EngrD 2190 – Lecture 31

Concept: Dimensional Analysis and Dynamic Scaling

Context: Four methods to obtain the value of a
non-core variable from a dimensionless correlation.

Defining Question: Which method is the more effective?
Which method is the more simple?

Last Reading: Chapter 5 pp. 454-467
Applications of Dimensional Analysis

EngrD 2190
Chemical Process Design & Analysis

Peer Rating of Team Members - Second Team Assignments

Due Friday 11/21

Name ______________________________ Team # ______

Please write the names of *all* your team members, *including yourself*, and rate the degree to which each member fulfilled his/her responsibilities for team-wide learning. Because the team's goal is to learn, questions are as important as answers, and critical analysis is as important as creative design. Use the following ratings:

Excellent	Consistently went above and beyond. Carried more than his/her fair share of the load. Prepared thought-provoking questions and/or educational explanations.
Very Good	Consistently fulfilled responsibilities for team-wide learning. Very well prepared and very cooperative.
Satisfactory	Often fulfilled responsibilities for team-wide learning. Sufficiently prepared and acceptably cooperative.
Ordinary	Usually fulfilled responsibilities for team-wide learning. Minimally prepared and narrowly cooperative.
Marginal	Sometimes failed to show up or contribute to the learning. Rarely prepared.
Unsatisfactory	Consistently failed to show up or contribute to the learning. Unprepared.
Superficial	Practically no participation.
No show	No participation.

These ratings should reflect each individual's level of participation, effort, and sense of responsibility, not his or her academic ability.

Name of team member **(include yourself)**	Rating
______________________________	______________
______________________________	______________
______________________________	______________
______________________________	______________

Include yourself

Please comment on your team and on the general concept of learning by working homework in teams. If your team was dysfunctional, was it a specific problem with team members or an inherent problem with the concept?

Prelim 3, Tuesday 11/25

- Prelim 3: Tuesday 11/25, 7:30-9:30 p.m., 128 and 245 Olin Hall.
  Graphical Modeling for Mass Balances - Part 2:
    - tie lines on H-(x,y) and ternary diagrams.
    - translating thermodynamic maps: T-(x,y) to x-y, for example.
    - operating lines for multistage absorbers and strippers.
    - operating lines for multistage distillation.
    - design using single-stage and countercurrent multi-stage units - may include use of T-(x,y), P-(x,y), H-(x,y) and ternary diagrams.

  Open notes and open exercise solutions.
  Bring a calculator and a ruler. Graphing calculators are allowed.

- Practice exercises: 4.37, 4.40, 4.47, 4.48, 4.58, 4.69, 4.70, 4.98, 4.109, and 4.110. *Solutions are posted.*

Three Skills in Dimensional Analysis / Dynamic Scaling

1. Derive a set of Dimensionless Groups given a List of Parameters.

    Lectures 28, 29, and 30 last week:

        Pendulums Swinging, People Walking, and Spheres Falling.

    Practice Exercises: 5.8, 5.13, 5.15, 5.20, and 5.22.

2. Use a Universal Correlation of Dimensionless Groups.

    Lecture 31 today.

    Practice Exercises: 5.28, 5.29, and 5.30.

3. Design a Dynamically Similar Model by Scaling.

    Lectures 32 and 33.

    Practice Exercises: 5.33, 5.34, and 5.39.

Solutions to practice exercises are posted at the EngrD 2190 homepage.

Recap:

The magnitude of a Π group describes the phenomenon.

Walking: $Fr < 2.5$ walking or trotting

$Fr > 2.5$ running or galloping

Falling Spheres: $Re < 1$ laminar flow

$Re > 1000$ turbulent flow

The Terminal Velocity of Falling Spheres - *Recap*

Table 5.7 The parameters of a sphere moving through a fluid

parameter	symbol	dimensions
terminal velocity	v	L/T
sphere diameter	D	L
buoyancy	$\rho_{\text{sphere}} - \rho_{\text{fluid}}$	M/L^3
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3
gravitational acceleration	g	L/T^2

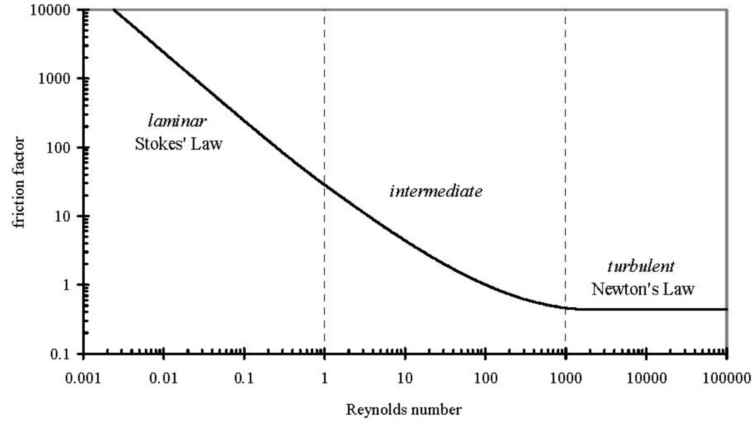
6 parameters - 3 dimensions = 3 core variables

