EngrD 2190 – Lecture 30

Concept: Dimensional Analysis and Dynamic Scaling

Context: Universal Scaling of the Terminal Velocity of a Sphere

Defining Question: What is *the* most famous dimensionless Group?

Read Chapter 5 pp. 447-454

Dynamic Similarity – How to Design a Model

Homework

- Homework 8 due today at noon.

  Write team code and names of all *contributing* team members on all solutions. Indicate this week's Team Coordinator.

  Submit *after* lecture or deliver to the EngrD 2190 mailbox in a cabinet in the hallway outside 116 Olin Hall (ChemE Business Office). **Not to my mailbox.**

- Homework 9 (the last homework) due Friday 11/21.

  5.12 deriving Pi groups

  5.26 & 5.31 analysis with dimensionless correlations.

Homework is your chief means of assessing your command of the material.

Homework 7 Excellence – Exercise 4.34 – Team 18

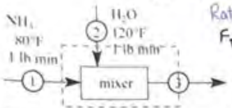
Amber Belk (Coordinator), Audrey Fu, Dolly Hritz

Team 18

Homework 7

4.34 A)

4.34 & 4.35 enthalpy-concentration phase diagram for ammonia+water mixtures at 300 psia (20.4 atm).



Rate Mass In = Rate Mass Out

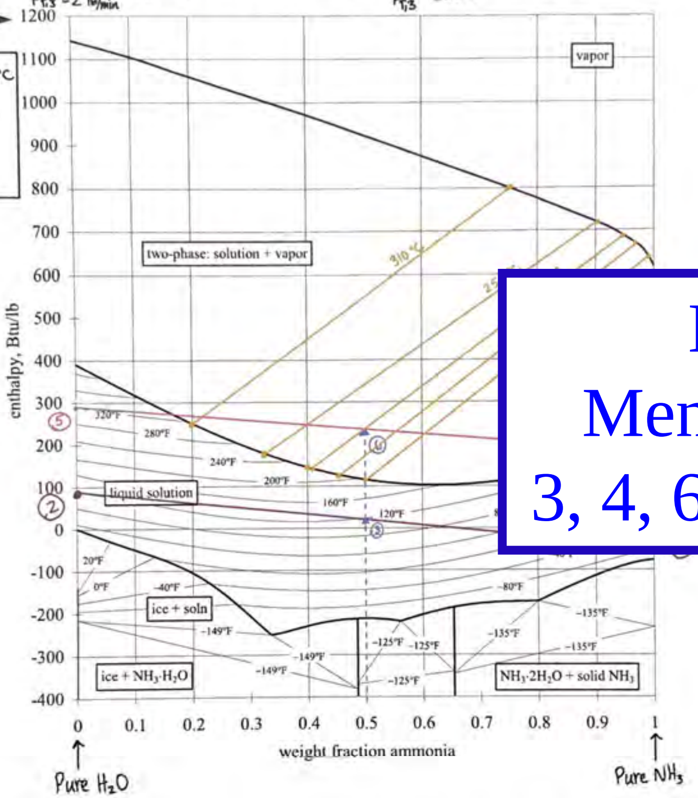
$F_{T,3} = F_{T,1} + F_{T,2} = 1\frac{lb}{min} + 1\frac{lb}{min}$

$F_{T,3} = 2$ lb/min

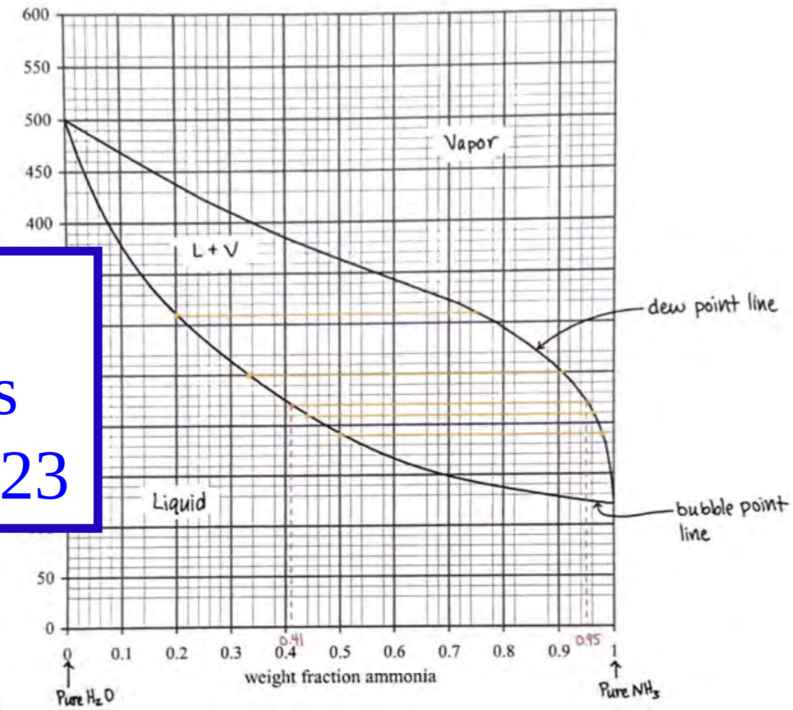
H_2O mass fraction $= \frac{F_{T,2}}{F_{T,3}} = \frac{1 \text{ lb/min}}{2 \text{ lb/min}} = 0.5$

NH_3 mass fraction $= 1 - 0.5 = 0.5$

Stream 3 Temperature: 120°C
- 50 mass% H_2O
  - ↳ 100% liquid
- 50 mass % HN_3
  - ↳ 100% liquid

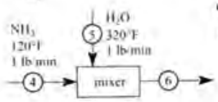


4.34 & 4.35 temperature-composition phase diagram for ammonia+water mixtures at 300 psia (20.4 atm).



Honorable Mention to Teams 3, 4, 6, 10, 19, and 23

B)



Just like in part A, $F_{T,6} = 2$ lb/min with 50 mass % H_2O and 50 mass percent NH_3.

Stream 6 Temperature: 220°C

50 mass % H_2O

50 mass % NH_3

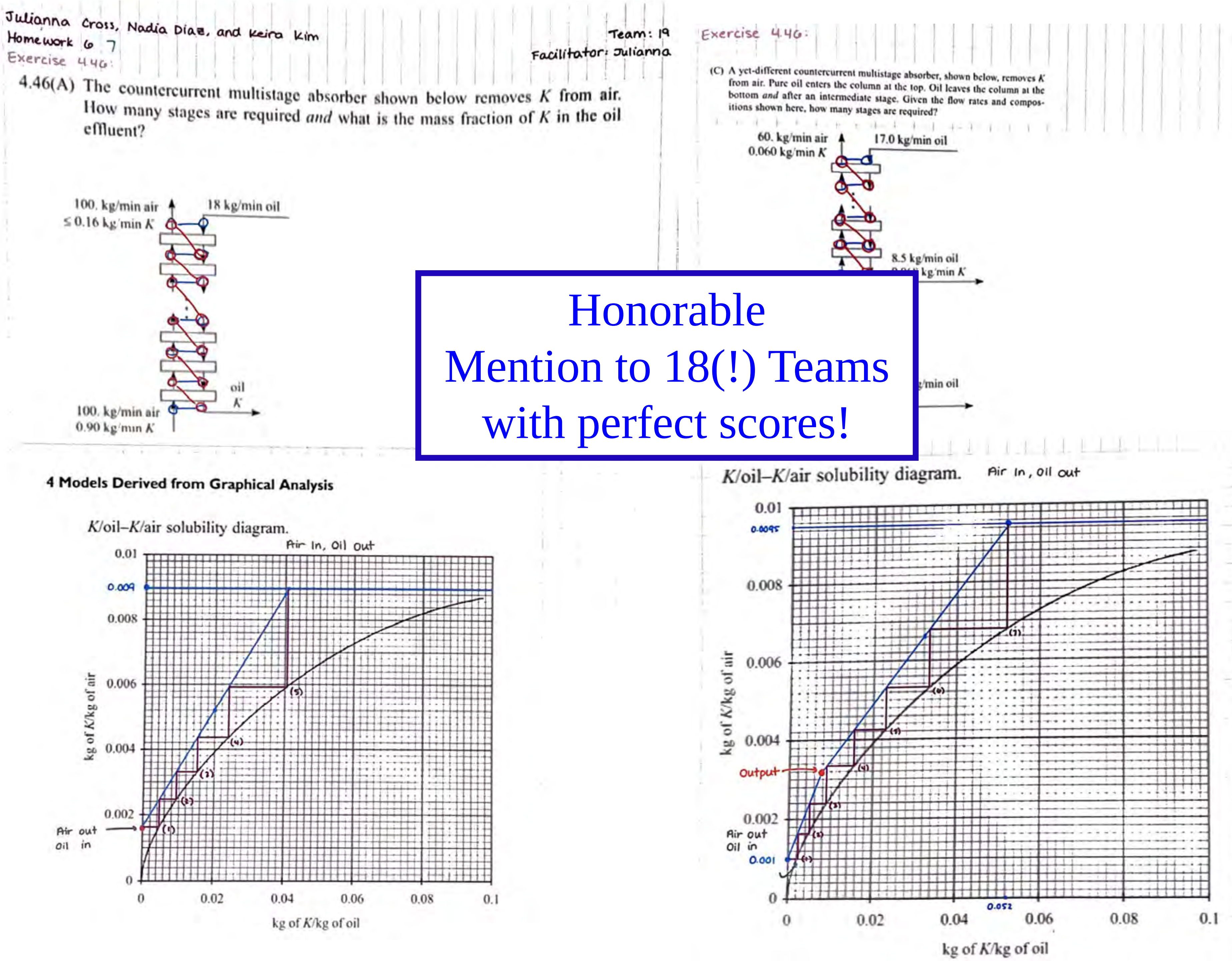
(see next page for work)

Vapor	Liquid
95 % NH_3	41 % NH_3
5 % H_2O	59% H_2O

Overall Takeaways:

- Temperature and enthalpy composition graphs can be used in tandem with each other to find the temperature, phases, and composition of mixed streams.
- You cannot use a temperature composition graph when mixing as temperature is not conserved.
- Always double check the temperatures when using an enthalpy composition graph to make sure you are on the correct temperature.

