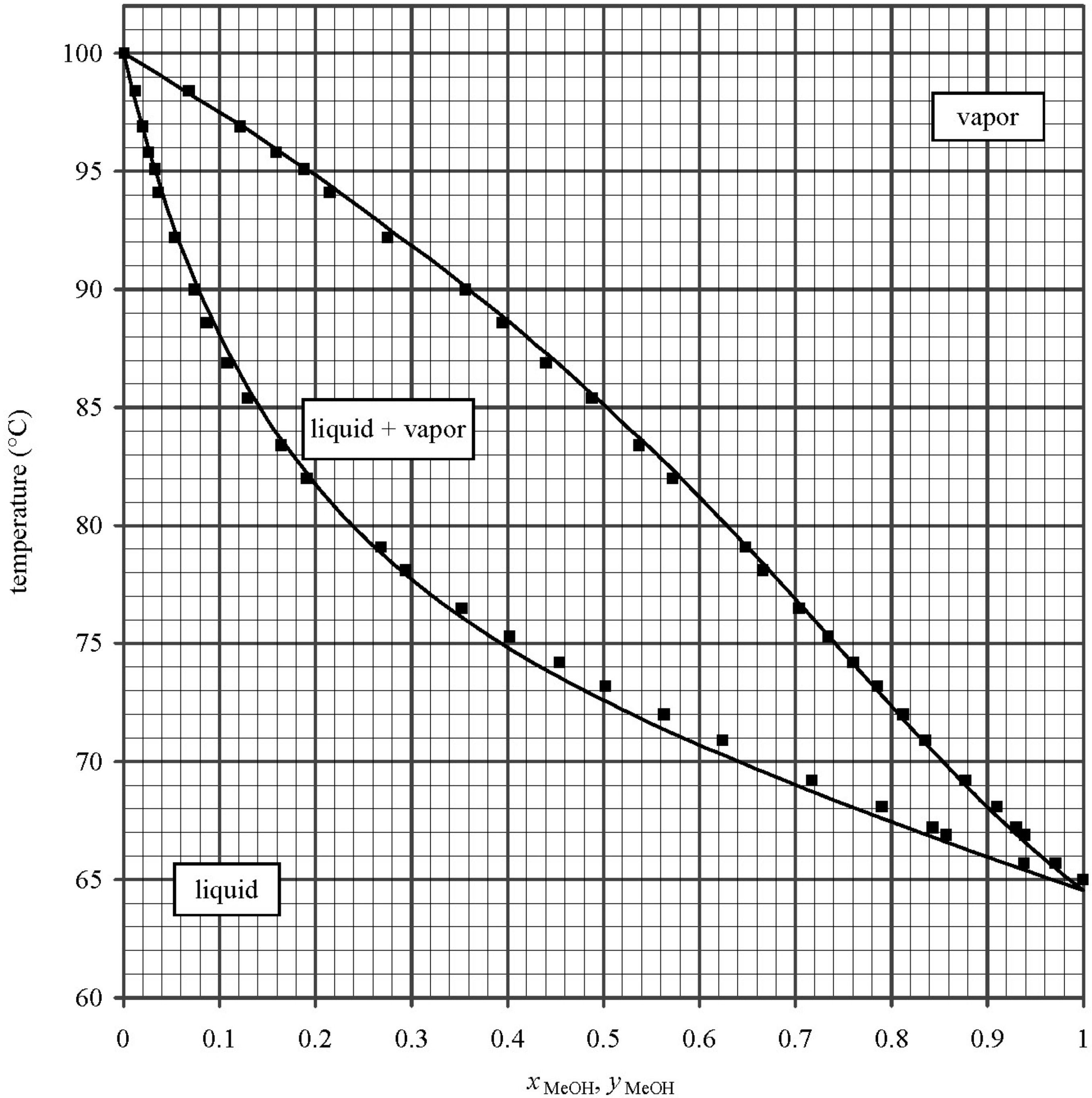
EngrD 2190 – Lecture 28

Concept: Dimensional Analysis and Dynamic Scaling

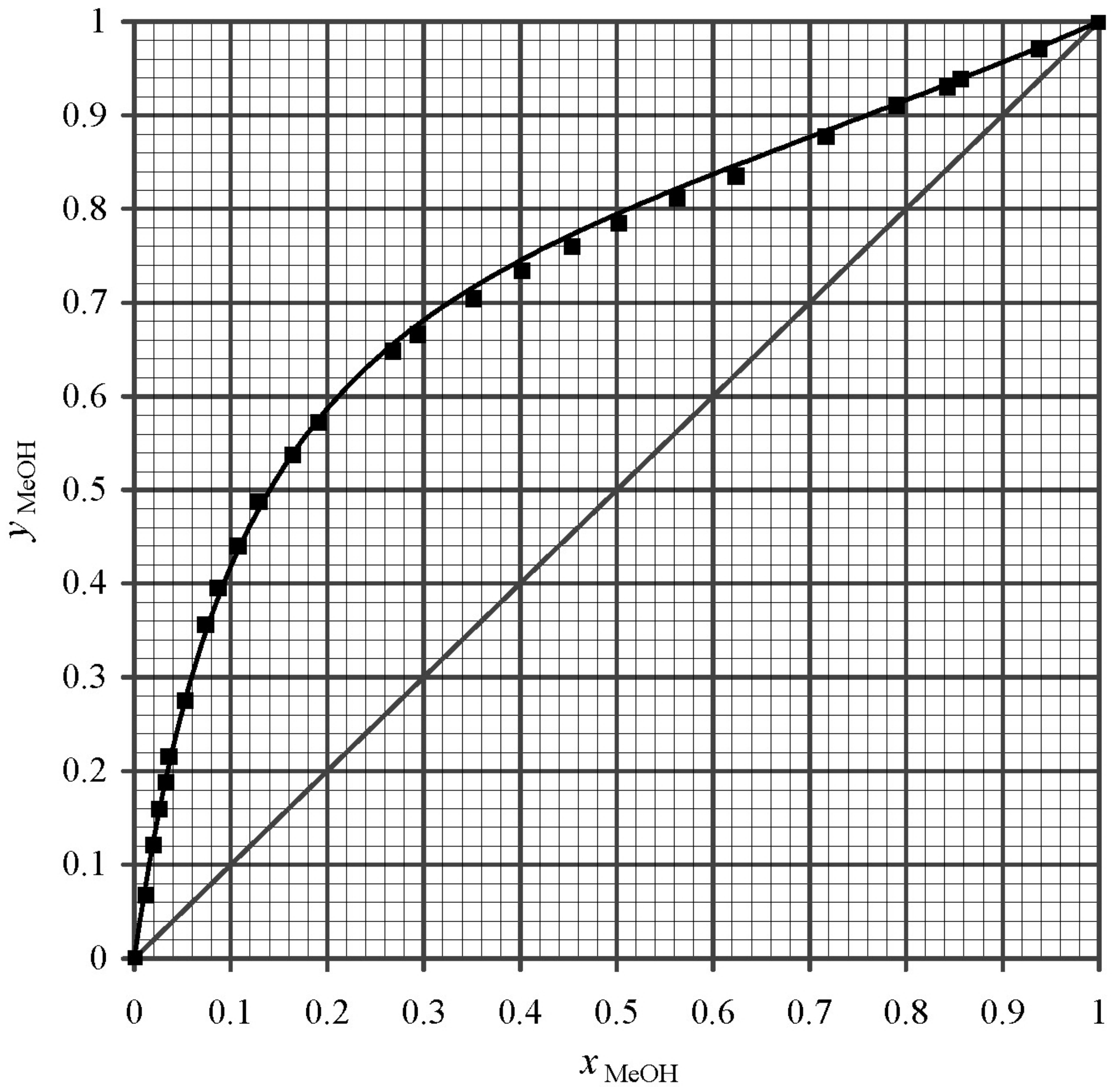
Context: Universal Scaling of a Pendulum

Defining Question: Why do you need not fear gigantic ants
or gargantuan mosquitos?

