EngrD 2190 – Lecture 29

Concept: Dimensional Analysis and Dynamic Scaling

Context: Universal Scaling of Walking and Running

Defining Question: What is a core variable?

Read Chapter 5 pp. 436-447

The terminal velocity of a sphere.

Lecture 30 will follow the textbook.

Calculation Session Today

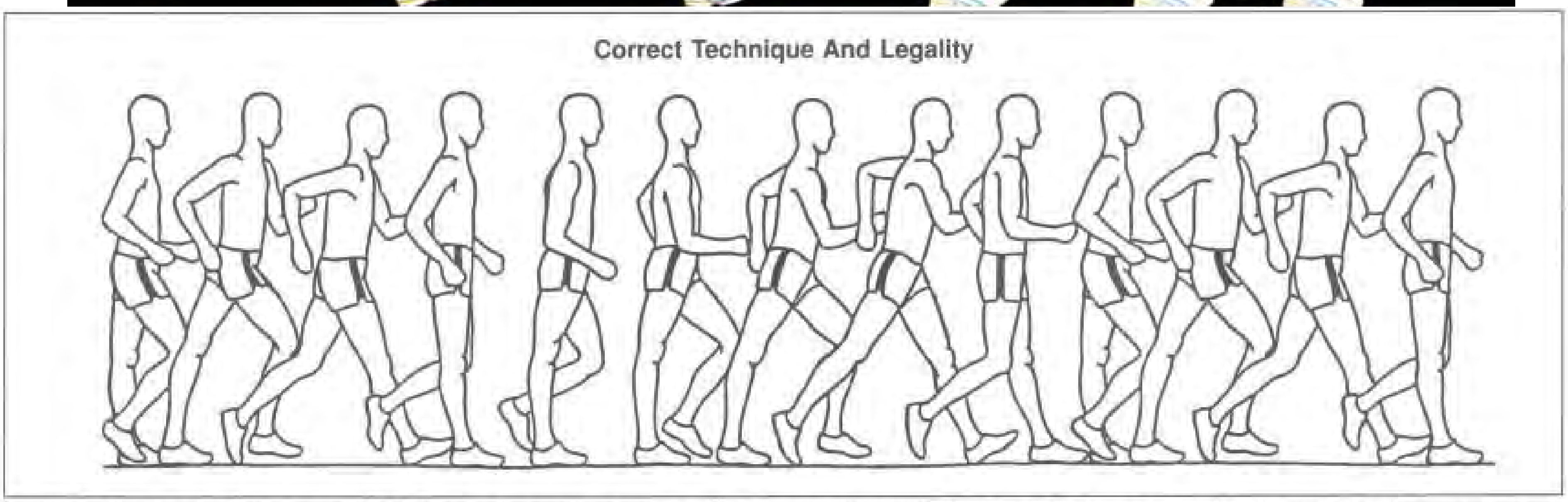
TA Panel on Pre-enrollment for Spring 2025

Prepare questions about the 4th semester and the ChemE curriculum.

Dimensional Analysis and Dynamic Scaling

A Universal Correlation for the Dynamics of Walking

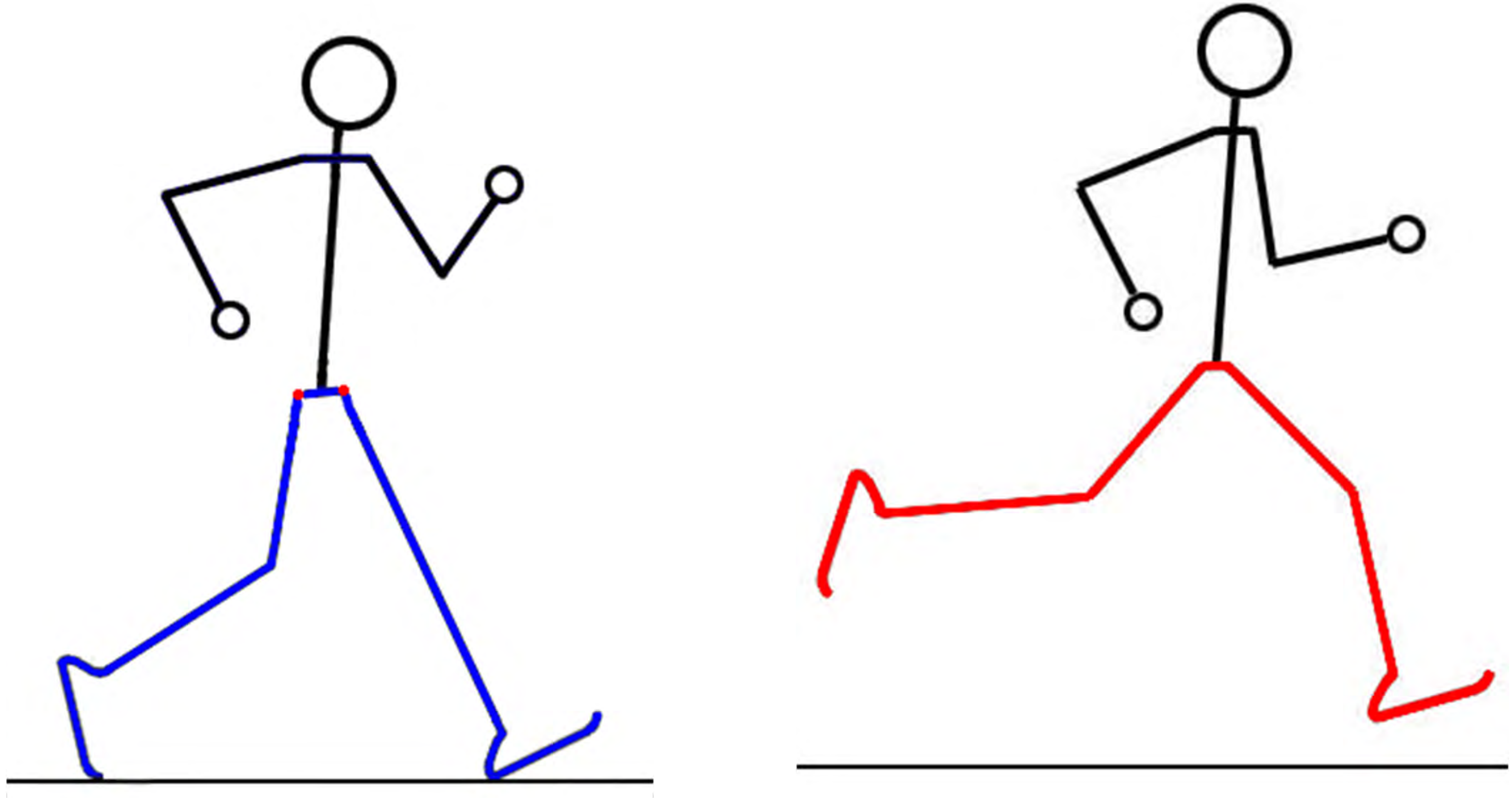
Race Walking



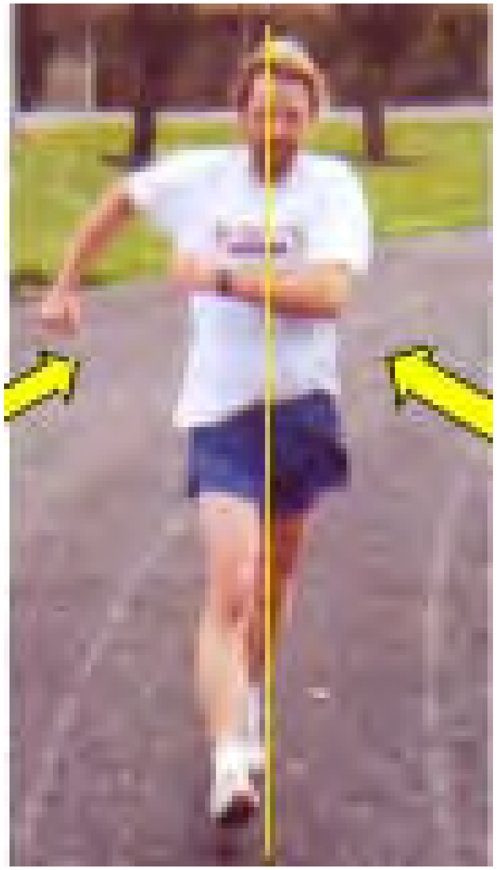
Note the heel-and-toe contact during the widest spread of the stride, and the straight leg as the heel contacts the ground.