Homework 7 Excellence – Exercise 4.46 – Team 19



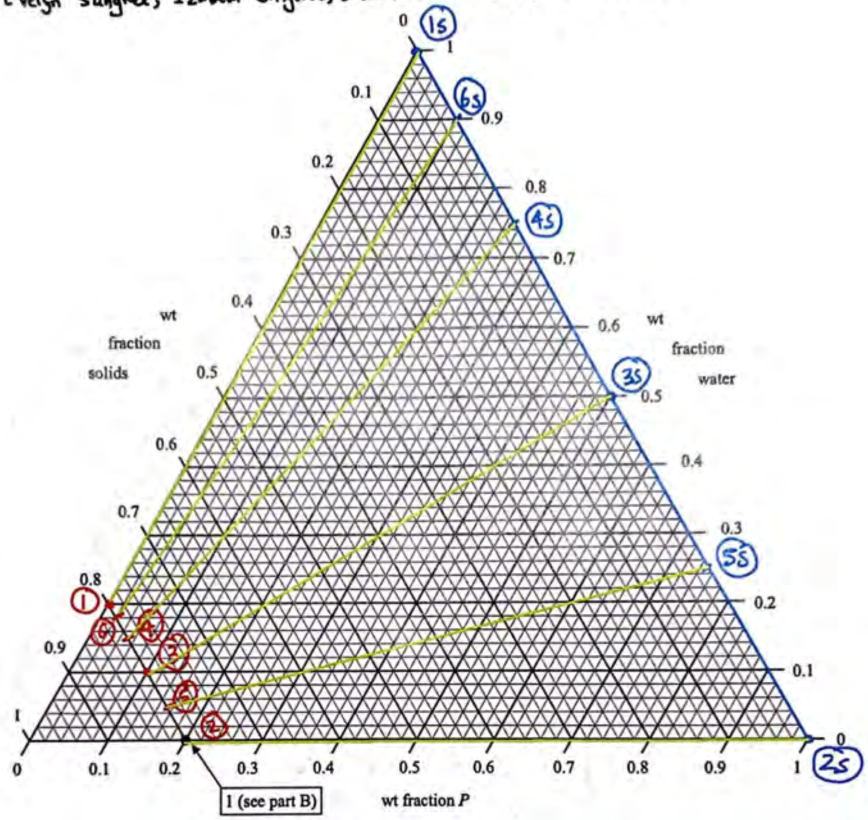
Honorable Mention to 18(!) Teams with perfect scores!

Homework 7 Excellence – Exercise 4.101 – Team 25

Homework 7 – 4.101

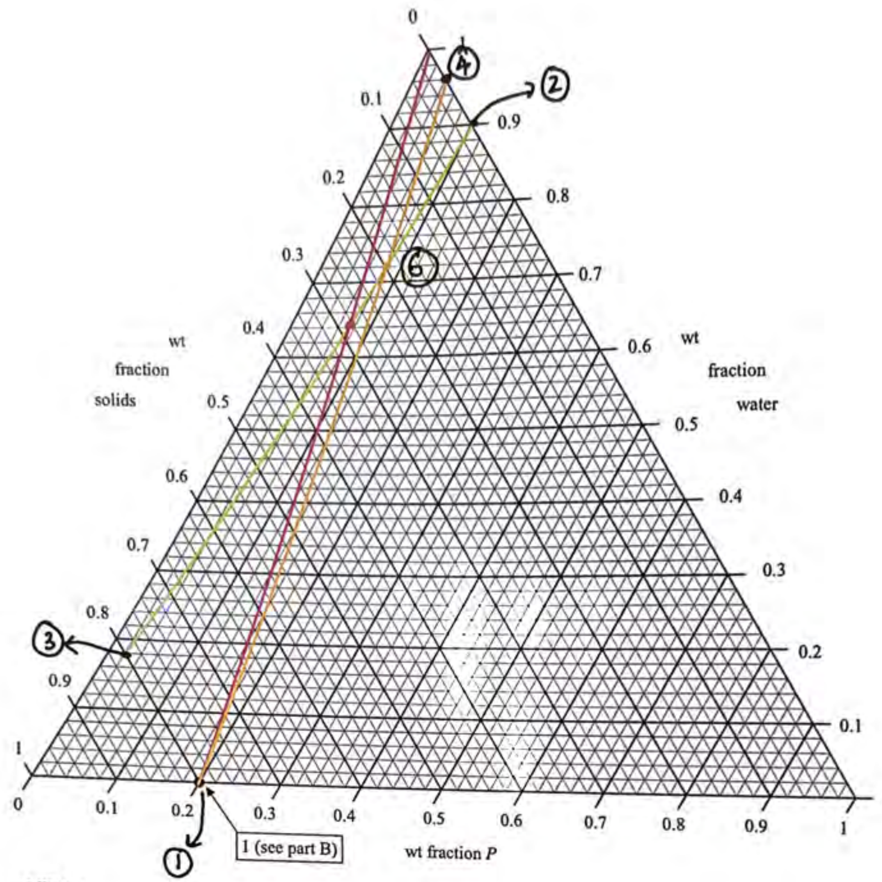
Team 25 – Evelyn Sangree, Izabela Grigorov, Lauren Hin ← weekly coordinator

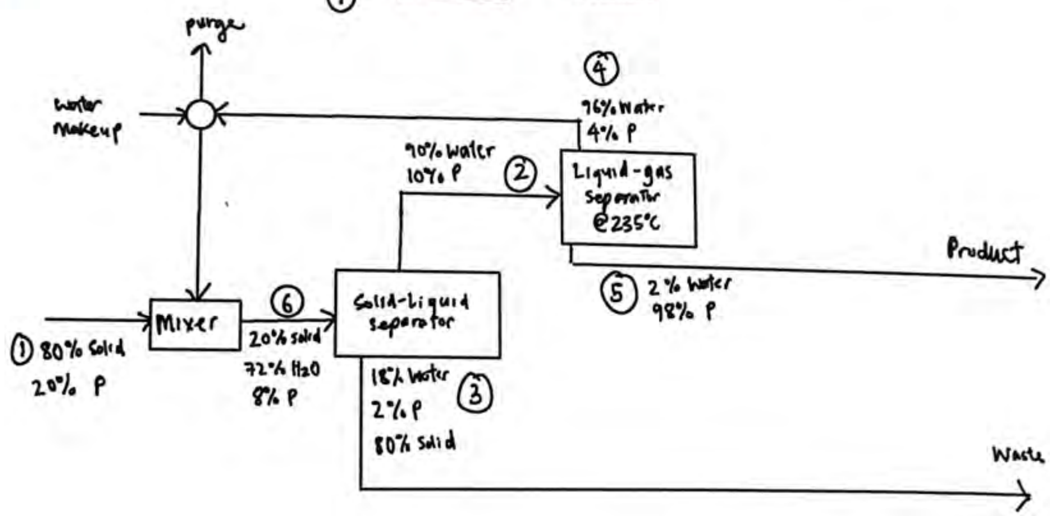
A)



4.101

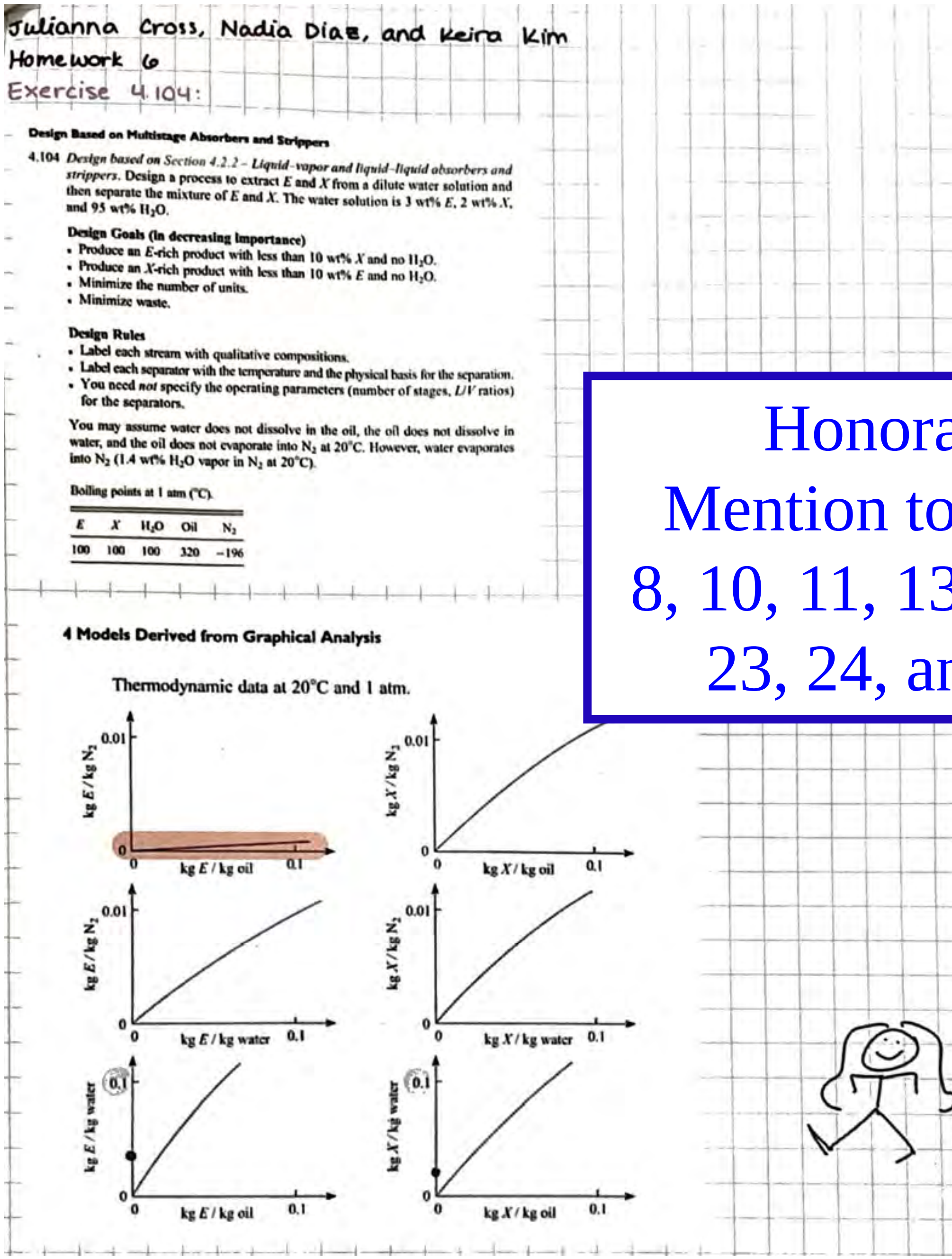
B)