Read Chapter 5 pp. 431-436
Dynamics of Walking and Running.
Lecture 29 will follow the textbook.



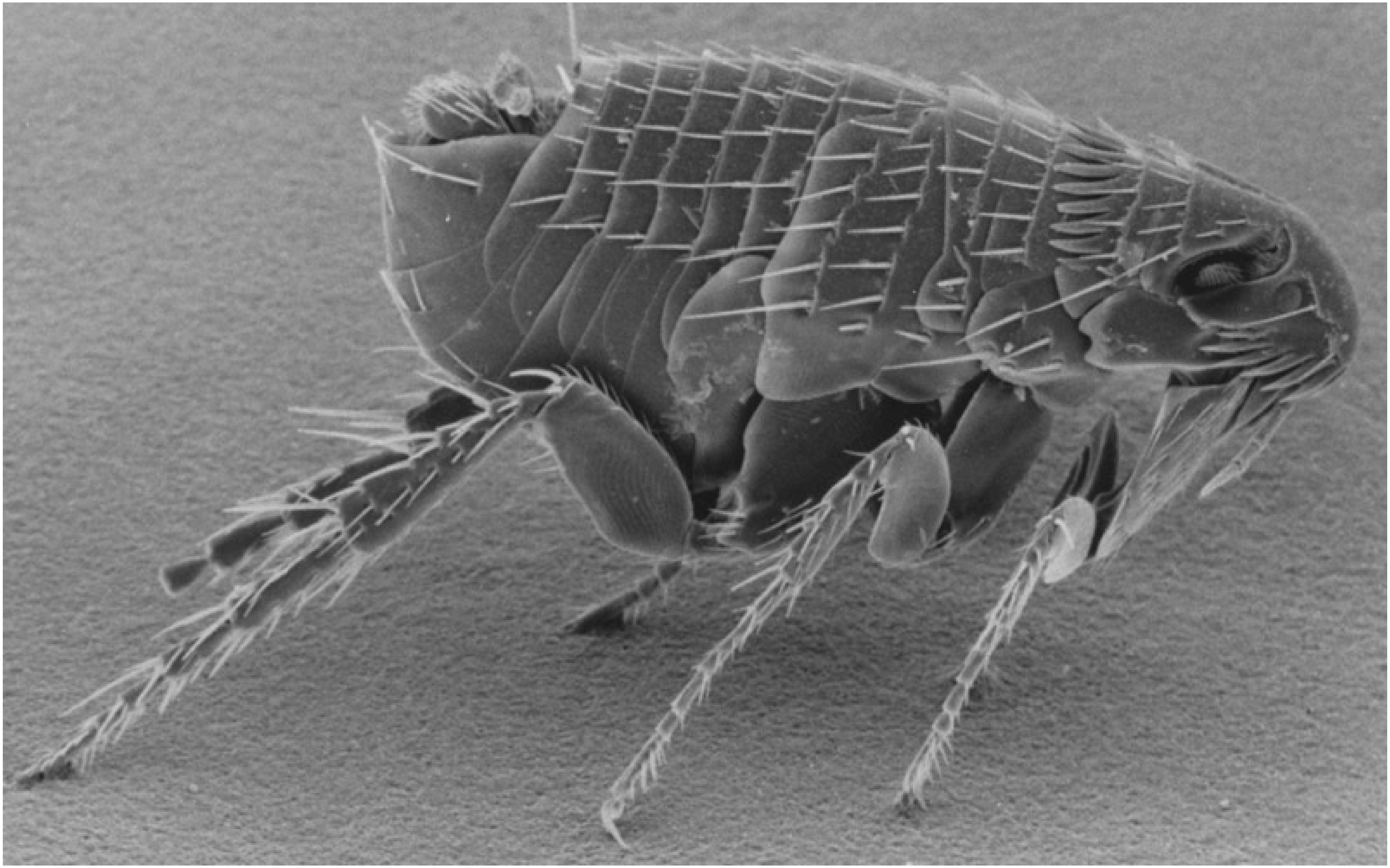
temperature-composition phase diagram for methanol+H_2O mixtures at 1 atm
(data from exercise 4.19)