Race Walking

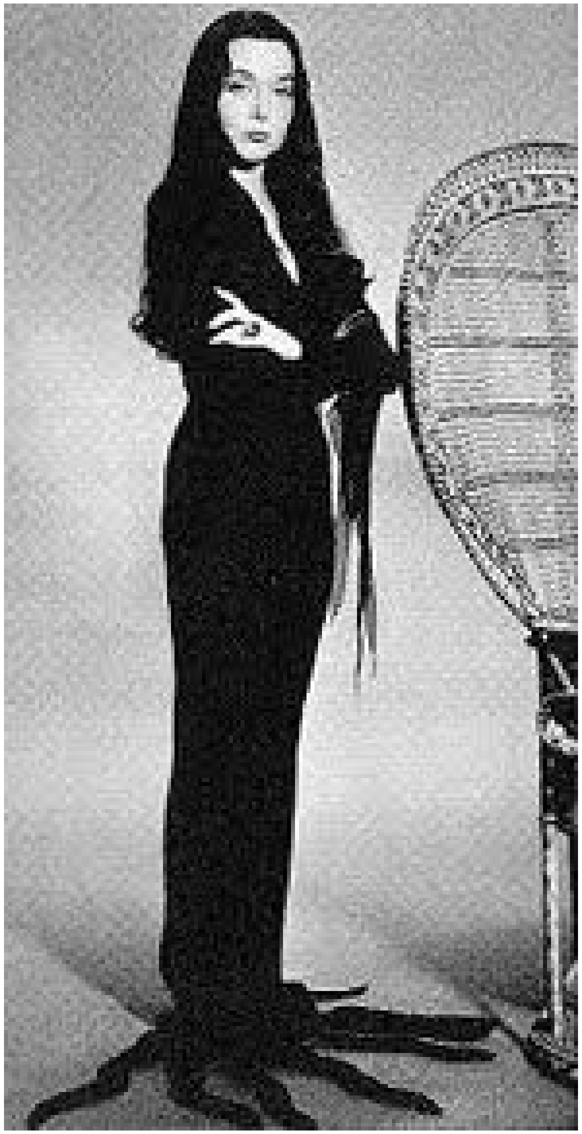
Race Walking vs. Running



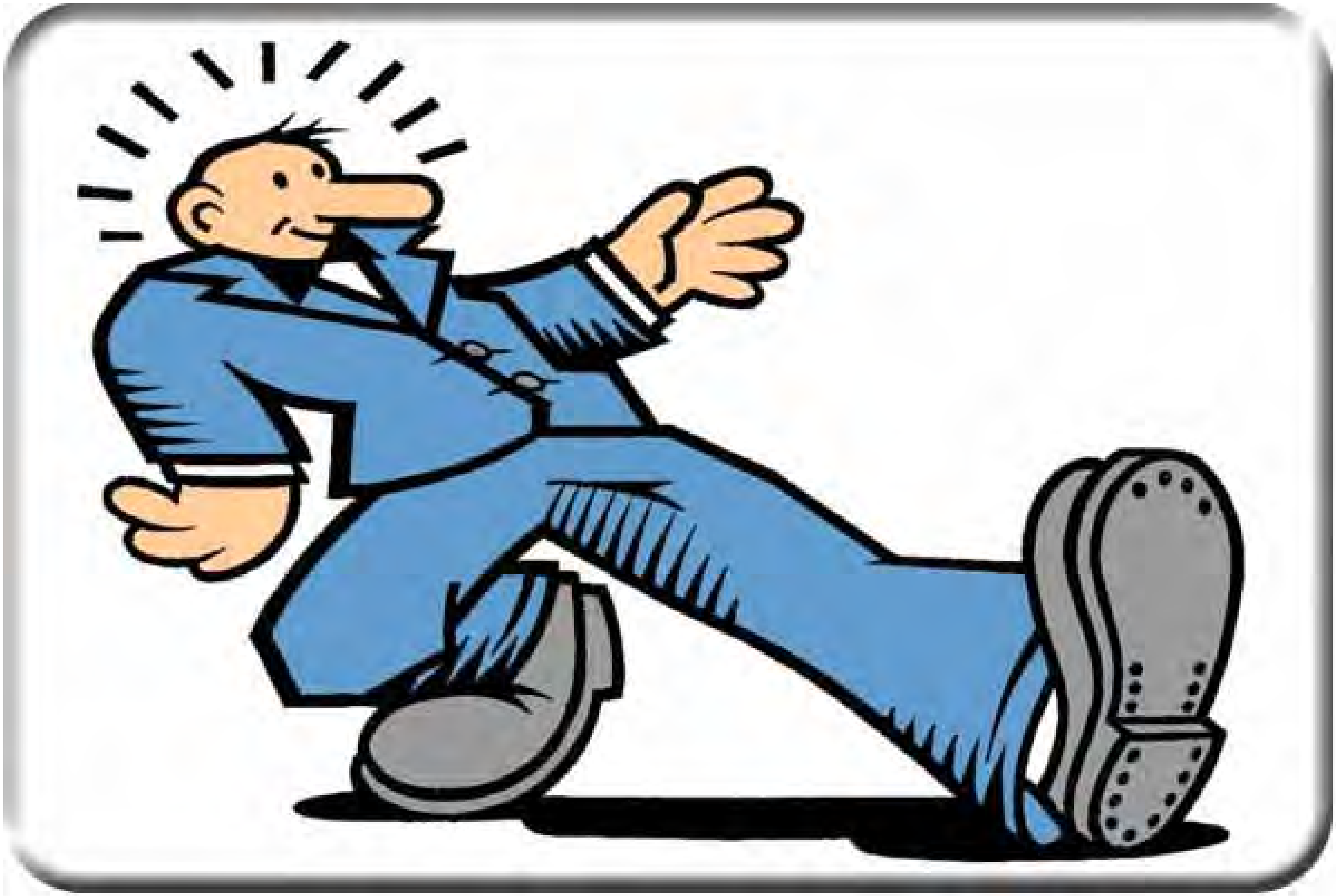
Power Walking



Short Strides



Long Strides



Long Strides

Silly Walking

Silly Walking

John Cleese, Andrew White Visiting Professor at Cornell, 1999-2006

More Brits Walking

The Dynamics of Walking

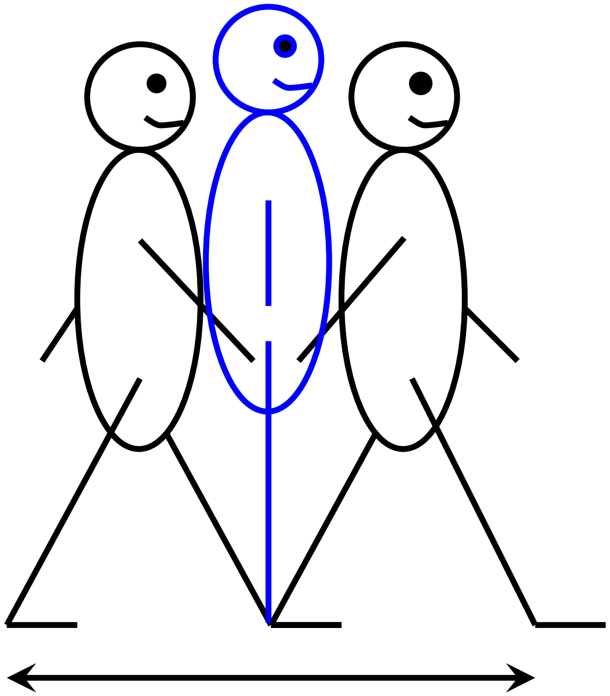


Table 5.6 p. 432.

Parameter	Symbol	Dimensions
velocity	v	L/T
leg length	ℓ	L
mass	m	M
gravity	g	L/T^2
stride	s	L

Goal: a function of the form: $v = f(\ell, m, g, s)$ $[v] =$ L/T

Better Goal: a function of the form: constant $= f(v, \ell, m, g, s)$ [constant] = (none)

We seek a *dimensionless* relation.