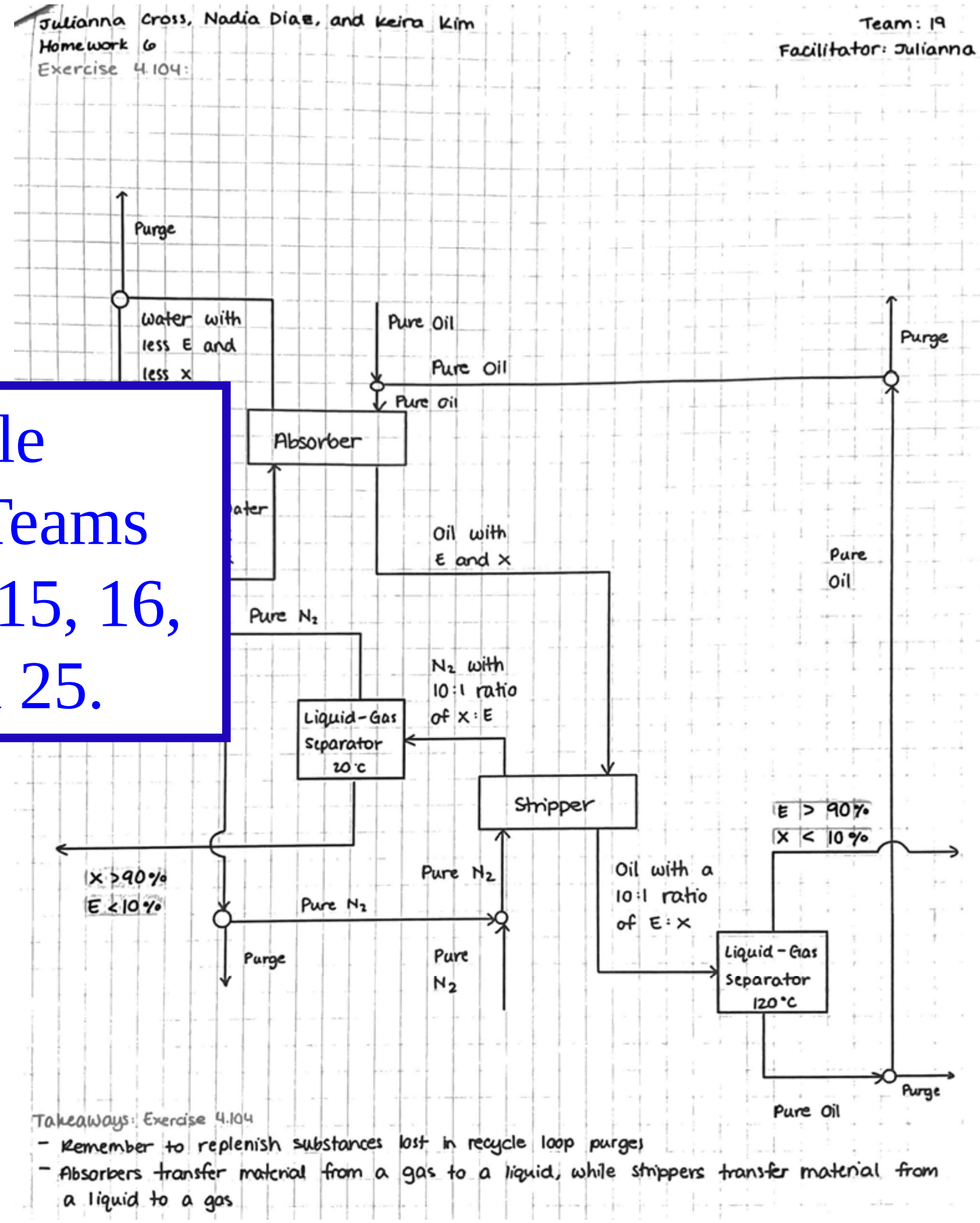




Honorable Mention to Teams 11, 15, 16, 18, 19, 23, and 26.

Homework 7 Excellence – Exercise 4.104 – Team 19





Honorable Mention to Teams 8, 10, 11, 13, 15, 16, 23, 24, and 25.

Recap

Motivation: to predict the behavior/performance of an inconvenient system.

Inconvenient system is too large or too small.

Inconvenient system is too fast or too slow.

Inconvenient system is too hot or too cold.

Plan: study a convenient model of the inconvenient system.

How to design a dynamically similar convenient model?

Dimensional Analysis and Dynamic Scaling!

Recap of Dimensional Analysis of Pendulums and Walking

Table 5.5. The parameters of a pendulum

physical quantity	symbol	dimensions
period of oscillation	t_p	T
length of pendulum	ℓ	L
mass of pendulum	m	M
gravitational acceleration	g	L/T^2
amplitude	α	(none)

One dimensionless group (of parameters): $\Pi = \dfrac{g t_{\text{period}}^2}{\ell}$ $\qquad [\Pi] = \dfrac{\frac{\text{L}}{\text{T}^2}\text{T}^2}{\text{L}} = \text{(none)}$

The dimensionless group reveals how parameters scale:

If ℓ is increased $\times 100$, t_{period} increases by $\times 100^{1/2} = \times 10$.

Must conduct experiments to obtain universal relation:

$$\frac{g t_{\text{period}}^2}{\ell} = 4\pi^2$$

Recap of Dimensional Analysis of Pendulums and Walking, cont'd

Table 5.6 The parameters of walking

parameter	symbol	dimensions
velocity	v	L/T
leg length	ℓ	L
mass	m	M
gravity	g	L/T^2
stride length	s	L

Two dimensionless groups: $\Pi_1 = \dfrac{v^2}{g\ell}$ $\Pi_2 = \dfrac{s}{\ell}$

$\dfrac{v^2}{g\ell}$ is called a Dimensionless Group (of Parameters)

aka a Dimensionless Number (The Froude Number) ← 'Number' is a misnomer

aka a Π Group ← upper case

The magnitude of a dimensionless group describes the phenomenon.

Froude Number < 2.5 $\Rightarrow$ bipeds walking or quadrupeds trotting

Froude Number > 2.5 $\Rightarrow$ bipeds running or quadrupeds galloping

Dimensional Analysis
and Dynamic Scaling

A Universal Correlation
for the Terminal Velocity of a Sphere

The Terminal Velocity of a Sphere - Applications

Solid-liquid separations

Muddy water - why does sand settle quickly but silt settles slowly? How slowly?

Solid-vapor separations

Dust settling in air - how long does volcanic ash remain in the atmosphere?

Vapor-liquid systems

How fast do air bubbles rise in a fermenter?

How fast do vapor bubbles rise through the liquid on an equilibrium stage in a distillation column?

Liquid droplets in air: mist and fog? Spray painting? Virus spreading?

Liquid-liquid systems

Oil-water absorbers and strippers - bouyancy effects?

The Method of Dimensional Analysis

recap from previous lecture:

1. List parameters — will always be given in this course.
2. Find dimensions of each parameter
3. Write equation for generic Π group
4. Choose core variables
5. Derive Π groups
6. Measure Data
7. Plot universal correlation

Dimensional Analysis of a Falling Sphere

1. List parameters
2. Find dimensions of each parameter

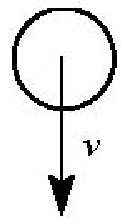


Figure 5.12. A solid sphere falling through a fluid.

Table 5.7. The parameters of a sphere moving through a fluid

	parameter	symbol	dimensions
dynamics	terminal velocity	v	L/T
the sphere	sphere diameter	d	L
	buoyancy	$\rho_{sphere}-\rho_{fluid}$	M/L^3
the fluid	fluid viscosity	μ	M/LT
	fluid density	ρ_{fluid}	M/L^3
physical constant	gravitational acceleration	g	L/T^2
	will be given	will *usually* be given	must derive

Why the density of the fluid is relevant

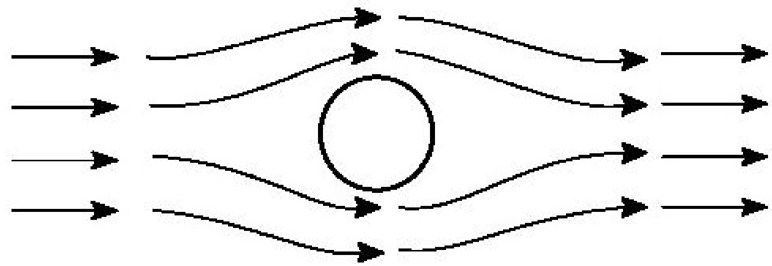
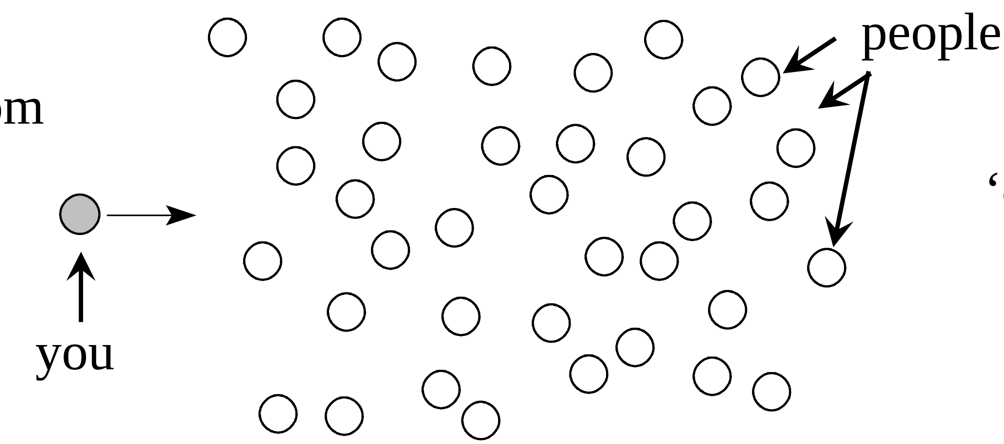
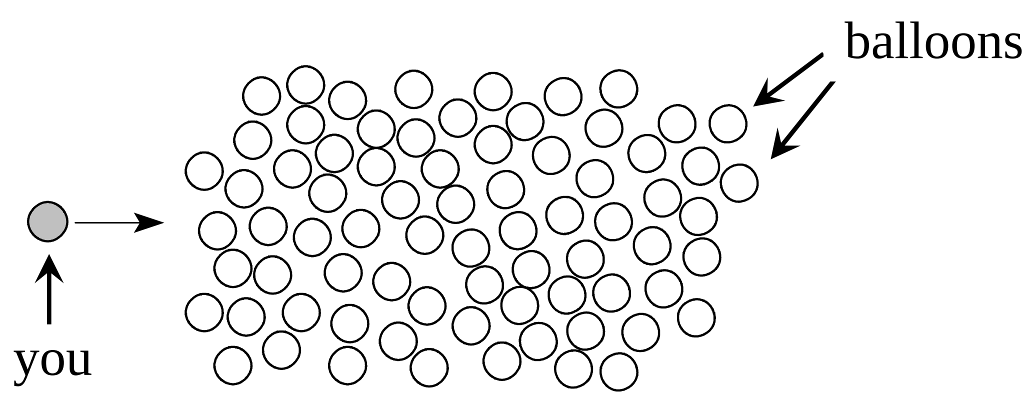


Figure 5.13. Fluid moving around a solid sphere.