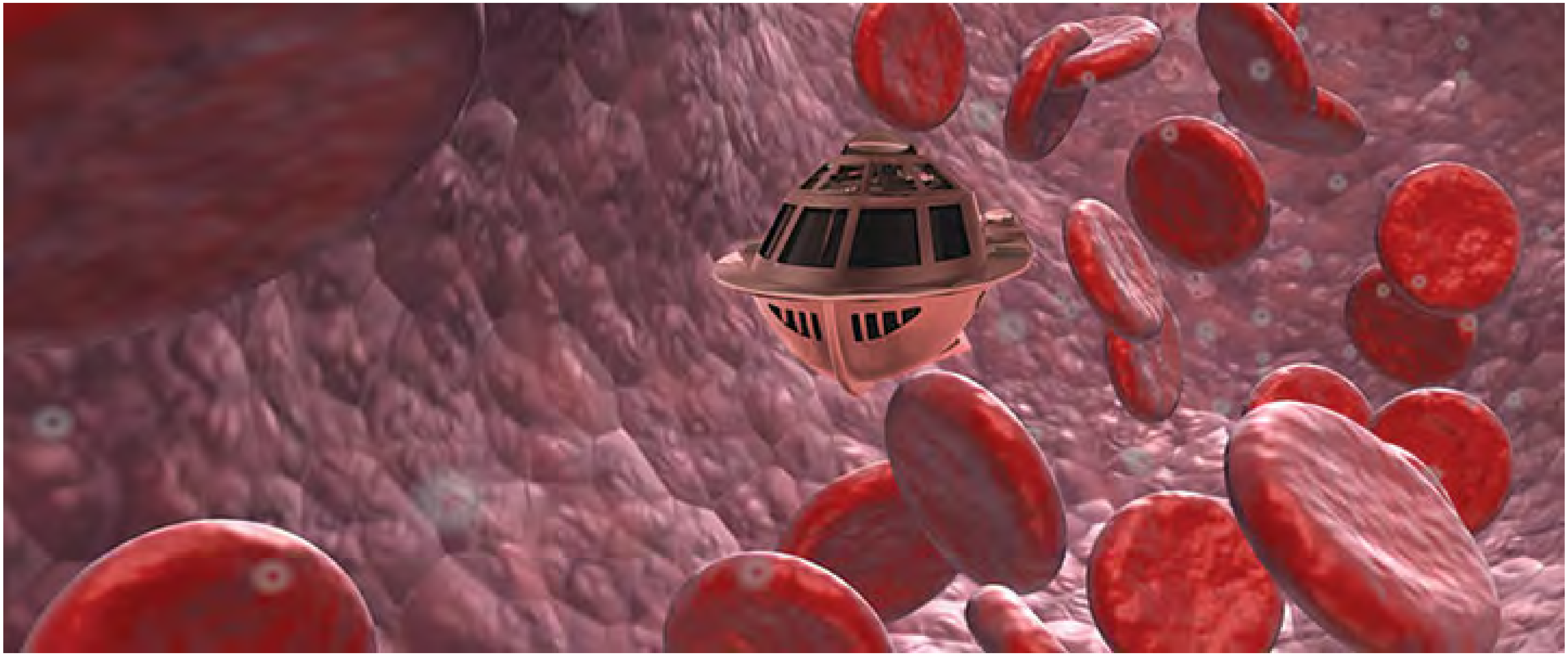


(data from exercise 4.19)

Dimensional Analysis and Dynamic Scaling

A ‘Water Bear’ (aka a Tardigrade) - Nature’s Toughest Animal





Fantastic Voyage (1966)

Fantastic Voyage (1966)

Mosquito! (1995)

Them! (1954)

The Deadly Mantis (1957)

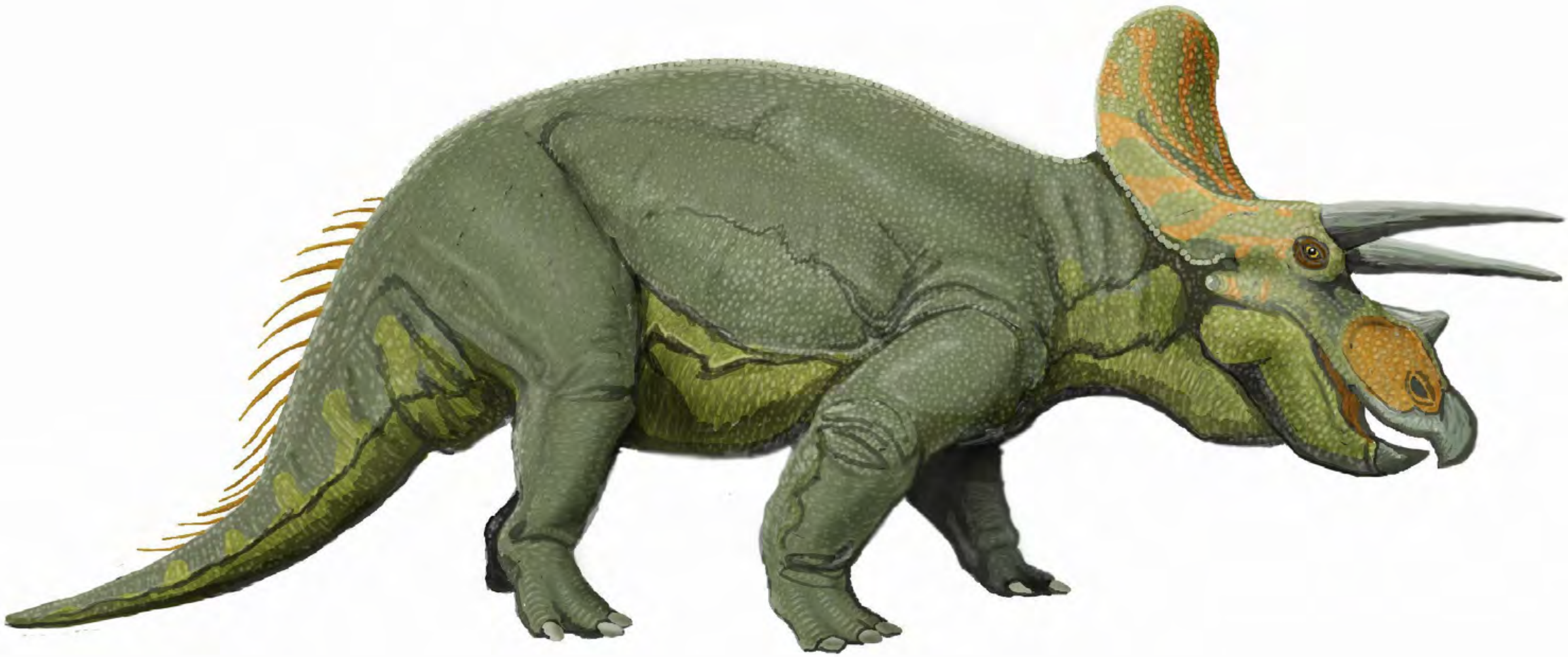
The Deadly Mantis (1957)

Starship Troopers (1997)