The Dynamics of Walking

Better Goal: a function of the form: constant = $f(v, \ell, m, g, s)$ [constant] = (none)

Every term in this function will be dimensionless

Every term will have the general form:

$$[v^a \ell^b m^c g^d s^e] = \text{(none)}$$

$$[v^a \ell^b m^c g^d s^e] = \left(\frac{\text{L}}{\text{T}}\right)^a \text{L}^b \text{M}^c \left(\frac{\text{L}}{\text{T}^2}\right)^d \text{L}^e$$

$$= \text{L}^{a+b+d+e} \text{T}^{-a-2d} \text{M}^c$$

Every term to be dimensionless, each dimension's exponent must be zero.

$$\text{L}: \quad a + b + d + e = 0$$

$$\text{T}: \quad -a - 2d = 0$$

$$\text{M}: \quad c = 0$$

Walking is independent of mass, like a pendulum.

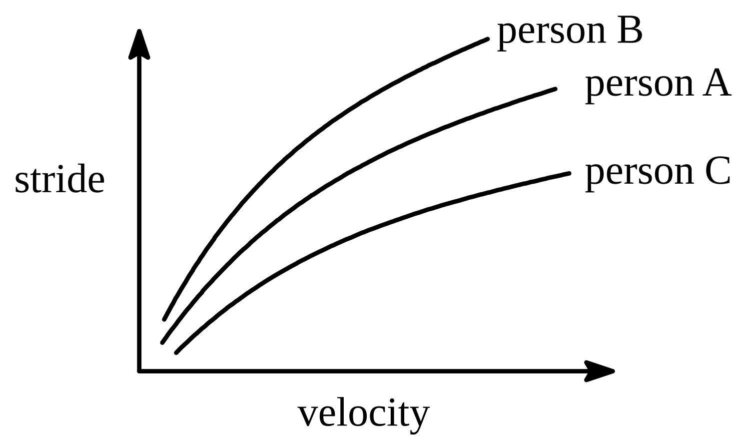
2 equations, 4 unknowns $\Rightarrow$ ∞ solutions. Must set the values of two exponents.

The parameters that correspond to these two exponents will be our *core variables*.

The Dynamics of Walking

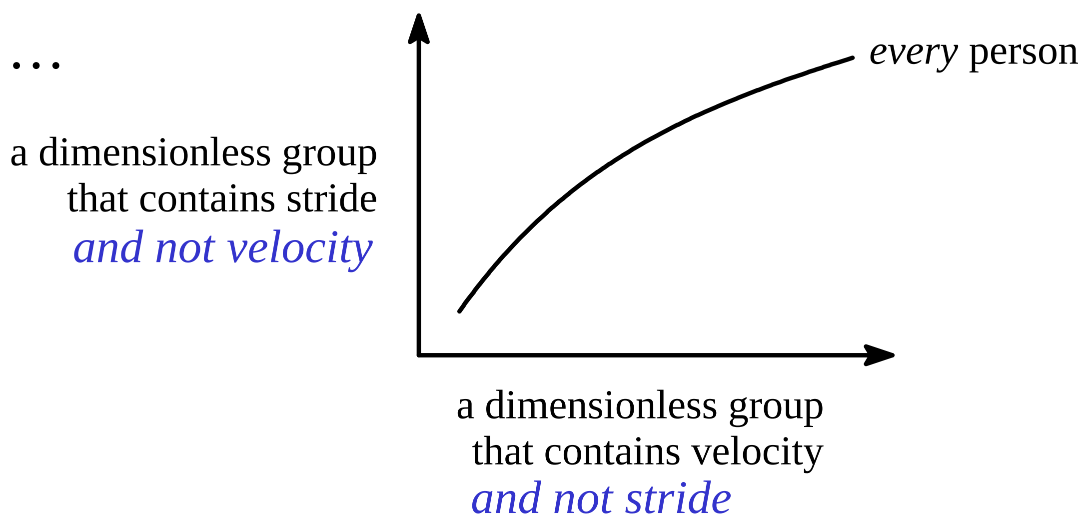
Choosing core variables.

We seek a correlation like …



but there will be a different line for each person.

Better: a universal correlation …



We choose velocity and stride for the core variables of walking.

The Dynamics of Walking

Deriving Dimensionless Groups (of Parameters)

First dimensionless group (of parameters): Π_1 aka 'a pi group.'

$$\Pi_1 = v^a \ell^b m^c g^d s^e$$

Π_1 contains velocity $\Rightarrow$ a = any non-zero number   Choose $a = 1$ for convenience.

Π_1 does not contain stride $\Rightarrow$ $e = 0$

$$\text{L}: \quad a + b + d + e = 0$$

$$\text{T}: \quad -a - 2d = 0$$

Substitute $a = 1$ into Time exponent: $-1 - 2d = 0 \Rightarrow d = -½$

Substitute $a = 1$, $d = -½$, $e = 0$ into Length exponent: $1 + b - ½ + 0 = 0 \Rightarrow b = -½$

$$\Pi_1 = v^1 \ell^{-½} m^0 g^{-½} s^0 = \frac{v}{(\ell g)^{½}}$$

Fractional exponents are inconvenient. We could have chosen $a = 2$, which would yield –

$$\Pi_1 = v^2 \ell^{-1} m^0 g^{-1} s^0 = \frac{v^2}{\ell g}$$

The Dynamics of Walking

Deriving Dimensionless Groups (of Parameters), continued.

Second dimensionless group (of parameters): Π_2

$$\Pi_2 = v^a \ell^b m^c g^d s^e$$

Π_2 contains stride $\Rightarrow$ e = any non-zero number   Choose $e = 1$ for convenience.

Π_2 does not contain velocity $\Rightarrow$ $a = 0$

$$\text{L}: \quad a + b + d + e = 0$$

$$\text{T}: \quad -a - 2d = 0$$

Substitute $a = 0$ into the Time exponent: $-0 - 2d = 0 \quad \Rightarrow \quad d = 0$

Substitute $a = 0$, $d = 0$, $e = 1$ into the Length exponent: $0 + b - 0 + 1 = 0 \quad \Rightarrow \quad b = -1$

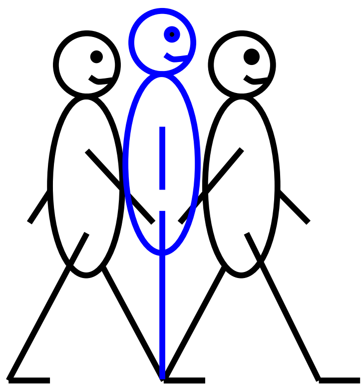
$$\Pi_2 = v^2 \ell^{-1} m^0 g^0 s^1 = \frac{s}{\ell}$$

The Dynamics of Walking

$$\Pi_1 = \frac{v^2}{\ell g}, \qquad \Pi_2 = \frac{s}{\ell}$$

Π_1 appears in dimensionless descriptions of other physical systems, whenever forward motion causes something to be lifted in a gravitational field.

$$\Pi_1 = \frac{v^2}{\ell g} = \propto \frac{\frac{1}{2} m v^2}{m \ell g} \propto \frac{\text{kinetic energy}}{\text{potential energy in a gravitational field}}$$



A body rises between strides.

A ship creates a bow wave.

William Froude first used this dimensionless group to characterize ship efficiencies.

$\Pi_1 = v^2/(\ell g)$ is called the "Froude Number"

"Froude" rhymes with food, not cloud.