Consider running
in a straight line
through a crowded room



'density' = people/area



Dimensional Analysis of a Falling Sphere

3. Write an equation
   for generic Π group

Table 5.7. The parameters of a sphere moving through a fluid

parameter	symbol	dimensions
terminal velocity	v	L/T
sphere diameter	d	L
buoyancy	$\rho_{sphere}-\rho_{fluid}$	M/L^3
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3
gravitational acceleration	g	L/T^2

$$\Pi = v^a d^b (\rho_{sphere}-\rho_{fluid})^c \mu^d \rho_{fluid}^e g^f \qquad (5.46)$$

$$[\Pi] = \left(\frac{L}{T}\right)^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d \left(\frac{M}{L^3}\right)^e \left(\frac{L}{T^2}\right)^f \qquad (5.47)$$

$$[\Pi] = M^{c+d+e} T^{-a-d-2f} L^{a+b-3c-d-3e+f} \qquad (5.48)$$

For Π to be dimensionless, each dimension's exponent must be zero.

$$\text{Mass, M}: \; c+d+e = 0 \qquad (5.49)$$

$$\text{Time, T}: \; -a-d-2f = 0 \qquad (5.50)$$

$$\text{Length, L}: \; a+b-3c-d-3e+f = 0 \qquad (5.51)$$

(6 parameters) – (3 dimensions) = 3 core variables

Dimensional Analysis of a Falling Sphere

4. Choose core variables

Table 5.7. The parameters of a sphere moving through a fluid

parameter	symbol	dimensions
terminal velocity	v	L/T
sphere diameter	d	L
buoyancy	$\rho_{sphere}-\rho_{fluid}$	M/L^3
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3
gravitational acceleration	g	L/T^2

Two Rules for Core Variables

1. The set of core variables must represent all dimensions. Easy to check.

Not velocity, sphere diameter, and gravity (dimension M is omitted).

2. The core variables must not form a dimensionless group. Not easy to check.

Not fluid density and buoyancy.

Not velocity, sphere diameter, and gravity:
$v^2/(dg)$ is a Froude Number; $[v^2/(dg)]$ = (none)

It is often easier to derive the Π groups. If a problem arises, this rule was likely violated.

Dimensional Analysis of a Falling Sphere

4. Choose core variables, cont'd

Table 5.7. The parameters of a sphere moving through a fluid

parameter	symbol	dimensions
terminal velocity	v	L/T
sphere diameter	d	L
buoyancy	$\rho_{sphere}-\rho_{fluid}$	M/L^3
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3
gravitational acceleration	g	L/T^2

Logical choices for the core variables:

What do we want to predict? velocity

What do we want to vary in our experiments? sphere diameter and fluid viscosity

But we are not the first to study spheres falling through fluids.
Use traditional core variables to relate our results to previous results.

Traditional core variables: buoyancy, gravity, and fluid viscosity.

Dimensional Analysis of a Falling Sphere

5. Derive Π groups

Table 5.7. The parameters of a sphere moving through a fluid

parameter	symbol	dimensions
terminal velocity	v	L/T
sphere diameter	d	L
buoyancy	$\rho_{sphere}-\rho_{fluid}$	M/L^3
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3
gravitational acceleration	g	L/T^2

$$\Pi = v^a d^b (\rho_{sphere} - \rho_{fluid})^c \mu^d (\rho_{fluid})^e g^f$$

Use the core variables - buoyancy, gravity, and fluid viscosity - to set the exponents.

	buoyancy	gravity	fluid viscosity	
Π_1	$c = 1$	$f = 0$	$d = 0$	$\Rightarrow \Pi_1 = \dfrac{\rho_{sphere} - \rho_{fluid}}{\rho_{fluid}}$
Π_2	$c = 0$	$f = \cancel{1}\ -1$	$d = 0$	$\Rightarrow \Pi_2 = \dfrac{v^2}{dg}$ the Froude Number
Π_3	$c = 0$	$f = 0$	$d = \cancel{1}\ -1$	$\Rightarrow \Pi_3 = \dfrac{\rho_{fluid} v d}{\mu}$ the Reynolds Number

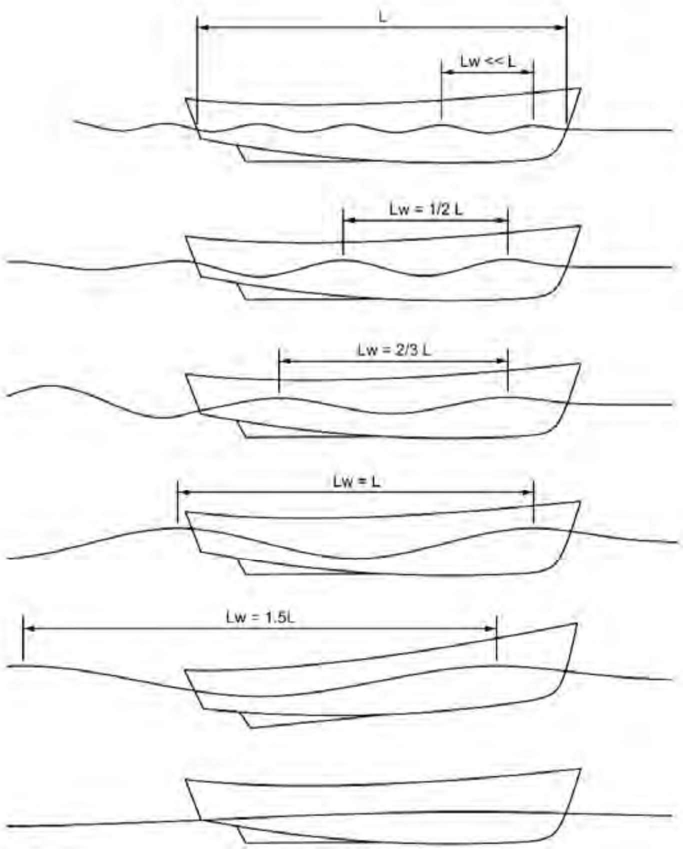
Core variables buoyancy, gravity, and fluid viscosity yield key dimensionless groups!

Core variables buoyancy, gravity, and fluid viscosity yield key dimensionless groups!

$$\Pi_2 = \frac{v^2}{dg} \quad \text{the Froude Number}$$

$$\Pi_2 = \frac{v^2}{dg} \propto \frac{\tfrac{1}{2}mv^2}{mgd} = \frac{\text{kinetic energy}}{\text{potential energy in a gravitational field}}$$

$$\Pi_2 = \frac{v^2}{dg} = \frac{m\frac{v^2}{d}}{mg} = \frac{\text{inertial force}}{\text{gravitational force}}$$



WAVELENGTH << SHIP LENGTH
SMALL WAVES SEEN ALONG SIDE OF HULL
MINIMAL WAVEMAKING RESISTANCE

Fn = 0.28
WAVELENGTH = 1/2 SHIP LENGTH
BOW WAVE SYSTEM HAS A CREST AT THE STERN
CREST PARTIALLY CANCELS STERN WAVE SYSTEM.
REDUCING WAVEMAKING

Fn = 0.33
WAVELENGTH = 2/3 SHIP LENGTH
BOW WAVE CREATES A TROUGH AT THE STERN,
WHICH ADDS TO THE STERN WAVE SYSTEM,
INCREASING WAVEMAKING

Fn = 0.40 or SPEED LENGTH RATIO = 1.34
WAVELENGTH = SHIP LENGTH
"HULL SPEED" LAST EFFICIENT SPEED
FOR DISPLACEMENT SHIPS

Fn = 0.5
WAVELENGTH = 1.5 SHIP LENGTH
"HUMP SPEED" - WORST SPEED TO OPERATE AT

Fn >> 0.5
WAVELENGTH >> SHIP LENGTH
HIGH SPEED PLANING CRAFT

The Reynolds Number

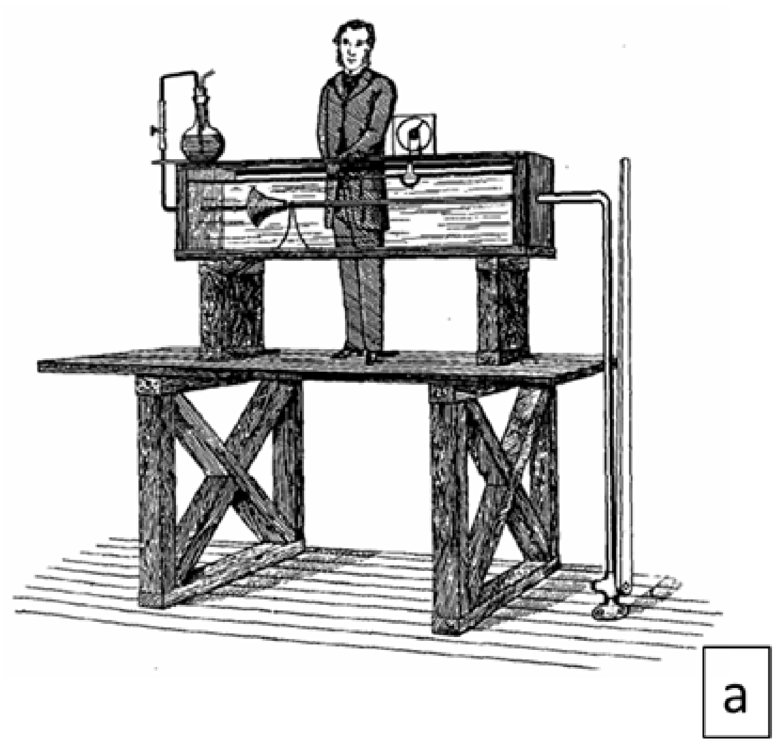
$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} \quad \text{the Reynolds Number}$$

density × volume = mass

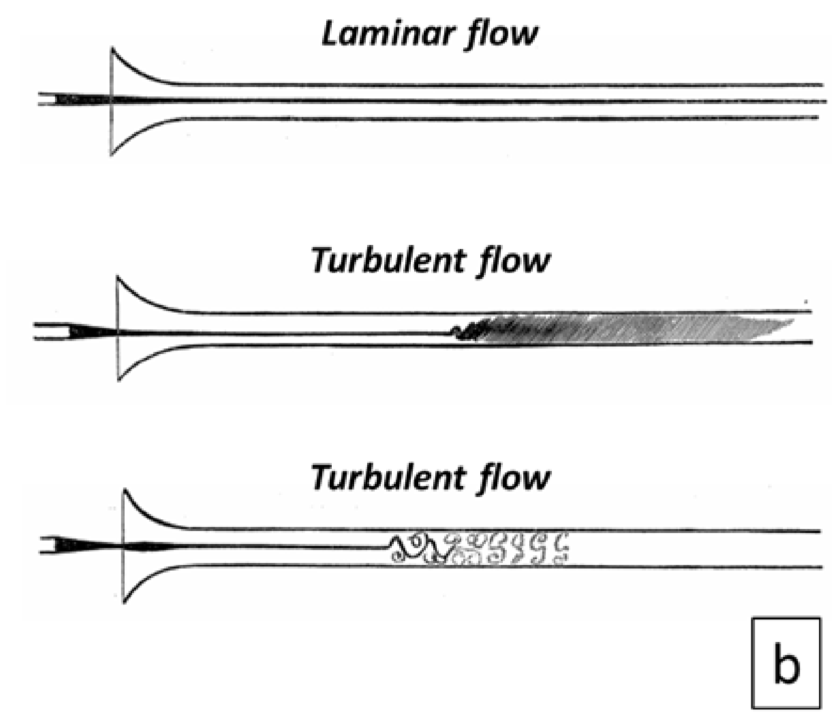
$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} \times \frac{v d^2}{v d^2} = \frac{(\rho_{\text{fluid}} d^3) v^2}{\mu v (d^2)} \propto \frac{\text{kinetic energy}}{\text{dissipated energy}}$$