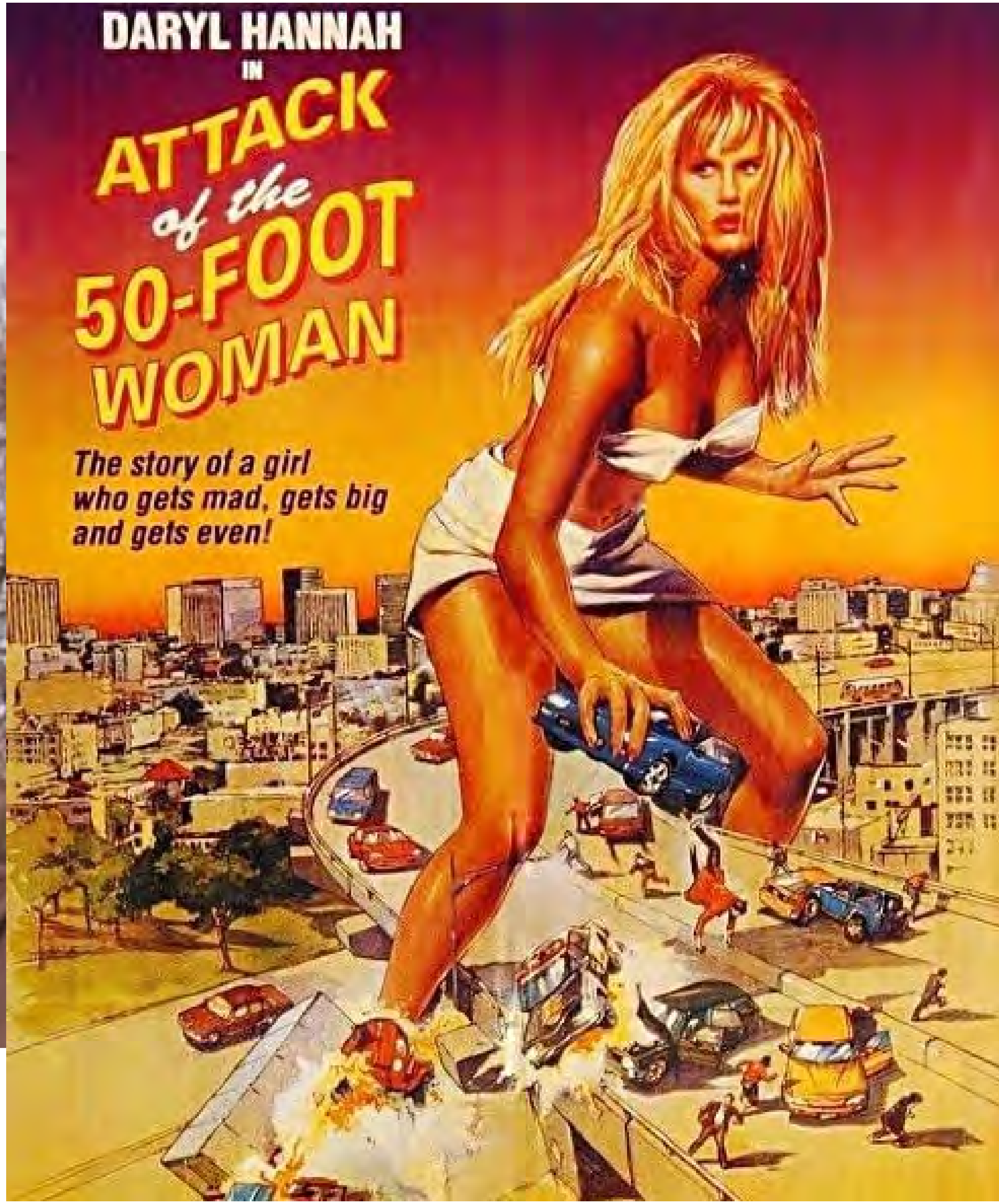
One Million BC (1940)

The Giant Gila Monster (1959)

The Lost World (1960)

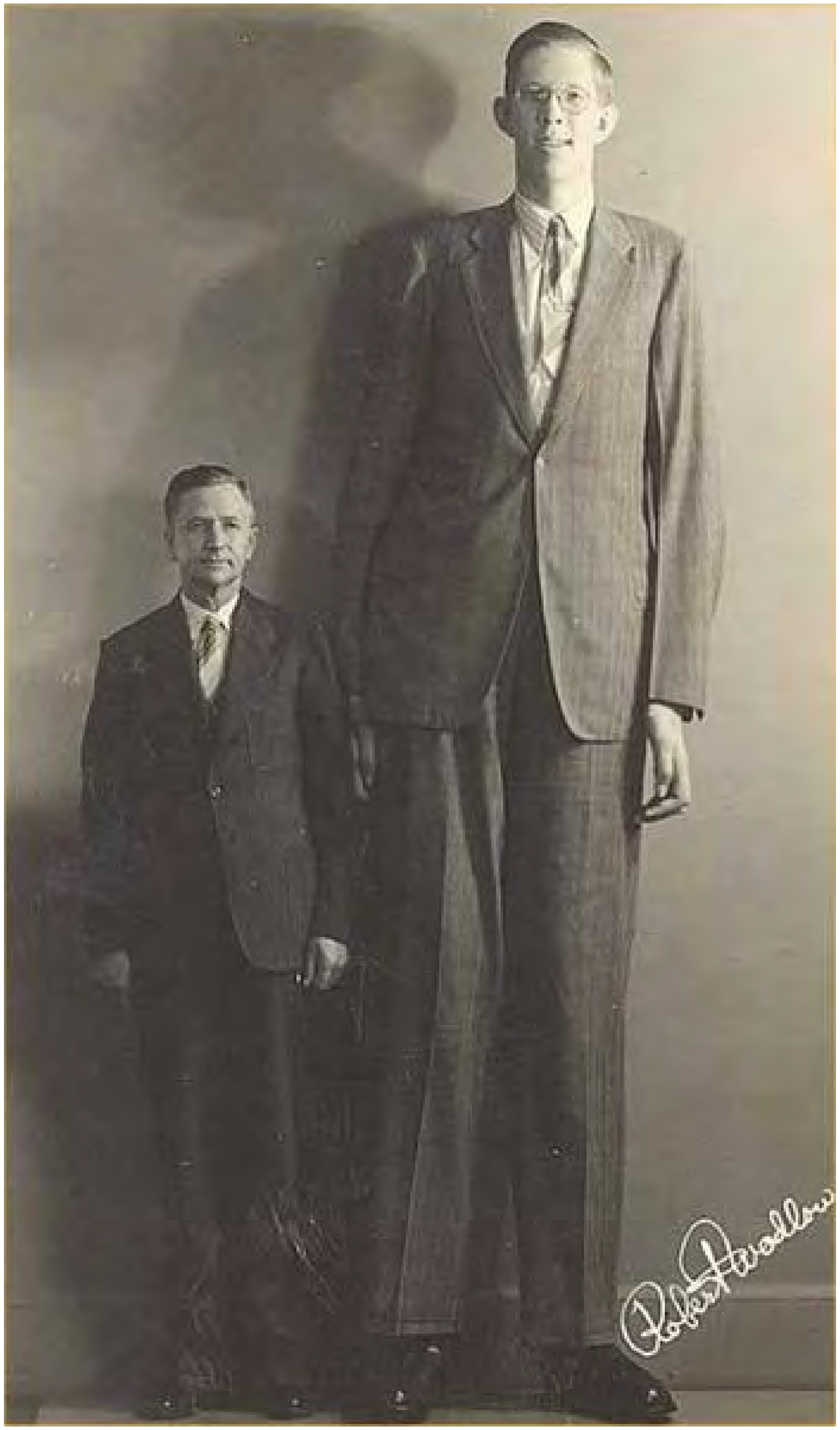



ALLIED ARTISTS PICTURES presents
ATTACK OF THE 50 FT. WOMAN
ALLISON HAYES · WILLIAM HUDSON · YVETTE VICKERS
BERNARD WOOLNER · NATHAN HERTZ · MARK HANNA


DARYL HANNAH
IN
ATTACK
of the
50-FOOT
WOMAN
The story of a girl
who gets mad, gets big
and gets even!


