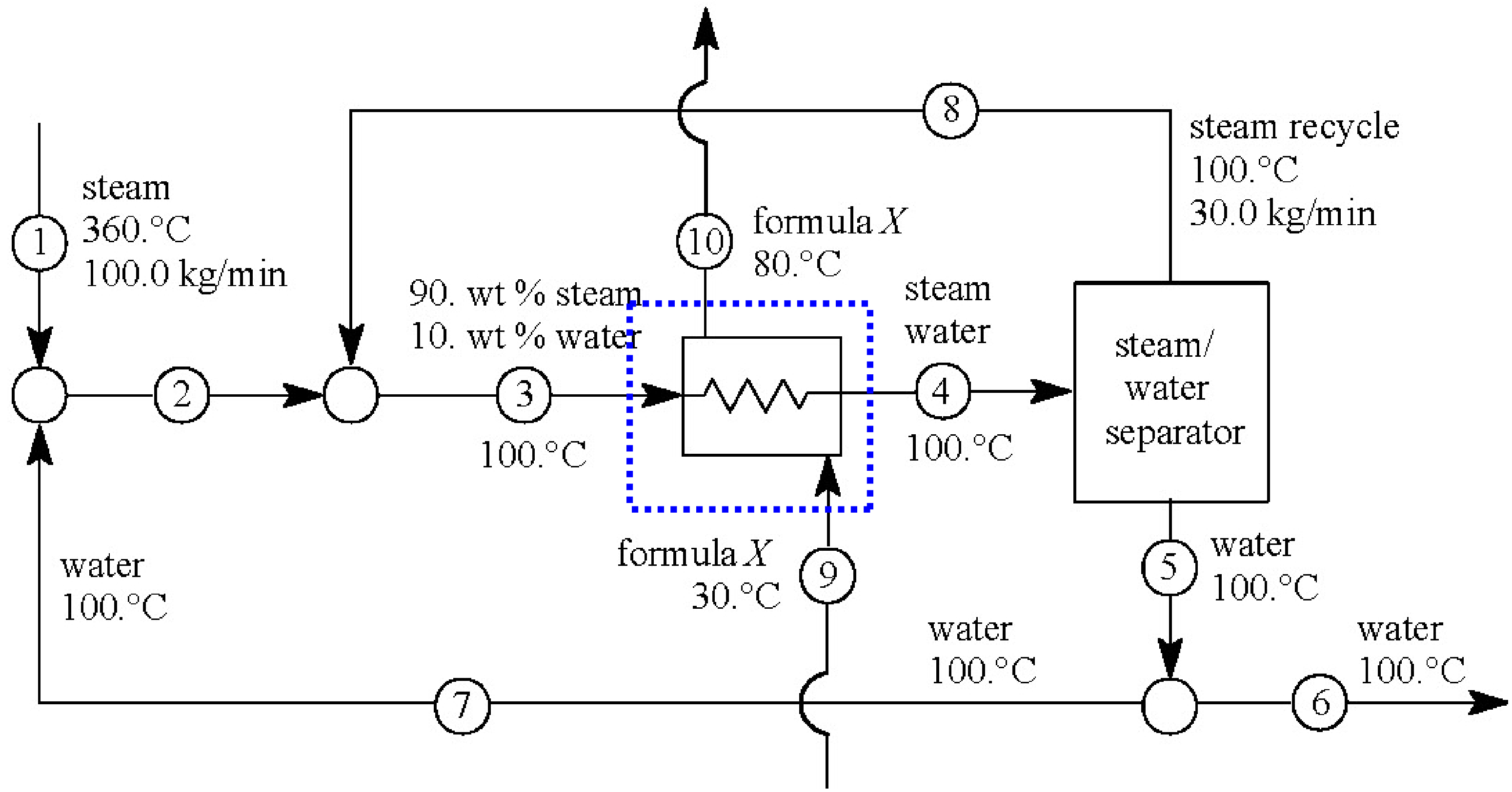
Steam in stream 3 = steam bypassed + steam condensed = 30. + 122.2 = 152.2 kg steam/min

Because stream 3 is 90% steam, $F_{T,3}$ = 152.2/0.90 = 169 kg/min.

Because stream 3 is 10% water: $F_{\text{water},3}$ = 169 × 0.10 = 16.9 kg water/min.

Water in stream 4 = steam condensed + water bypassed = 122.2 + 16.9 = 139.1 kg water/min.

Exercise 3.78 - Summary



Water in stream 4 = 139.1 kg water/min.

Water in stream 5 = 139.1 kg water/min.

Water in stream 7 = 139.1 – 100 = 39 kg water/min.

Energy Balances

Process units with simple mass flows may have complex energy flows.
Consider drawing an equivalent unit that comprises elementary energy units.

When a mass balance fails, use an energy balance to obtain a mass flow rate.