∝ Shear stress (Newton's Law of Viscosity)

∝ surface area of sphere



Osborne Reynolds 1883



Re < 1

1 < Re < 1000

Re > 1000

The Reynolds Number

$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} \quad \text{the Reynolds Number}$$

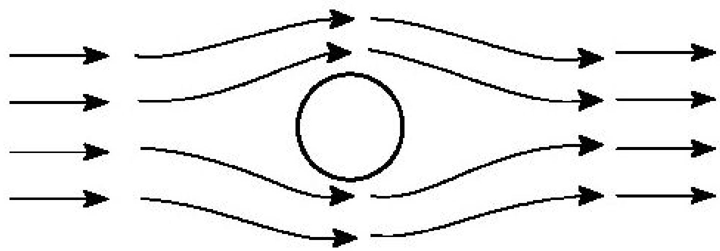


Figure 5.13. Fluid moving around a solid sphere.

$$\text{Re} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\text{(mass)(acceleration)}}{\text{(viscosity)(velocity gradient)(area)}} = \frac{m\frac{dv}{dt}}{\mu\frac{dv}{d\ell}d^2}$$

$$\text{apply the chain rule:} \quad \frac{\frac{dv}{dt}}{\frac{dv}{d\ell}} = \frac{d\ell}{dt} = v$$

$$\text{mass} = \rho_{\text{fluid}} d^3$$

$$\text{Re} = \frac{m\frac{dv}{dt}}{\mu\frac{dv}{d\ell}d^2} = \frac{(\rho_{\text{fluid}} d^3)v}{\mu d^2} = \frac{\rho_{\text{fluid}} d v}{\mu}$$

Dimensional Analysis of a Falling Sphere

6. Measure Data

$$\frac{\rho_{sphere} - \rho_{fluid}}{\rho_{fluid}} = f\left(\frac{v^2}{dg}, \frac{\rho_{fluid} v d}{\mu}\right)$$

Measure v, d, ρ_{sphere}, μ, and ρ_{fluid}.

Calculate $\frac{\rho_{sphere} - \rho_{fluid}}{\rho_{fluid}}$, $\frac{v^2}{dg}$, and $\frac{\rho_{fluid} v d}{\mu}$

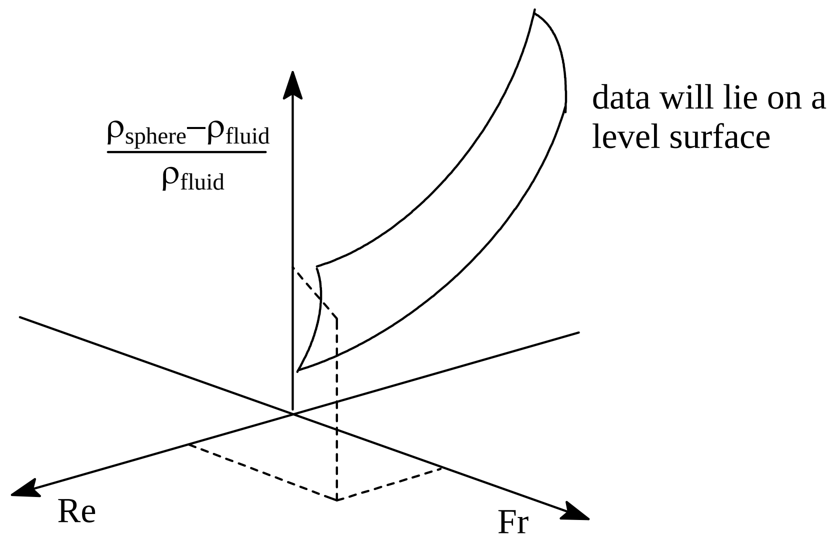
fluid	ball material	dia (mm)	mass (g)	height (m)	time (s)	viscosity (Pa s)	fluid density (kg/m3)	sphere density (kg/m3)	sph-fl density (kg/m3)	velocity (m/s)	Reynolds Number	Froude Number	Reduced Buoyancy
glycerin	nylon	3.18	0.019	0.05	64.94	1.2	1250	1128	122	0.0007	0.0023	0.000	0.10
glycerin	polypropylene	3.18	0.014	0.09	41.76	1.2	1250	831	419	0.0022	0.0073	0.000	0.33
glycerin	glass	2.80	0.038	0.11	22.56	1.2	1250	3306	2056	0.0049	0.0142	0.001	1.64
glycerin	teflon	3.18	0.038	0.09	20.04	1.2	1250	2257	1007	0.0046	0.0152	0.001	0.81
glycerin	steel	3.18	0.133	0.11	3.91	1.2	1250	7899	6649	0.0281	0.093	0.025	5.32
glycerin	aluminum	6.35	0.361	0.23	7.91	1.2	1250	2693	1443	0.0291	0.192	0.014	1.15
oil	nylon	3.18	0.019	0.64	34.72	0.05	930	1128	198	0.0183	1.08	0.011	0.21
oil	steel	1.59	0.016	0.64	5.05	0.05	930	7602	6672	0.1257	3.72	1.015	7.17
oil	glass	2.80	0.038	0.51	5.42	0.05	930	3306	2376	0.0937	4.88	0.320	2.55
oil	teflon	3.18	0.038	0.64	7.66	0.05	930	2257	1327	0.0829	4.90	0.221	1.43
oil	polypropylene	9.53	0.381	0.51	17.63	0.05	930	841	89	0.0288	5.11	0.009	0.10
oil	lucite	6.35	0.172	0.51	8.05	0.05	930	1283	353	0.0631	7.45	0.064	0.38
oil	aluminum	6.35	0.361	0.51	2.23	0.05	930	2693	1763	0.2278	26.9	0.834	1.90
water	nylon	3.18	0.019	0.50	5.53	0.0010	1000	1128	128	0.0904	288	0.262	0.13
water	lucite	3.18	0.021	0.50	4.62	0.0010	1000	1247	247	0.1082	344	0.376	0.25
water	glass	2.80	0.038	0.51	1.50	0.0010	1000	3306	2306	0.3387	948	4.180	2.31
water	steel	1.59	0.016	0.51	0.85	0.0010	1000	7602	6602	0.5976	950	22.923	6.60
water	teflon	3.18	0.038	0.51	1.70	0.0010	1000	2257	1257	0.2988	950	2.865	1.26
water	polypropylene	6.35	0.113	0.51	3.30	0.0010	1000	843	157	0.1539	978	0.381	0.16

Dimensional Analysis of a Falling Sphere

7. Plot universal correlation

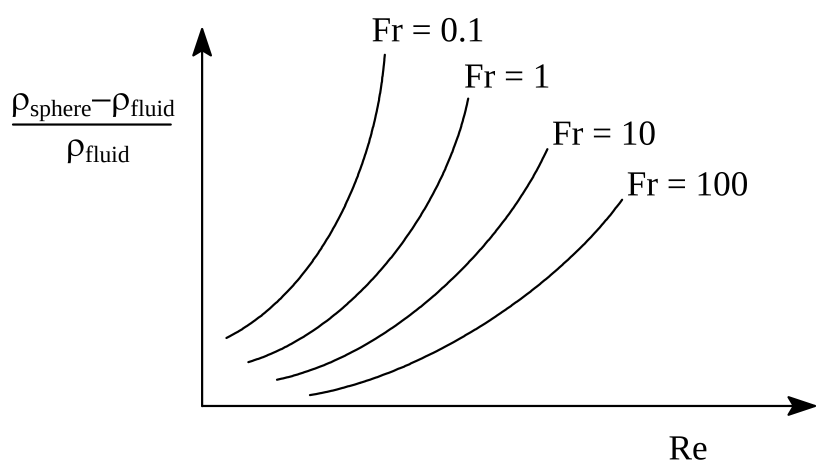
$$\frac{\rho_{sphere} - \rho_{fluid}}{\rho_{fluid}} = f\left(\frac{v^2}{dg}, \frac{\rho_{fluid} v d}{\mu}\right)$$

How to plot? A 3-D plot of a level surface?



Difficult to read quantitative values.

Better to plot as a family of curves for values of Fr.



Dimensional Analysis of a Falling Sphere

7. Plot universal correlation, cont'd

Divide reduced buoyancy by Froude number

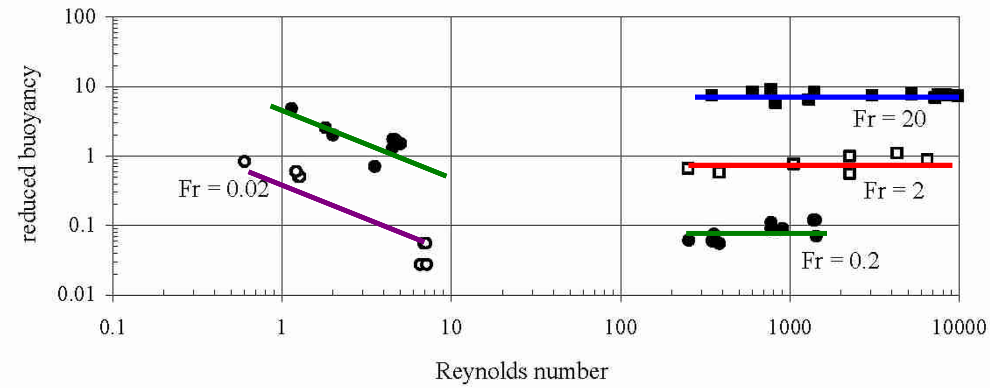


Figure 5.14. Data for a sphere moving through a fluid, plotted for Froude numbers of 0.02, 0.2, 2 and 20, ± 10%.

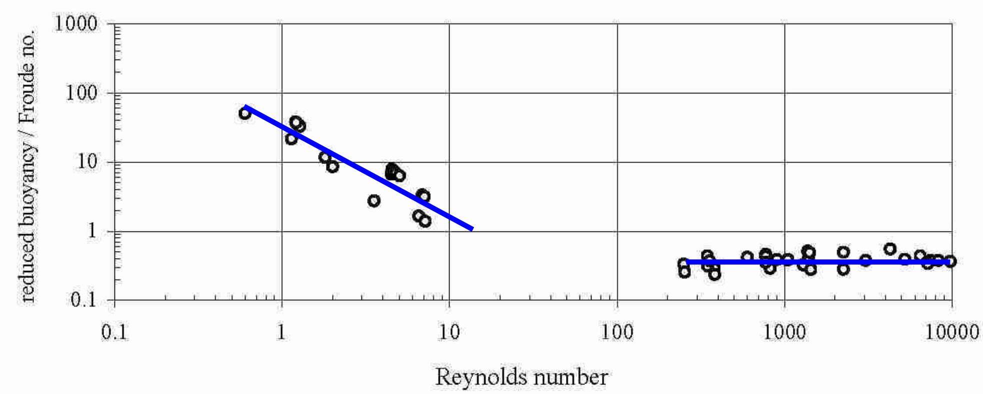


Figure 5.15. Data for a sphere moving through a fluid.