The Shobijin from *Mothra* (1961)

Gulliver and the Lilliputians





Dynamic Scaling Example 1 - Motion Through a Fluid

Swimmer stops swimming - swimmer glides for 1-3 body lengths

Dynamic Scaling Example 1 - Motion Through a Fluid

Propellers stop - ship glides for ~100 ship lengths

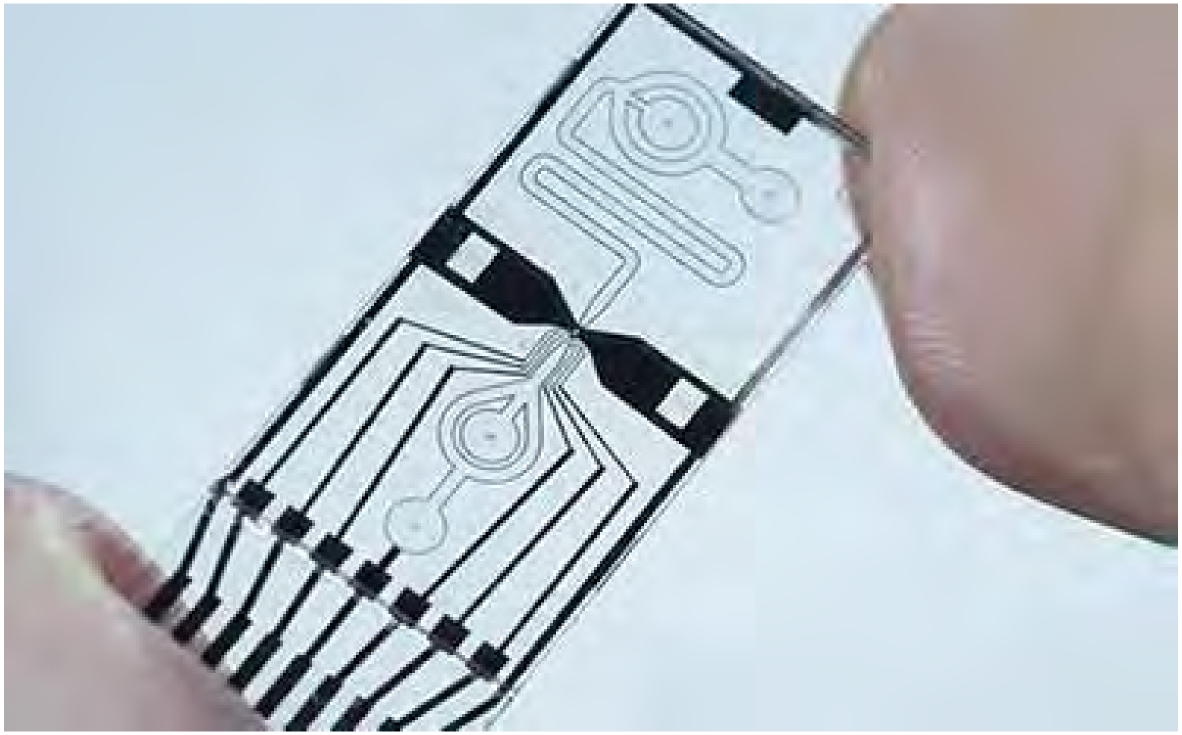
Dynamic Scaling Example 1 - Motion Through a Fluid

Propulsion stops - paramecium glides for ~0 body lengths

Dynamic Scaling Example 1 - Motion Through a Fluid

Key ratio for motion through fluids: $\dfrac{\text{inertial forces}}{\text{frictional forces}}$

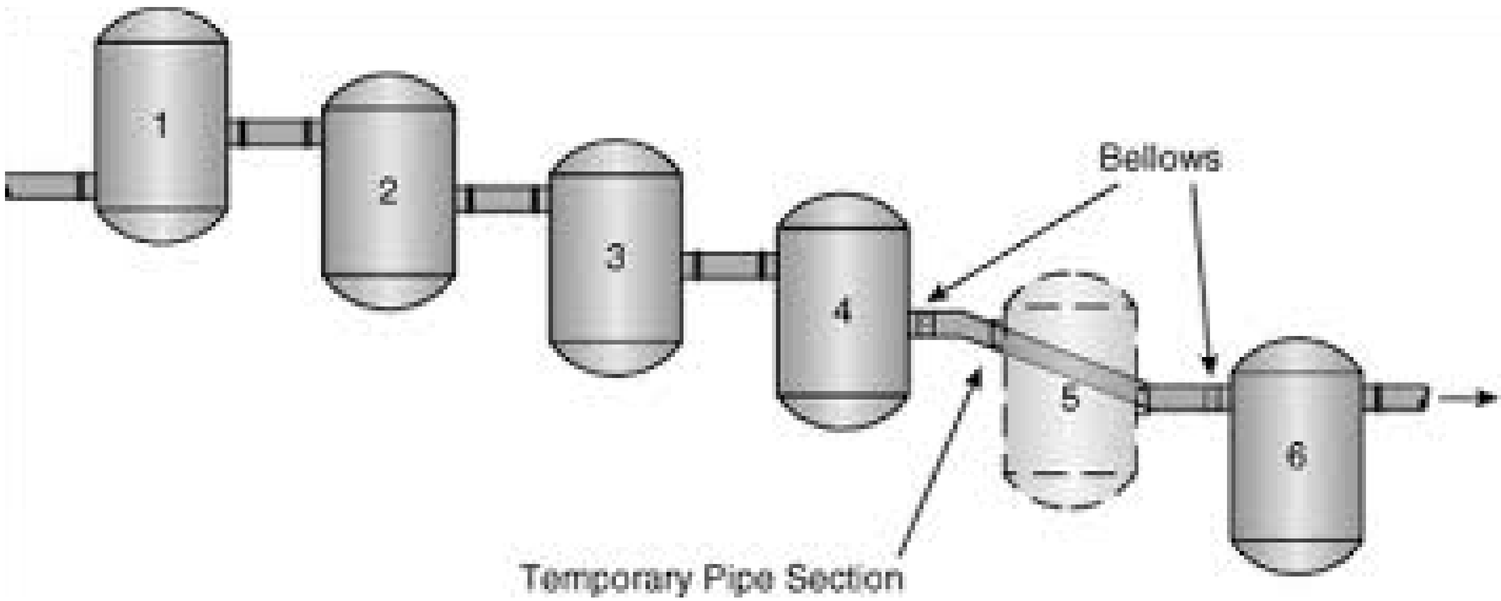
Ratio is negligible for microfluidics



Dynamic Scaling Example 1 - Motion Through a Fluid

Key ratio for motion through fluids: $\frac{\text{inertial forces}}{\text{frictional forces}}$