The Dynamics of Walking - Summary

$$\text{constant} = f(v, \ell, m, g, s)$$

$\downarrow$ dimensional analysis

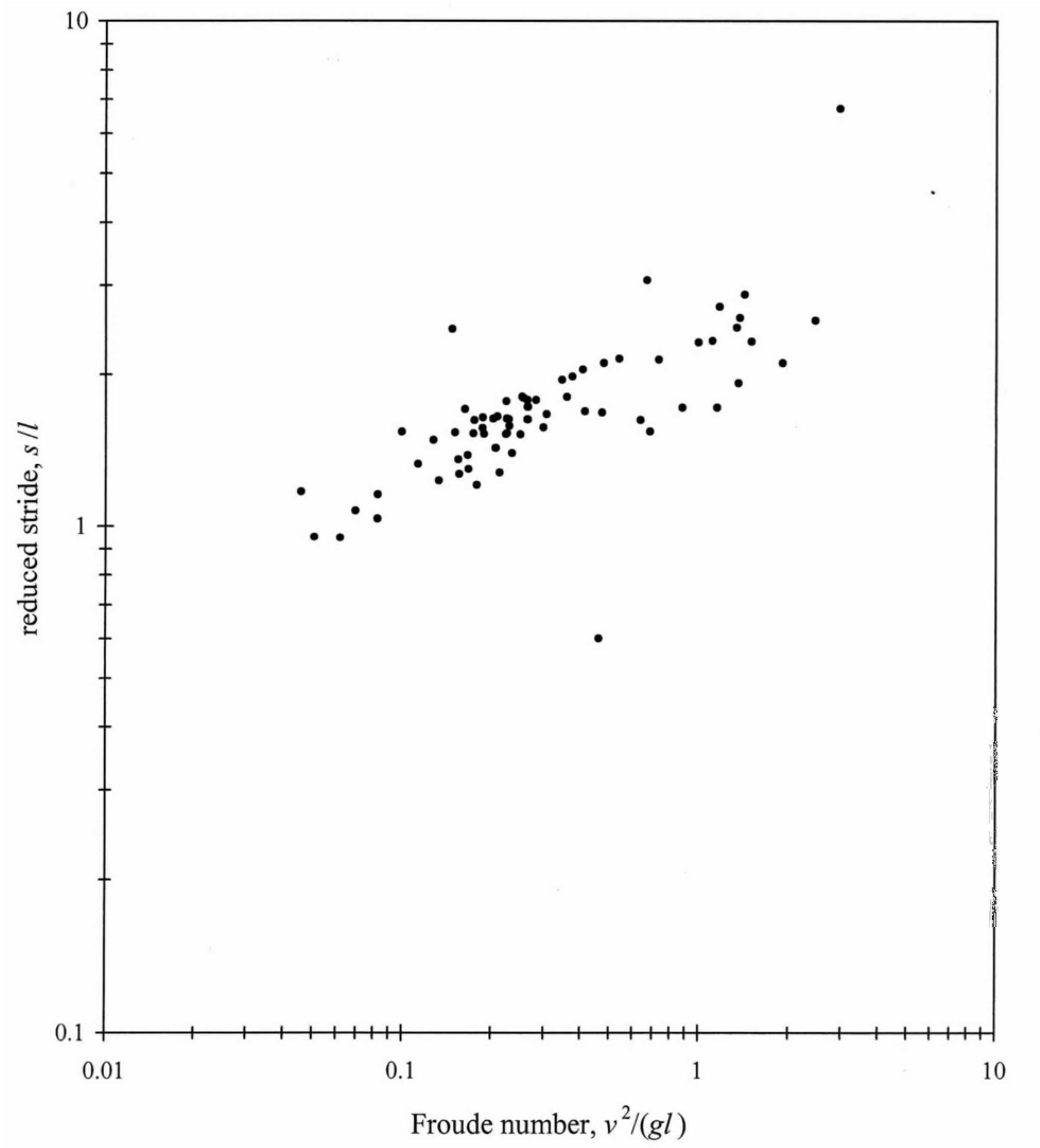
$$\frac{s}{\ell} = f\left(\frac{v^2}{\ell g}\right)$$

$v^2/(\ell g)$ = Fr is called the "Froude Number."

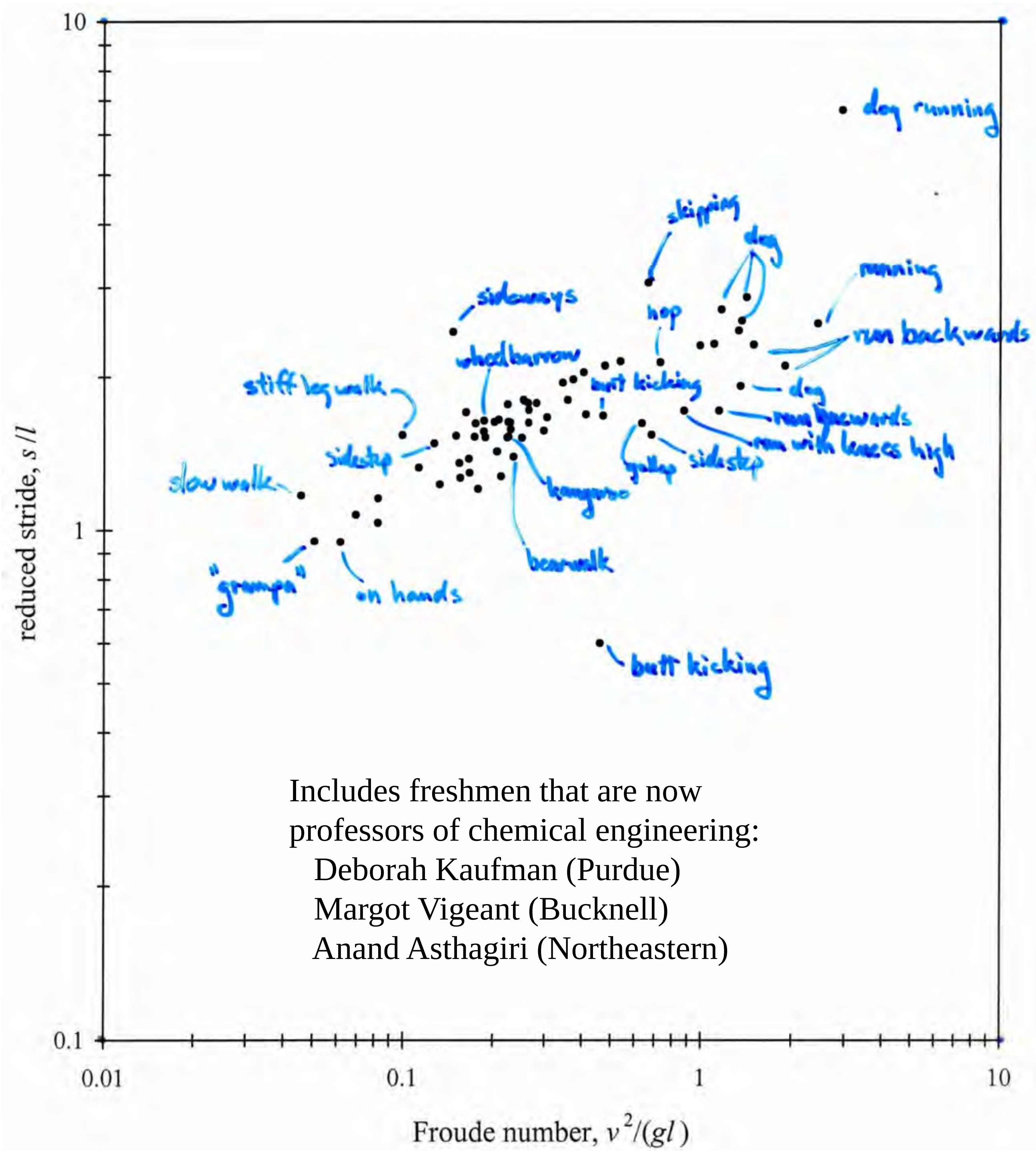
s/ℓ is not named for anyone. s/ℓ is called 'reduced stride.'

To obtain the specific function, we must conduct experiments.