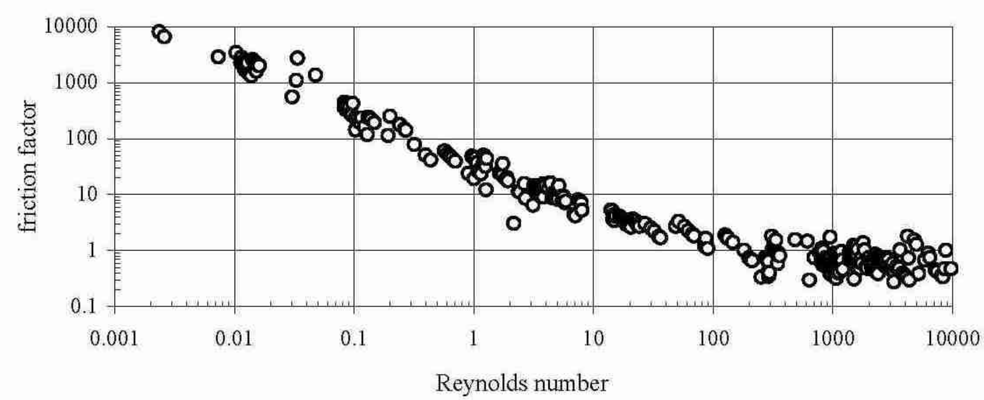
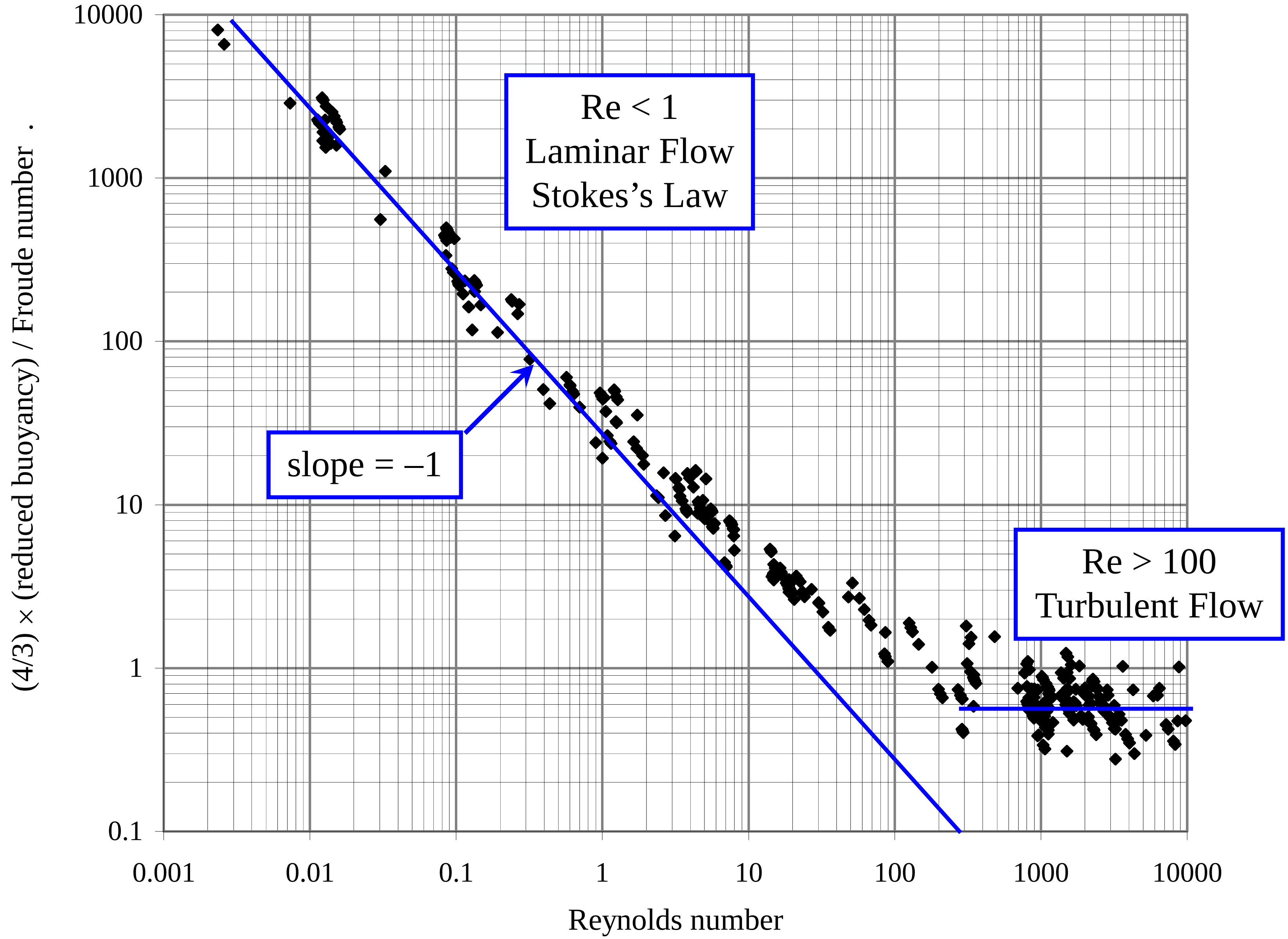


Figure 5.16. Data for a sphere moving through a fluid.

Fluid Flow around a Sphere



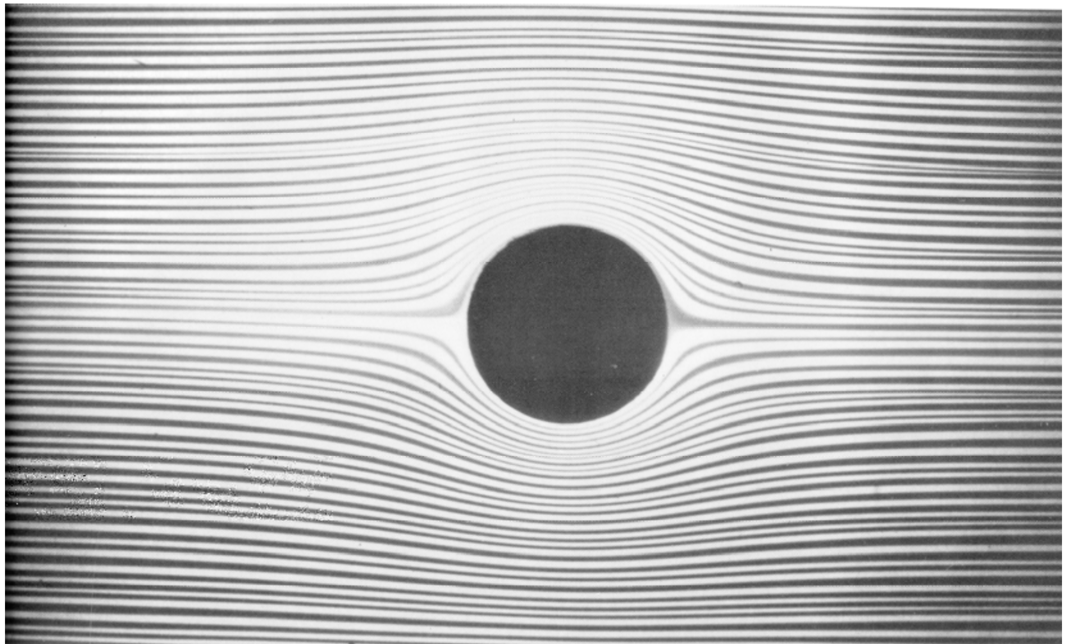
Dynamically Similar Spheres Falling at their Terminal Velocity

sphere	sphere density	sphere diameter	fluid	fluid density	fluid viscosity	terminal velocity	Re
steel	7900 kg/m^3	6.9 mm	glycerin	1250 kg/m^3	1.2 Pa·sec	14 cm/sec	1
		0.90	vegetable oil	930	0.05	6	1
		0.065	water	1000	0.001	1.5	1
		0.040	air	1.3	1.8×10^{-5}	38	1
silica	2600 kg/m^3	12 mm	glycerin	1250	1.2	8.2	1
		1.5	vegetable oil	930	0.05	3.7	1
		0.11	water	1000	0.001	0.9	1
inconvenient →		0.057	air	1.3	1.8×10^{-5}	25	1
lucite	1260 kg/m^3	60. mm	glycerin	1250	1.2	1.6	1
		2.5	vegetable oil	930	0.05	2.2	1
		0.20	water	1000	0.001	0.51	1
		0.073	air	1.3	1.8×10^{-5}	20	1

Use convenient models to predict results for inconvenient systems.