Ratio is large for commercial chemical processes

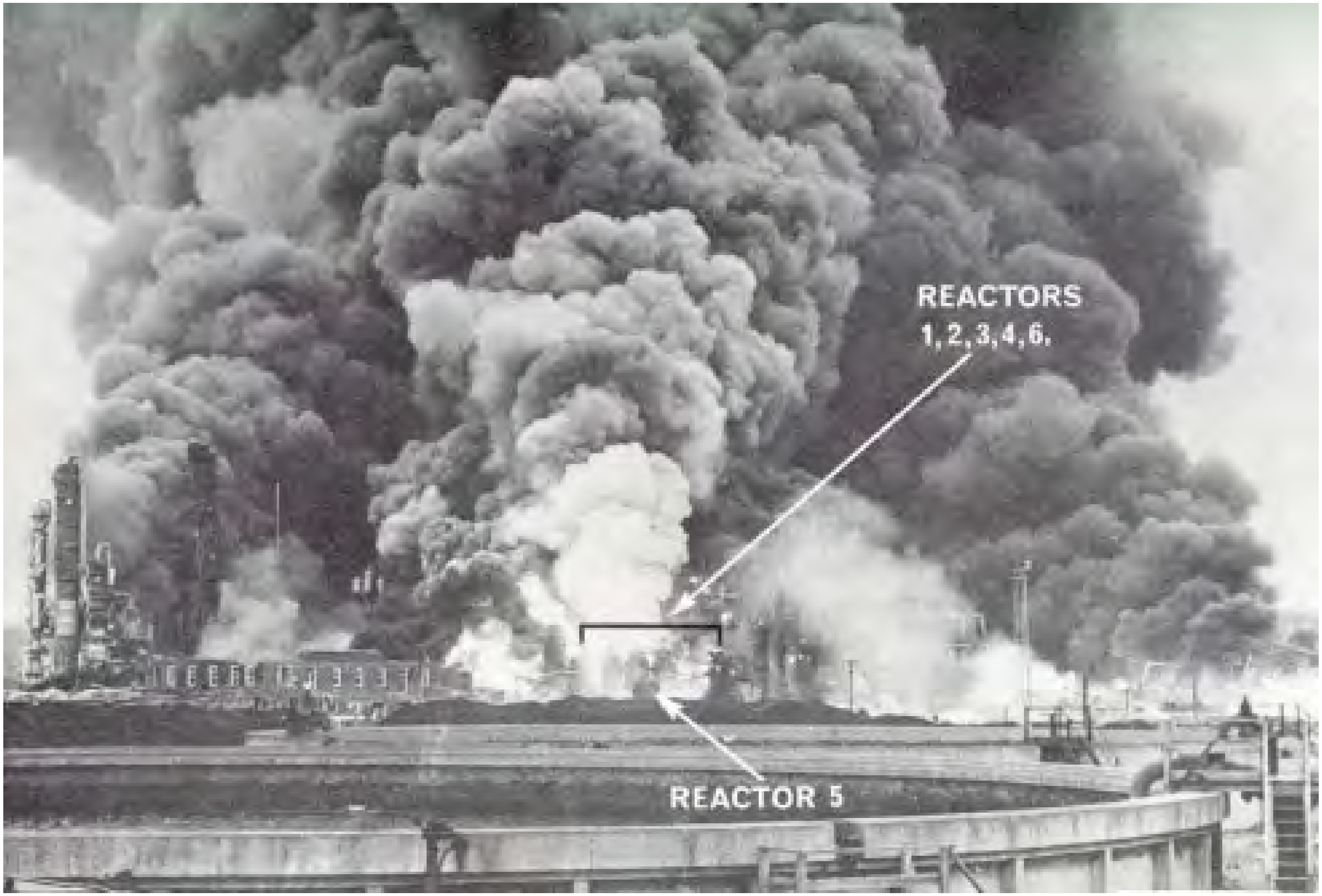


Partial oxidation of cyclohexane to cyclohexanol.
First step in process to synthesize Nylon.

Reactor cascade connected by 20” pipes.

Flow is 40 tons/minute

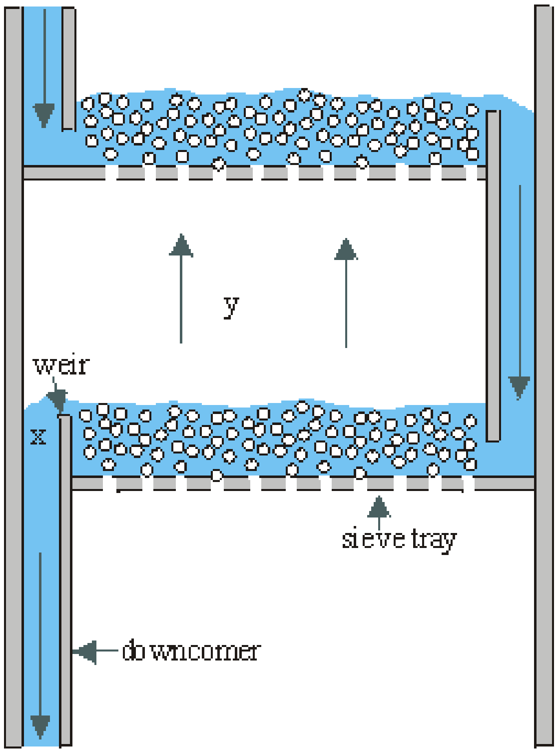
Flixborough England 1974



Explosion killed 28 and caused $450,000,000 in damages.

Dynamic Scaling Example 2 - Distillation Column Scale-Up

McCabe-Thiele analysis: 10 equilibrium stages

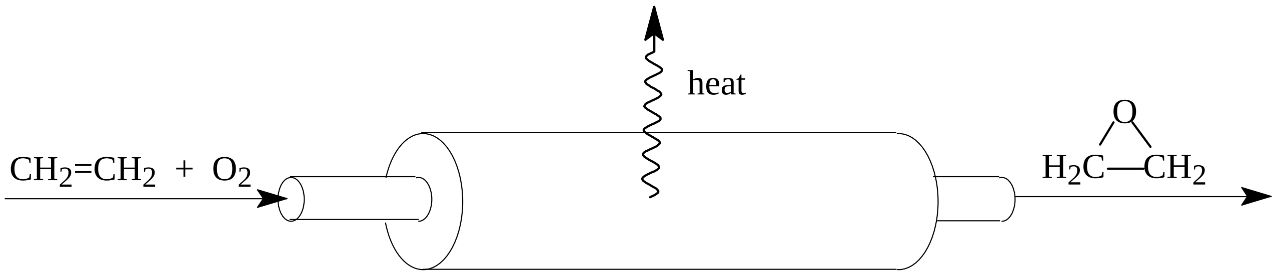


	bench scale model	actual column (×100)
total height:	60 cm	60 m
tray diameter:	10 cm	10 m
vapor holes in sieve tray:	3 mm	30 cm (~14 inches)
Liquid depth on tray:	2 cm	2 m (~6½ feet)

The bench-scale model worked well. The commercial-scale unit failed. Why?