Experiments in Walking – EngrI 112, Fall 1994



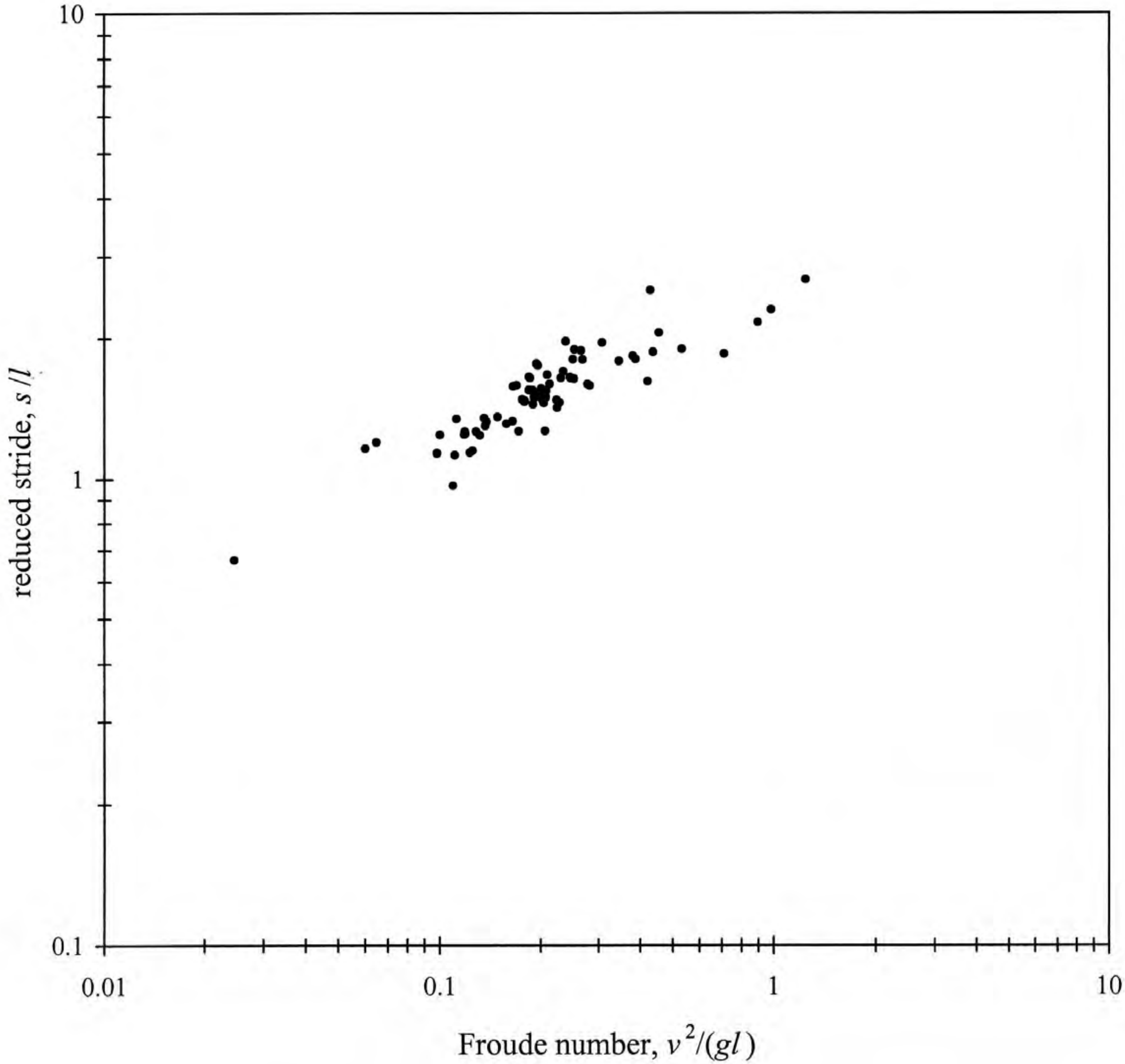
Experiments in Walking – EngrI 112, Fall 1994



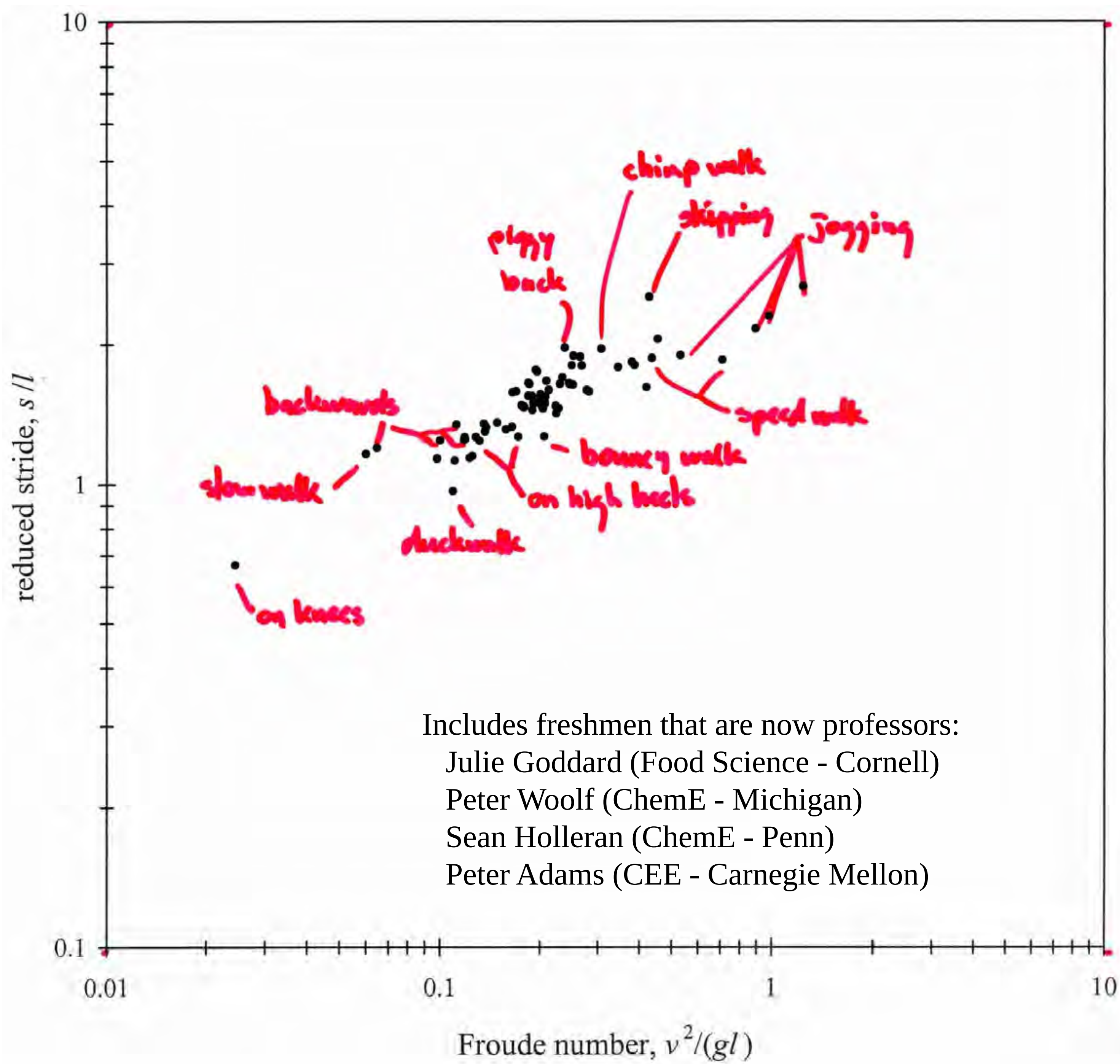
Includes freshmen that are now professors of chemical engineering:

- Deborah Kaufman (Purdue)
- Margot Vigeant (Bucknell)
- Anand Asthagiri (Northeastern)

Experiments in Walking – EngrI 112, Fall 1995



Experiments in Walking – EngrI 112, Fall 1995



Includes freshmen that are now professors:

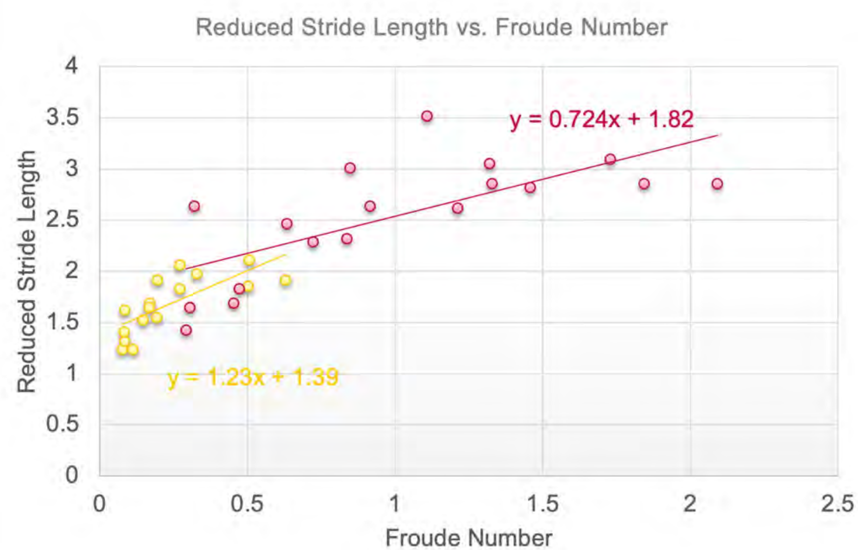
- Julie Goddard (Food Science - Cornell)
- Peter Woolf (ChemE - Michigan)
- Sean Holleran (ChemE - Penn)
- Peter Adams (CEE - Carnegie Mellon)