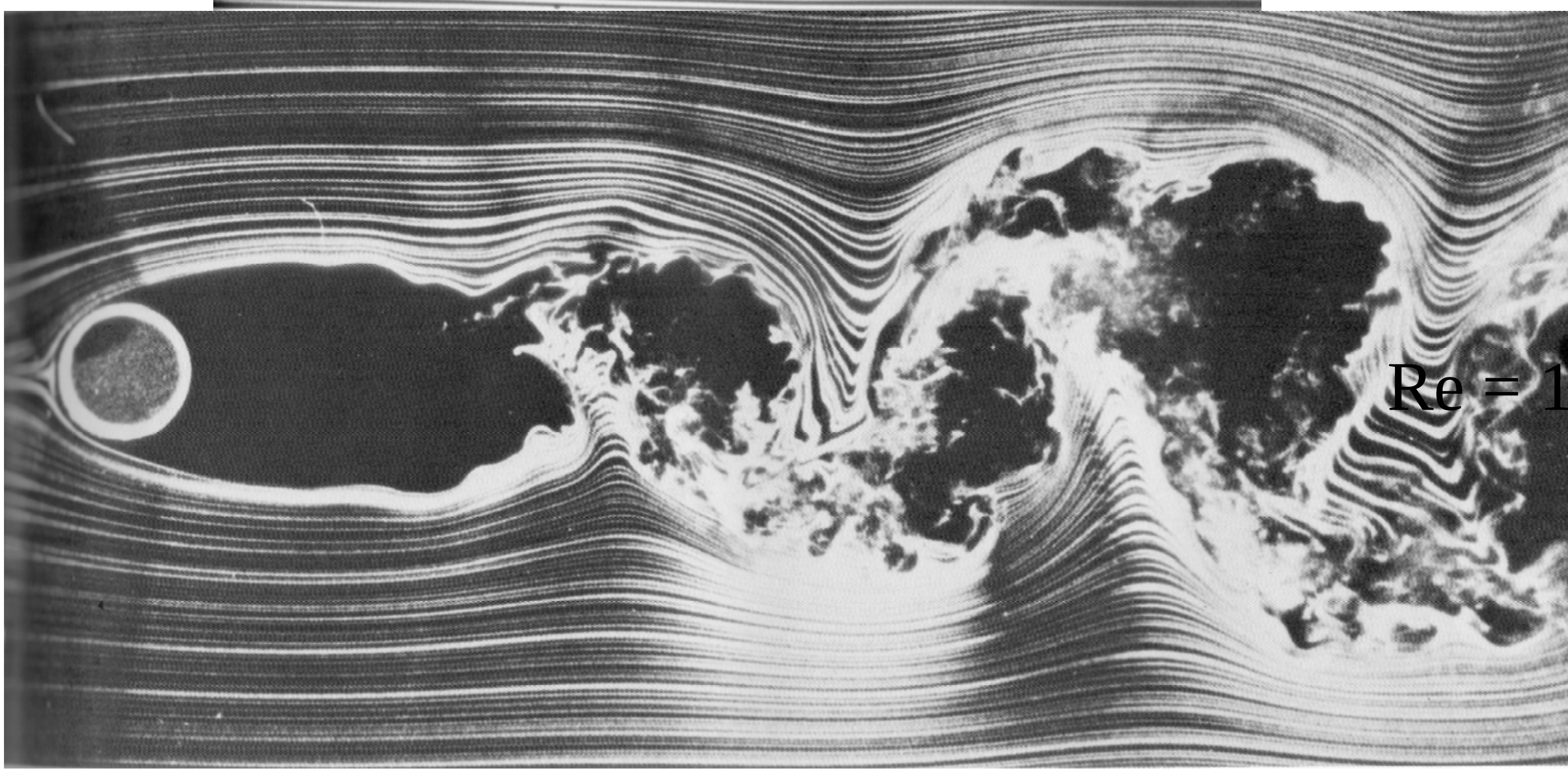
Extrapolation? No! All systems are dynamically similar.