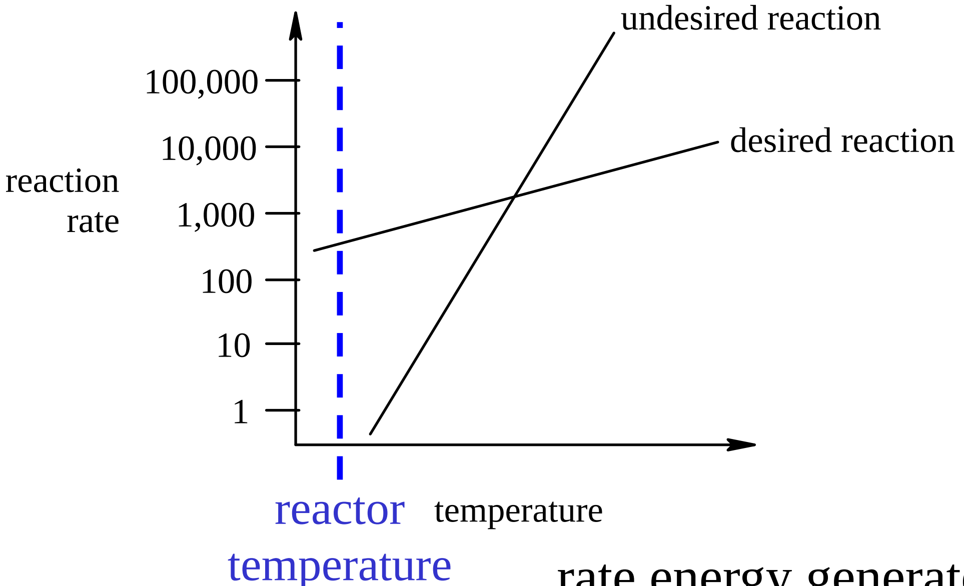
Flow behavior depends on viscosity, density, and surface tension, which cannot be scaled.

Dynamic Scaling Example 3 - Chemical Reactor Scale-Up



$$\text{desired reaction: } CH_2=CH_2 + \tfrac{1}{2}O_2 \rightarrow CH_2(O)CH_2 + 105\ \text{kJ/mol}$$

$$\text{undesired reaction: } CH_2=CH_2 + 2O_2 \rightarrow 2CO_2 + 2H_2O + \mathbf{1320\ kJ/mol}$$



	bench scale model	actual reactor (×100)
reactor diameter:	1 cm	1 m
reactor length:	10 cm	10 m
	worked fine	blowed up!

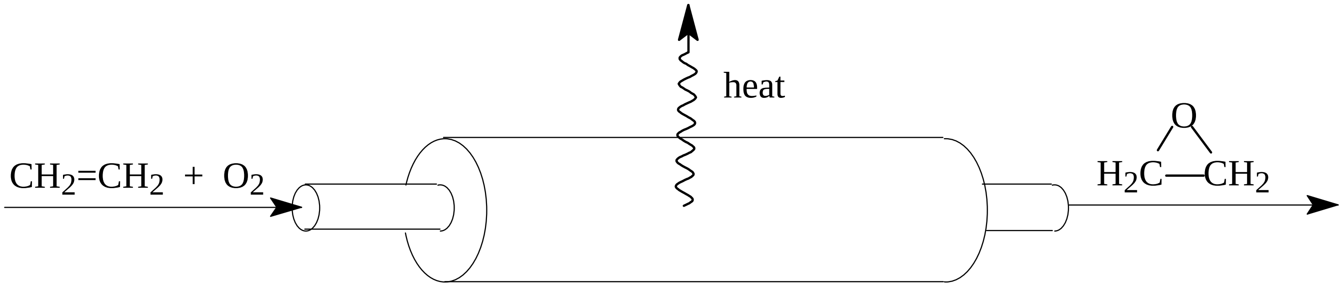
Why?

rate energy generated $\propto$ mass in reactor $\propto$ reactor volume $= L\times(\pi r^2)$

rate energy removed $\propto$ reactor surface area $= L\times(2\pi r)$

So how do we scale-up the reactor?

Dynamic Scaling Example 3 - Chemical Reactor Scale-Up



So how do we scale-up the reactor?

Many small reactors!

Chemical Process Modeling and Analysis

Mathematical Modeling

process flowsheet → equations

Graphical Modeling

process flowsheet → paths on phase maps

process unit → operating lines

Dimensional Analysis

bench-scale unit → commercial-scale unit	scale up
bench-scale unit → micro-scale unit	scale down

moderate time interval (minutes) → long time interval (years)

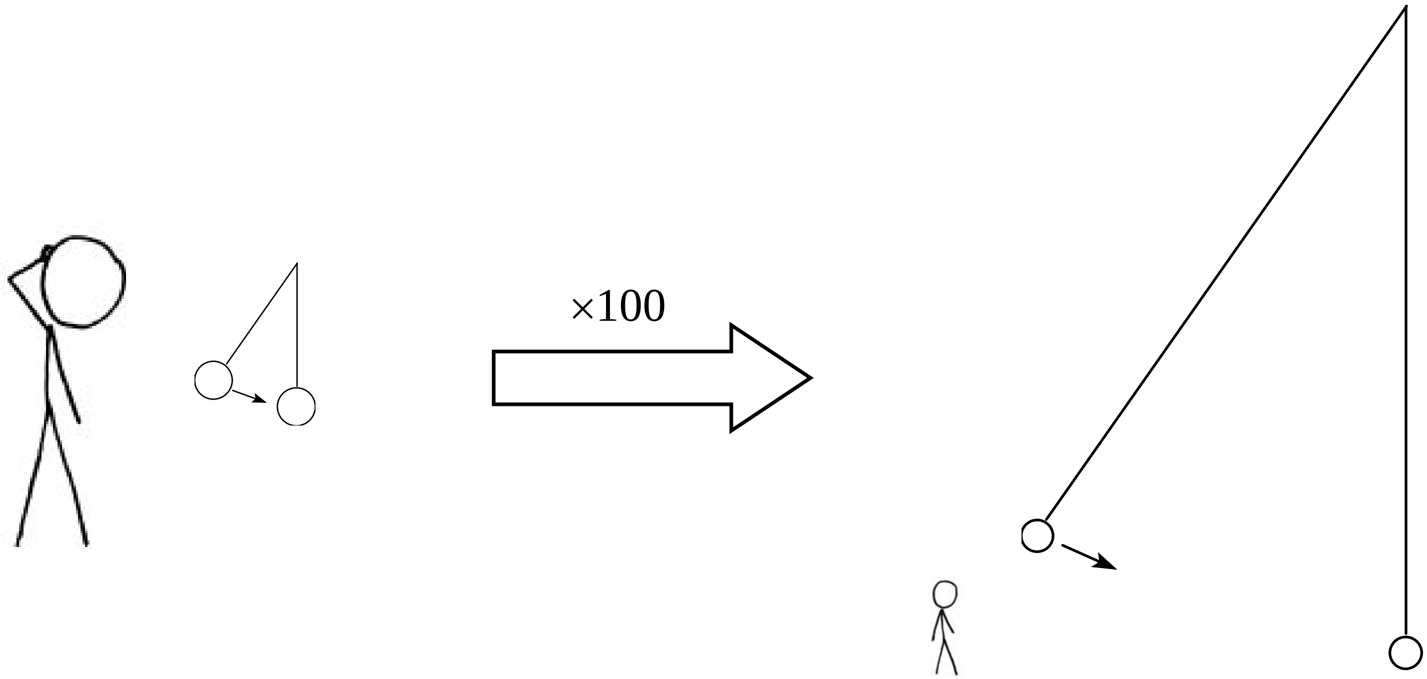
moderate time interval (minutes) → short time interval (msec)

In general: convenient size, duration, or cost → inconvenient or inaccessible

How? What are the rules?