Dynamic Scaling Experiential Module - 2024

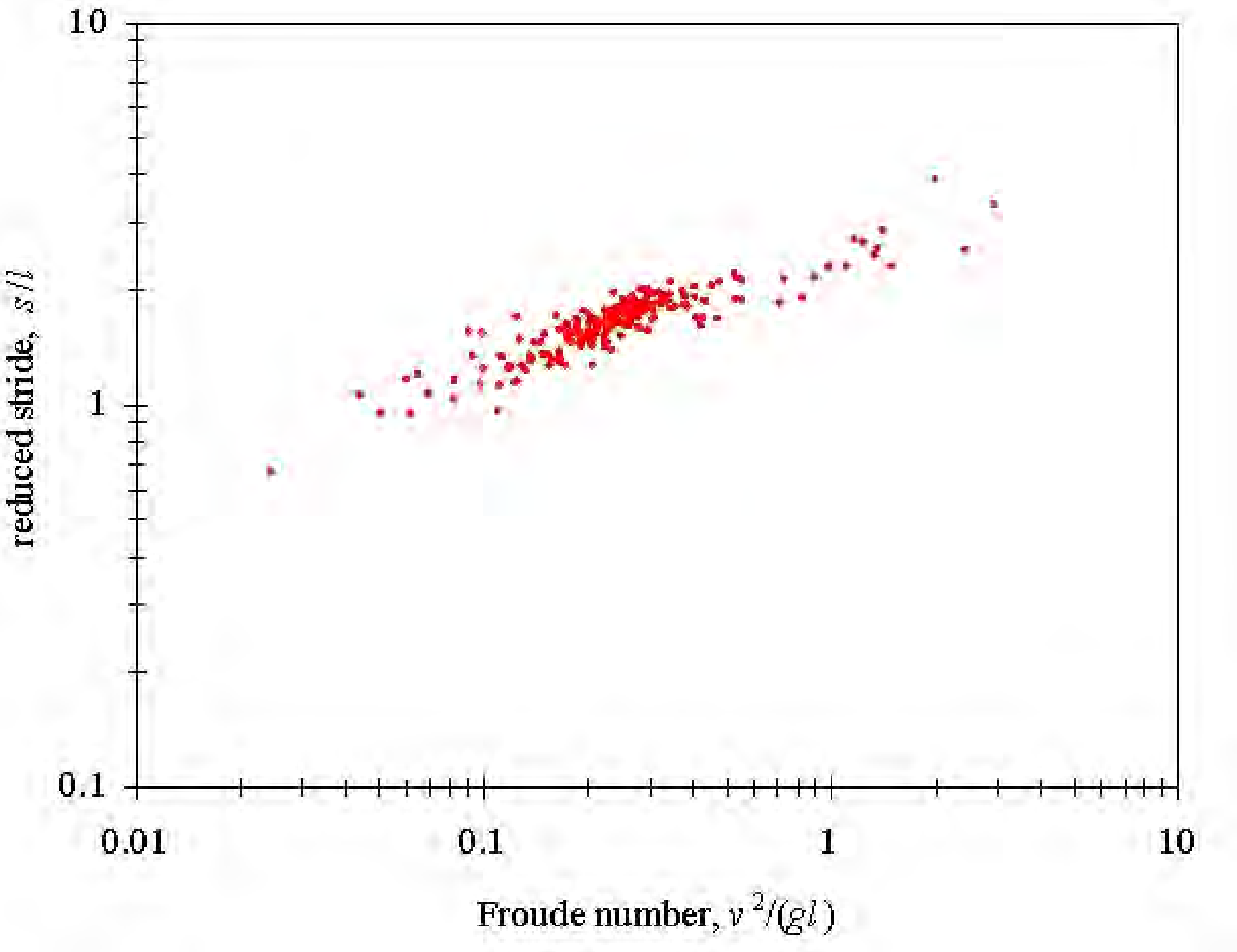
Analysis & Discussion

Equations:
$y=1.23x+1.39$
$y=0.724x+1.82$
Setting the equations equal and solving for x (Froude Number), we found that **Fr = 0.839**.

Compared with the value of 0.5 we found online, we are close enough to the expected value given the available equipment and precision.



Experiments in Walking – Freshmen in EngrI 112, 1994 - 95



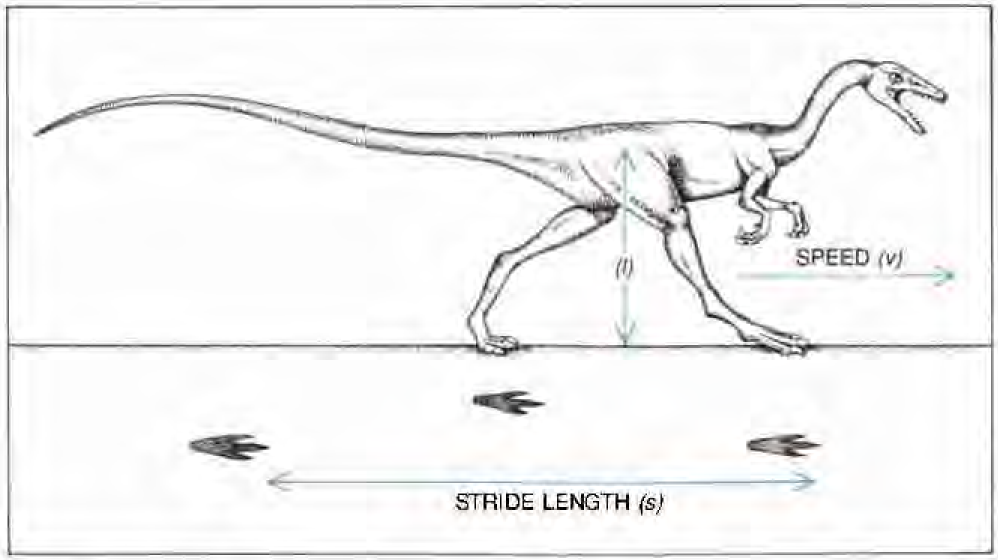
“How Dinosaurs Ran”

R. N. Alexander, Scientific American, April 1991.

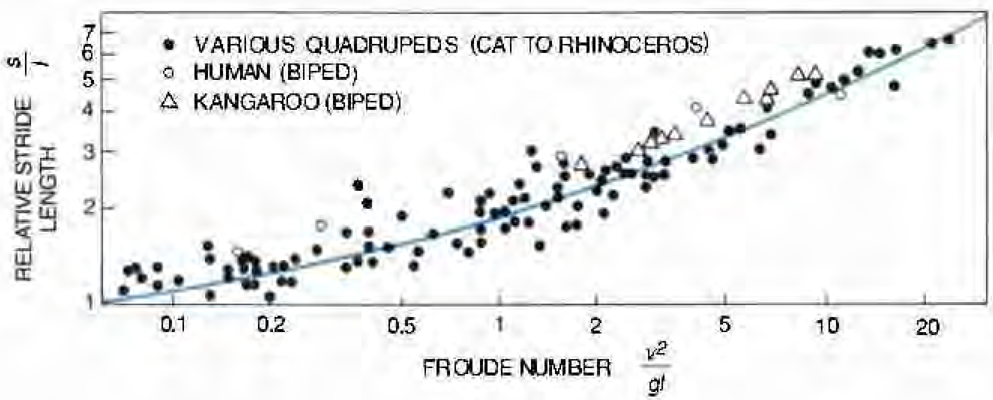
DINOSAUR TRACKS provide a record of stride length and speed. A small, three-toed carnivore may have pursued a larger sauropod along this Texan trail. This pair of footprints was discovered by Ronald T. Bird at Paluxy Creek in 1944.