Flow Around a Cylinder



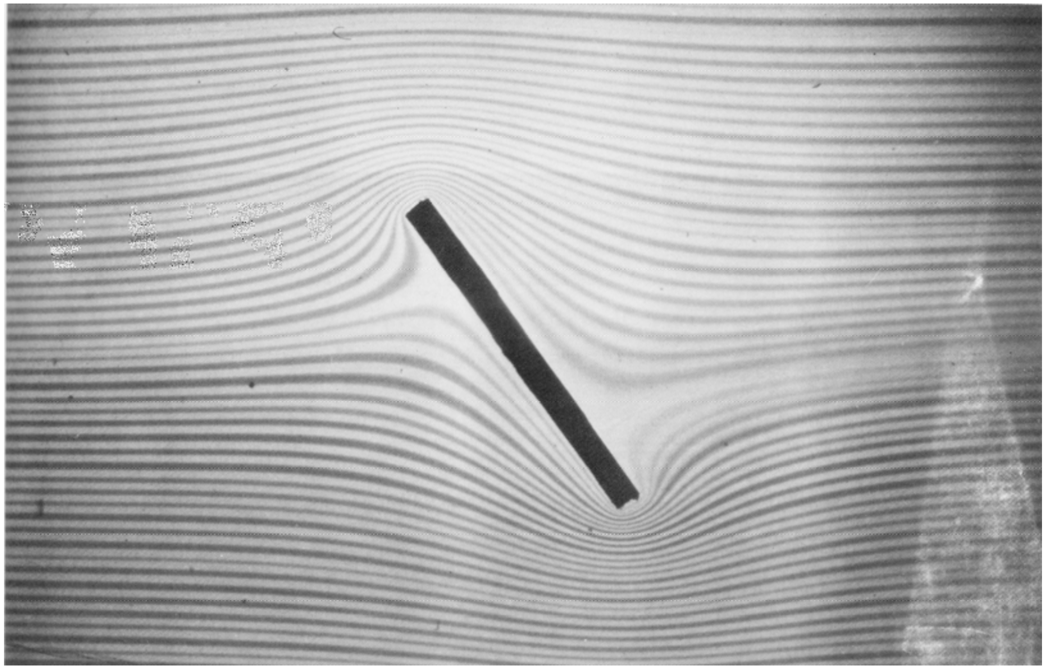
Re « 1

Re = 10,000

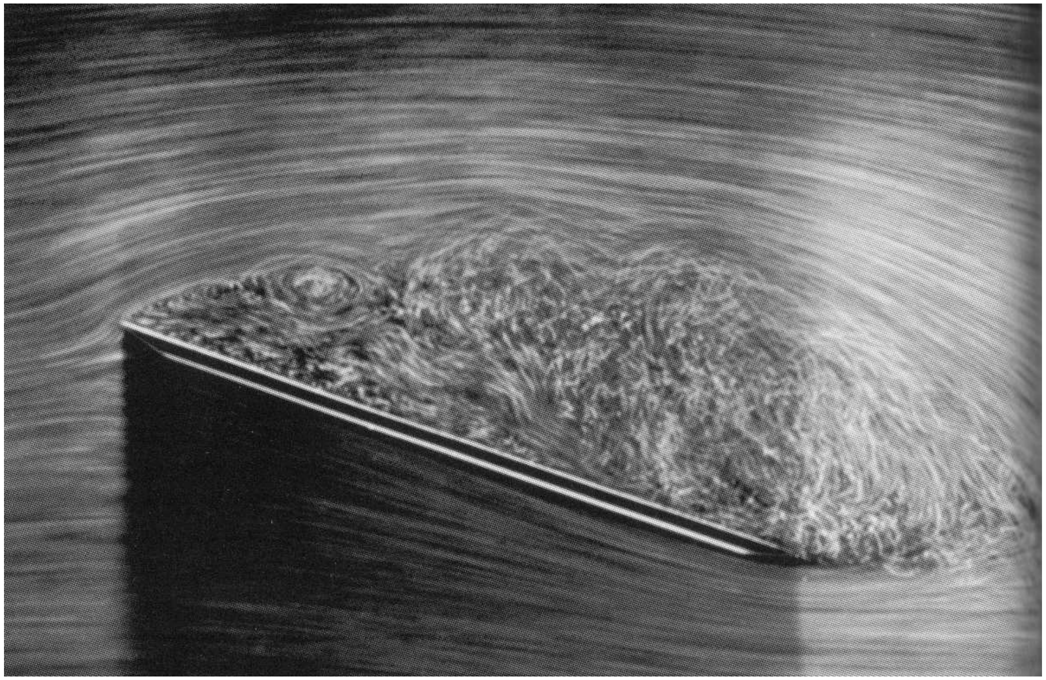


Re = 100

Flow Past a Plate

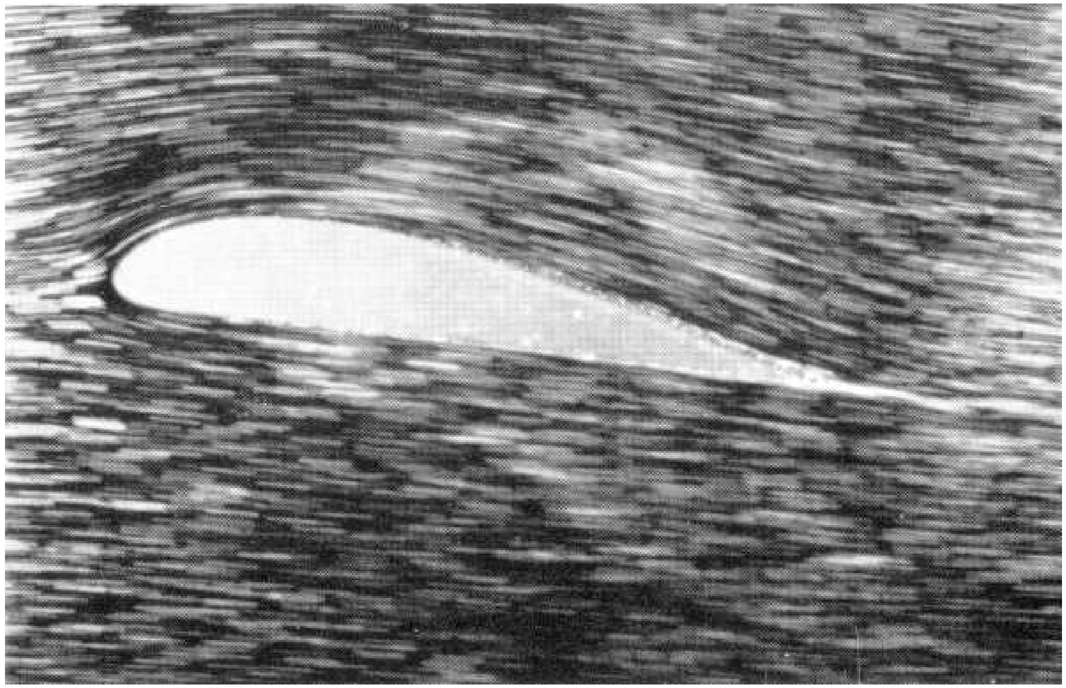


Re = 0.1

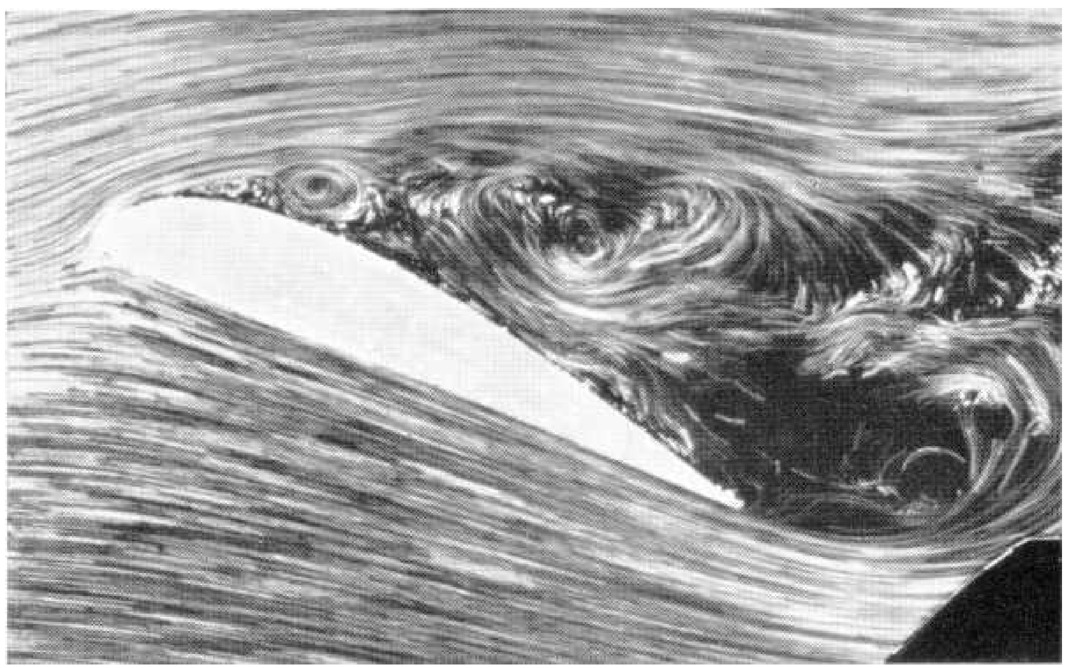


Re = 10,000

Flow Past an Airfoil

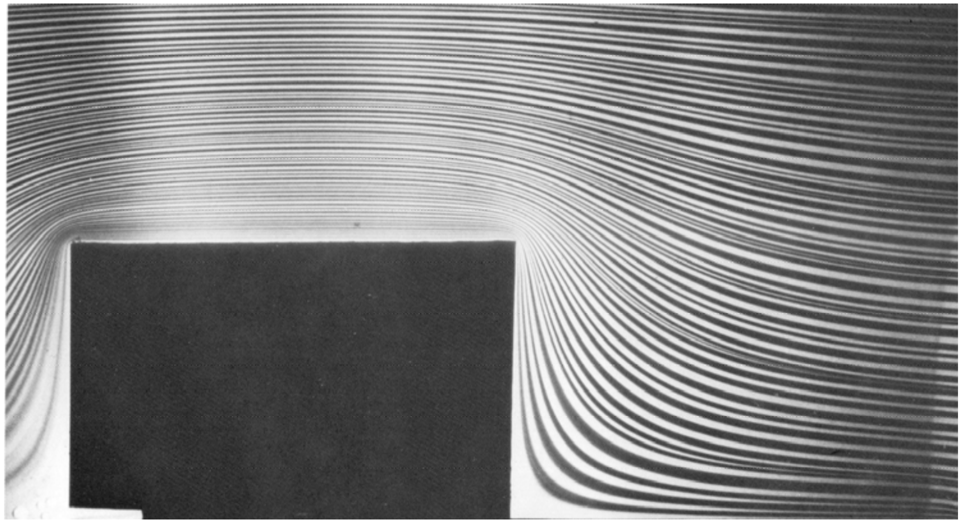


Re = 100



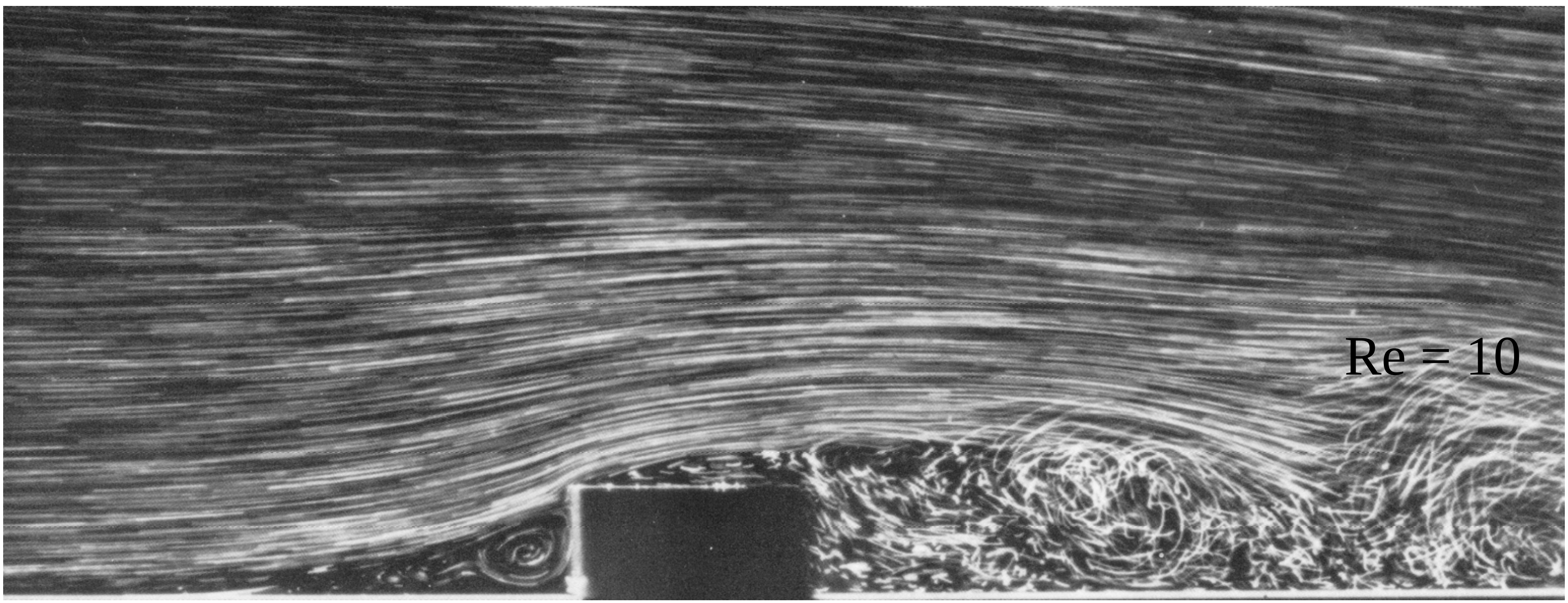
Re = 1000

Flow Past a Block

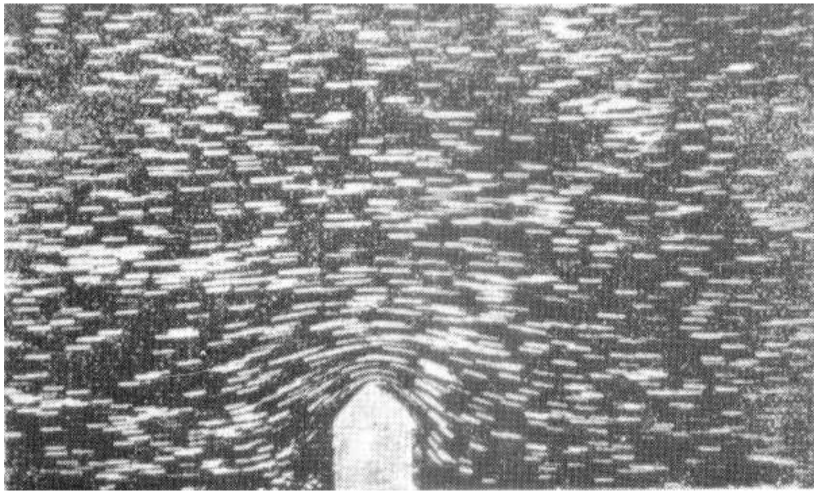


Re = 0.01

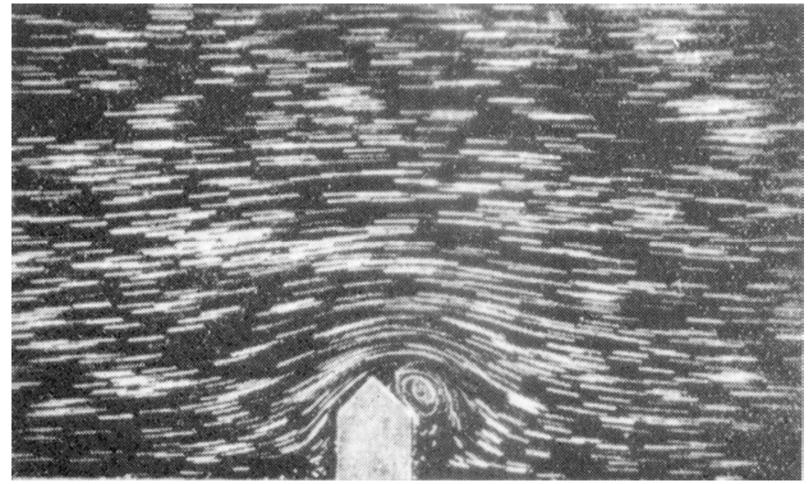
Re = 10,000



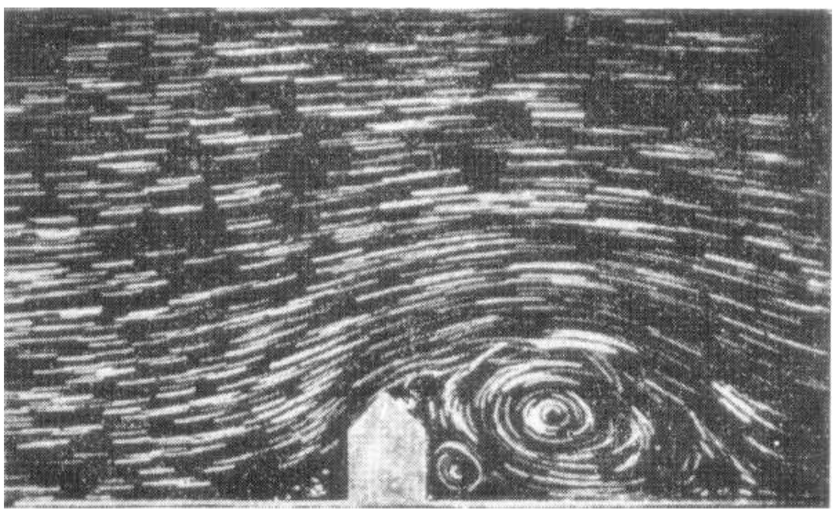
Flow Past a House



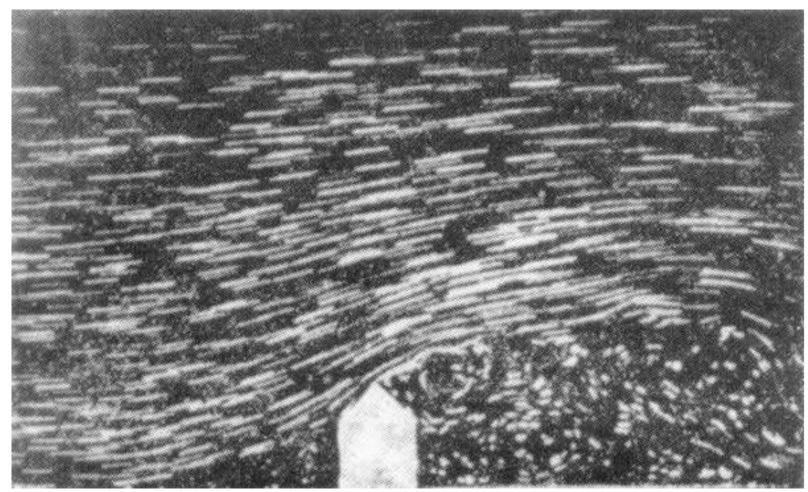
Re = 0.1



Re = 10



Re = 100



Re = 10,000