Dimensional Analysis Example 1: A Pendulum

How does period change with pendulum length? Mass? Angle?



Dimensional Analysis and Dynamic Scaling!

Dimensions and Units

Table 5.1 Base dimensions

base dimension	symbol
length	L
mass	M
time	T
temperature	Θ
amount	N
electric charge	Q
luminous intensity	I

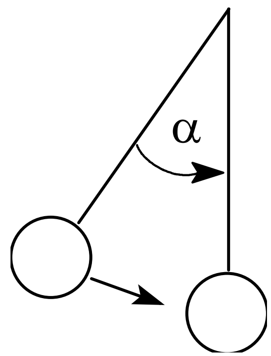
Table 5.2 The SI and English systems of base units

base dimension	SI (mks)	English
length, L	meter, m	foot, ft
mass, M	kilogram, kg	pound-mass, lb_m
time, T	second, s	second, s
temperature, Θ	Celsius, °C, or Kelvin, K	Fahrenheit, °F, or Rankine, °R
amount, N	mole	mole

Table 5.4 Derived units and dimensions

quantity	dimensions	units in SI
volume	L^3	m^3
velocity	L/T	m/s
acceleration	L/T^2	m/s^2
momentum (= mass × velocity)	ML/T	kg·m/s
force (= mass × acceleration)	ML/T^2	$kg \cdot m/s^2 \equiv$ newton (N)
pressure (= force/area)	M/LT^2	$kg/m \cdot s^2 = N/m^2 \equiv$ pascal (Pa)
energy (= $\frac{1}{2}mv^2$ or force × distance)	ML^2/T^2	$kg \cdot m^2/s^2 = N \cdot m \equiv$ joule (J)
power (= energy/time)	ML^2/T^3	$kg \cdot m^2/s^3 = J/s \equiv$ watt (W)

Dimensional Analysis Example 1: A Pendulum



$$\alpha = \frac{\text{arc length}}{\text{radius}}$$

$$[\alpha] = \frac{L}{L} = \text{(none)}$$

Table 5.5. The parameters of a pendulum

physical quantity	symbol	dimensions
period of oscillation	t_p	T
length of pendulum	ℓ	L
mass of pendulum	m	M
gravitational acceleration	g	L/T^2
amplitude	α	(none)

will always be given

Dimensional Analysis Example 1: A Pendulum

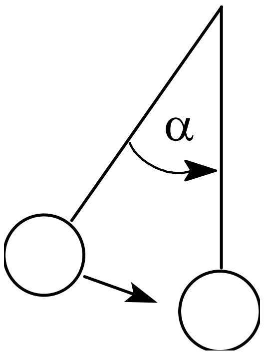


Table 5.5. The parameters of a pendulum

physical quantity	symbol	dimensions
period of oscillation	t_p	T
length of pendulum	ℓ	L
mass of pendulum	m	M
gravitational acceleration	g	L/T^2
amplitude	α	(none)

We seek an equation of the form $t_p = f(\ell, m, g, \alpha)$

t_p has dimensions of time; $f(\ell, m, g, \alpha)$ must also have dimensions of time

The function cannot contain *m*. Why? Because no other parameter has dimensions of mass.

Can the function contain ℓ ? Yes, because we can cancel ℓ's dimensions with *g*.

$$[\ell] = \mathrm{L}, \quad [g] = \frac{\mathrm{L}}{\mathrm{T}^2} \quad \Rightarrow \quad \left[\frac{\ell}{g}\right] = \frac{\mathrm{L}}{\mathrm{L}/\mathrm{T}^2} = \mathrm{T}^2$$

If ℓ is in the function, it is present as the ratio ℓ/g.

Furthermore, the ratio must be present as its square root.

$$\left[\left(\frac{\ell}{g}\right)^{1/2}\right] = \left(\mathrm{T}^2\right)^{1/2} = \mathrm{T}$$

Dimensional Analysis Example 1: A Pendulum

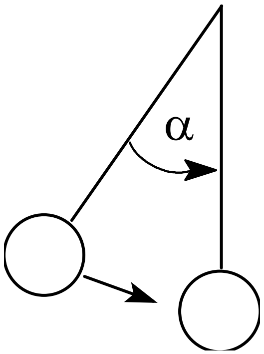


Table 5.5. The parameters of a pendulum

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amplitude	α	(none)

Dimensional analysis yields $$t_p = \left(\frac{\ell}{g}\right)^{1/2} f(\alpha)$$

The pendulum period of oscillation scales as the square root of the length.

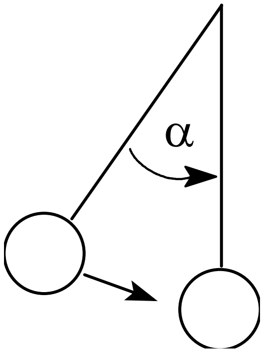
For our model pendulum (ℓ = 10 cm) we measure t_p = 0.64 sec.

For large pendulum (ℓ = 10 m) we predict $t_p = 100^{1/2} \times 0.64$ sec = 6.4 sec.

Without dynamic scaling one would naively predict $t_p = 100 \times 0.64$ sec = 64 sec.

This prediction is valid only for the same angle α in the model and the large pendulum.

Dimensional Analysis Example 1: A Pendulum



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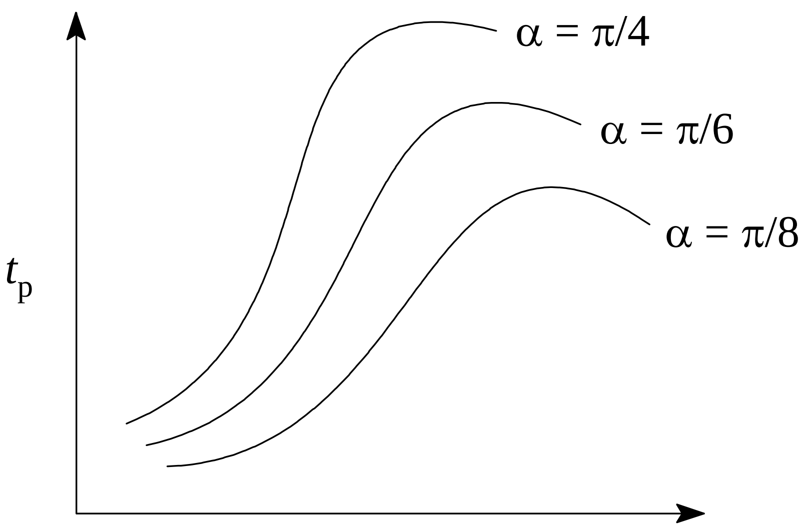
amplitude	α	(none)

Dimensional analysis yields $t_p = \left(\frac{\ell}{g}\right)^{1/2} f(\alpha)$

The pendulum period depends on ℓ and α only.



To find the function f we must conduct experiments, but our experimental agenda is shortened by dimensional analysis; vary only ℓ and α .

Experiments show the period is independent of α .

$$t_p = 2\pi\left(\frac{\ell}{g}\right)^{1/2}$$