“How Dinosaurs Ran”

R. N. Alexander, Scientific American, April 1991.



STRIDE LENGTH is the distance between two successive prints from the same foot. ***Compsognathus,*** **a carnivore the size of a contemporary chicken, is depicted here.**

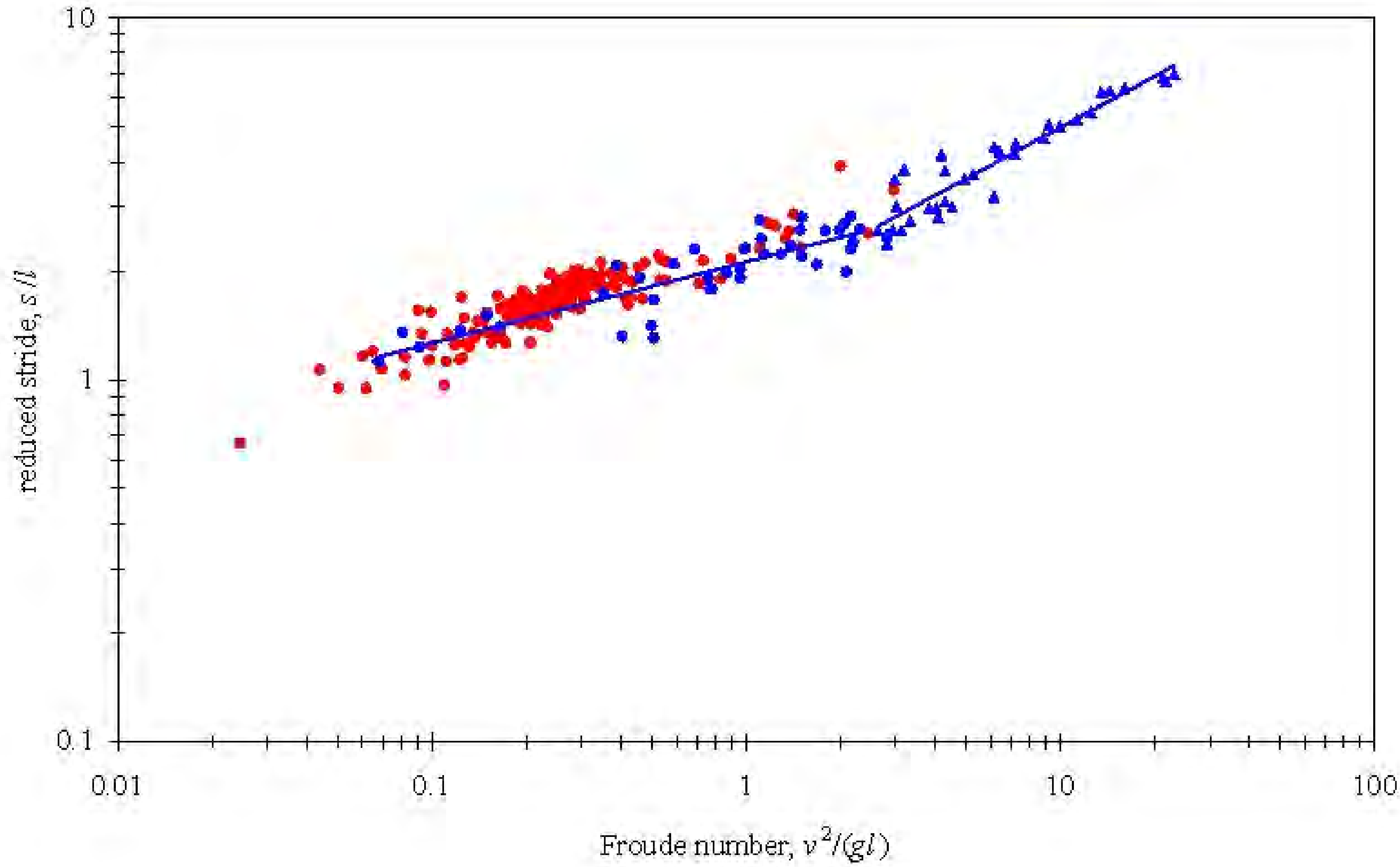


FROUDE NUMBERS for kangaroos, humans and quadrupeds, such as rhinoceroses, are plotted against the animals' relative stride lengths. The numbers increase logarithmically—so that the difference between Froude numbers 0.1 and 20 is clear.

Experiments in Walking

Freshmen in EngrI 112 – Fall 1994-95 (bipeds)

Alexander's data (quadrupeds)



Experiments in Walking

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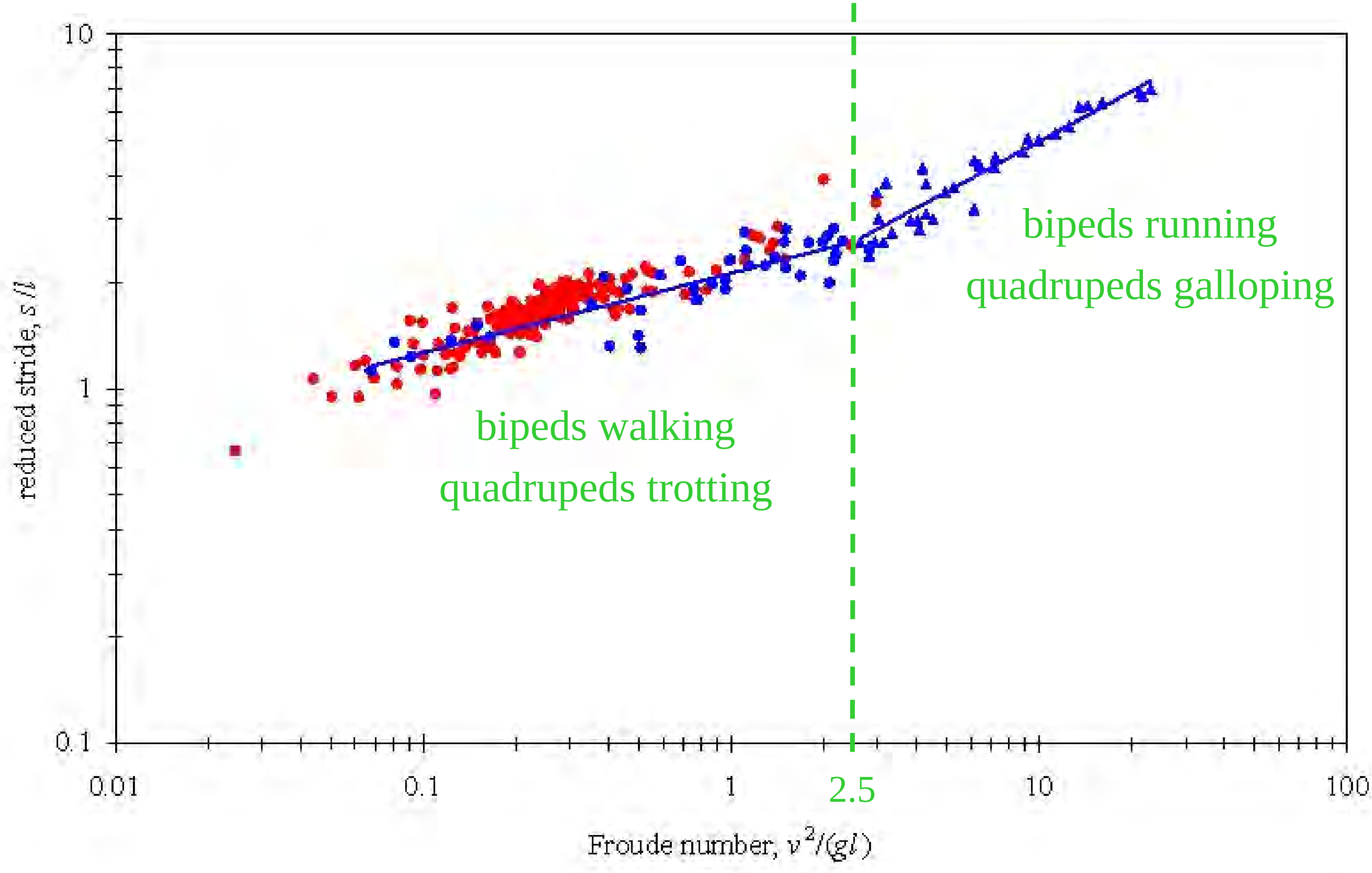

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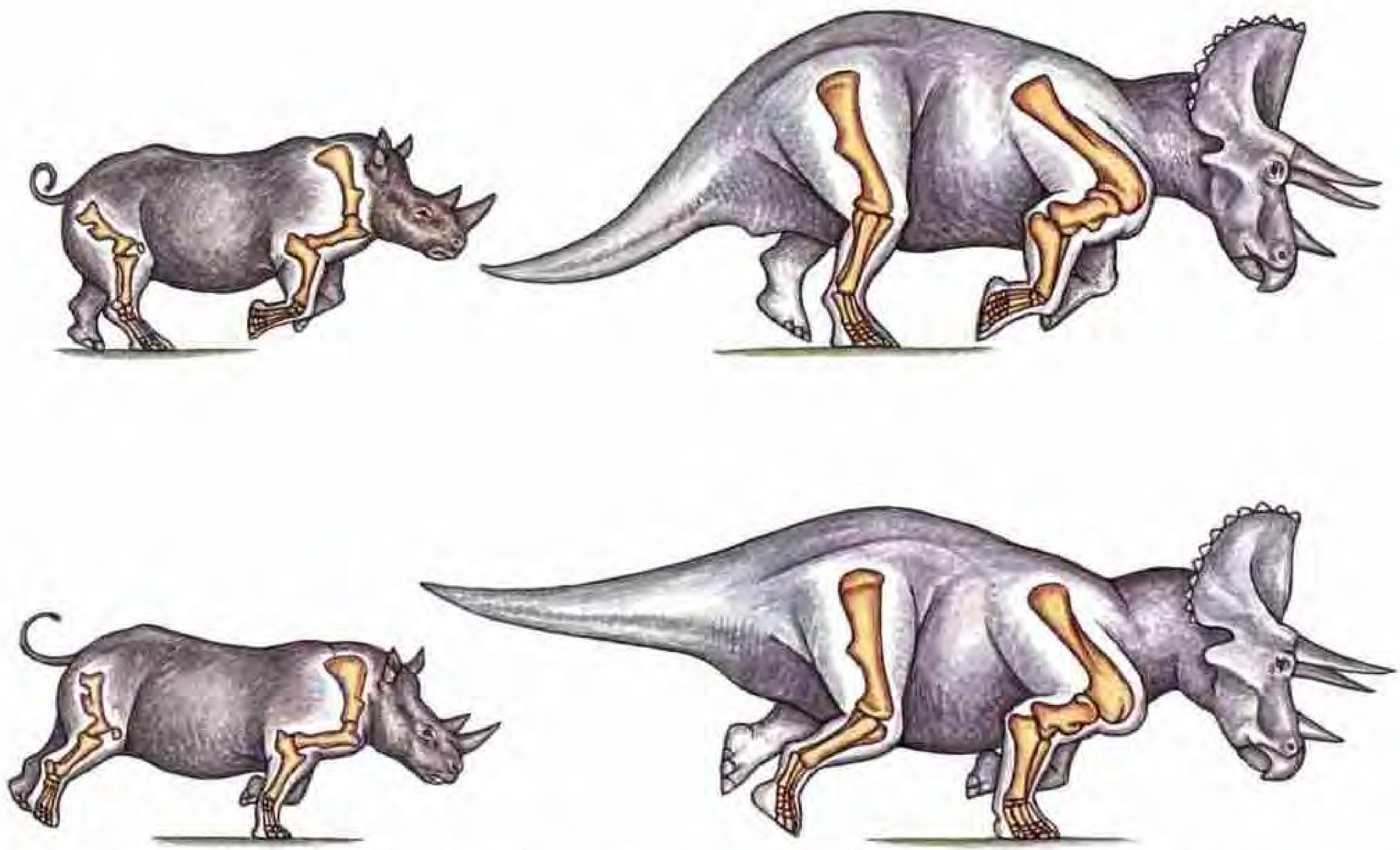


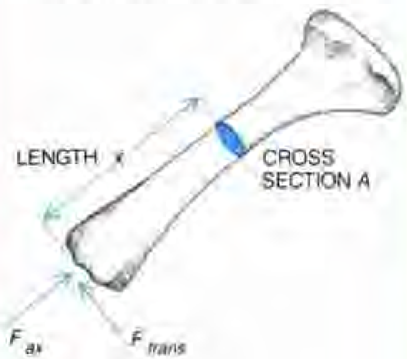
TRICERATOPS may have moved like the White rhinoceros, a modern, horned herbivore. The White rhinoceros here, which was sketched from a film, is galloping at seven meters per second—which is about the speed of a fast human run.

“How Dinosaurs Ran”

R. N. Alexander, Scientific American, April 1991.

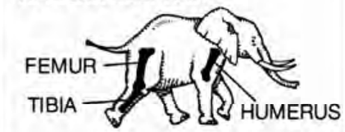
How Forces Act on Bones

When an animal moves, forces are exerted on the ends of its bones and cause stresses in the bone shaft. These forces can be broken down into different components: the axial force (F_{ax}) and the transverse force (F_{trans}). In the



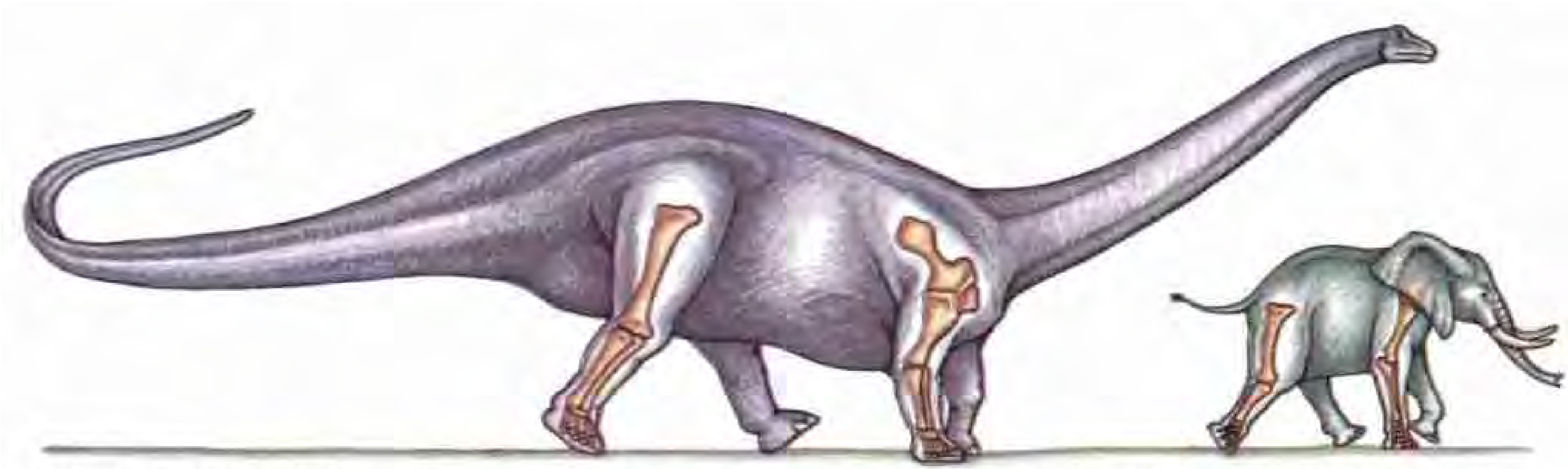
cross section (*area A*) of the diagram, F_{ax} produces a stress, $-F_{ax}/A$ (the stress is negative because it is compressive). This stress is weaker than that stress produced by F_{trans}: $F_{trans}x/Z$. (Z represents the section modulus, an engineering term, which describes properties of the cross section.)

Bone Strength in Large Animals

	BODY MASS (METRIC TONS)	STRENGTH INDICATOR FEMUR	TIBIA	HUMERUS
AFRICAN ELEPHANT 	2.5	7	9	11
AFRICAN BUFFALO 	0.5	22	27	21
APATOSAURUS 	34	9	6	14
DIPLODOCUS 	12–19	3–5	NO DATA	NO DATA
TRICERATOPS 	6–9	13–19	NO DATA	14–22
TYRANNOSAURUS 	7.5	9	NO DATA	NO DATA

“How Dinosaurs Ran”

R. N. Alexander, Scientific American, April 1991.



ELEPHANT AND *APATOSAURUS*, a plant-eating dinosaur, both have approximately the same strength indicators for their leg bones. This similarity means that the extinct giant probably ran much the same way our contemporary does.

Recommended reading in dynamic scaling.