$$\text{buoyancy:} \quad \frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}$$

$$\text{gravity:} \quad \text{Fr} = \frac{v^2}{gD} = \frac{\text{kinetic energy}}{\text{potential energy in a gravitational field}}$$

$$\text{viscosity:} \quad \text{Re} = \frac{\rho_{\text{fluid}} v D}{\mu} = \frac{\text{kinetic energy}}{\text{dissipated energy}}$$

$$\frac{\text{reduced buoyancy}}{\text{Fr}} = \text{Friction Factor}$$



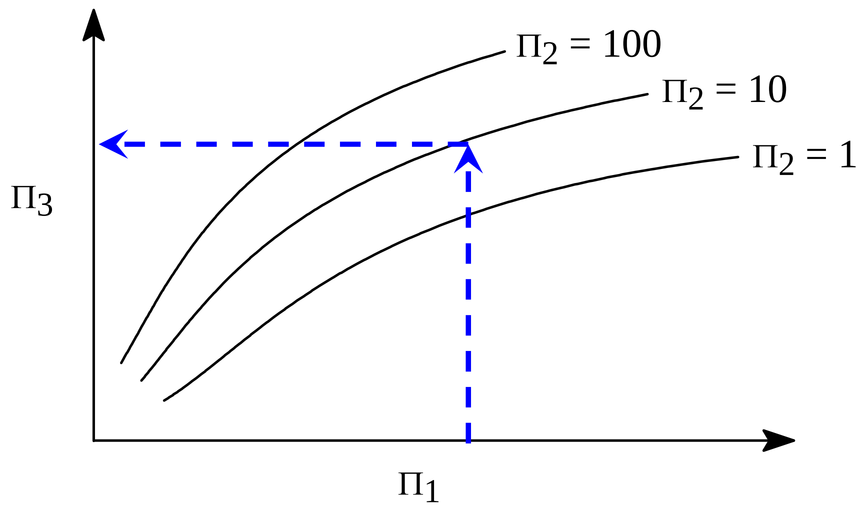
Re

Using a Universal Correlation

Parameter we seek is the core variable for Π_3.

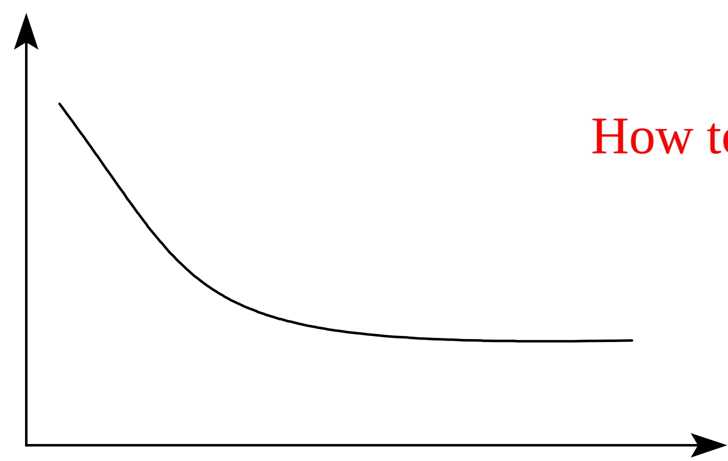
Use experimental data to calculate Π_1 and Π_2.

Read Π_3 from graph, calculate value of core variable.



But for the terminal velocity of a sphere …

$$\frac{\dfrac{\rho_{sphere} - \rho_{fluid}}{\rho_{fluid}}}{\dfrac{v^2}{gd}}$$



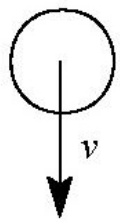
How to calculate velocity?

friction factor contains velocity.

$$\mathrm{Re} = \frac{\rho_{fluid} v d}{\mu}$$

Reynolds number contains velocity.

Using Dimensionless Correlations to find the Terminal Velocity



$$D = 60\ \mu\text{m} = 0.06\ \text{mm} = 6 \times 10^{-5}\ \text{m}$$

$$\rho_{\text{sphere}} = 2.6\ \text{g/cm}^3 = 2600\ \text{kg/m}^3$$

$$\rho_{\text{air}} = 1.3\ \text{kg/m}^3$$

$$\mu_{\text{air}} = 1.8 \times 10^{-5}\ \text{Pa}\cdot\text{sec}$$

Method 1: Guess 'n' Check

1. Guess v.
2. Use v to calculate Re.

$$\text{Re} = \frac{\rho_{\text{fluid}} v D}{\mu} = \frac{(1.3\ \text{kg/m}^3)v(6 \times 10^{-5}\ \text{m})}{1.8 \times 10^{-5}\ \text{Pa}\cdot\text{sec}} = 4.3v$$

3. Use Re and Dimensionless Correlation to find the Friction Factor.
4. Use the Friction Factor to calculate v.

$$\text{friction factor} \equiv \text{FF} = \frac{4}{3}\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{sphere}}} \Big/ \frac{v^2}{gD}$$

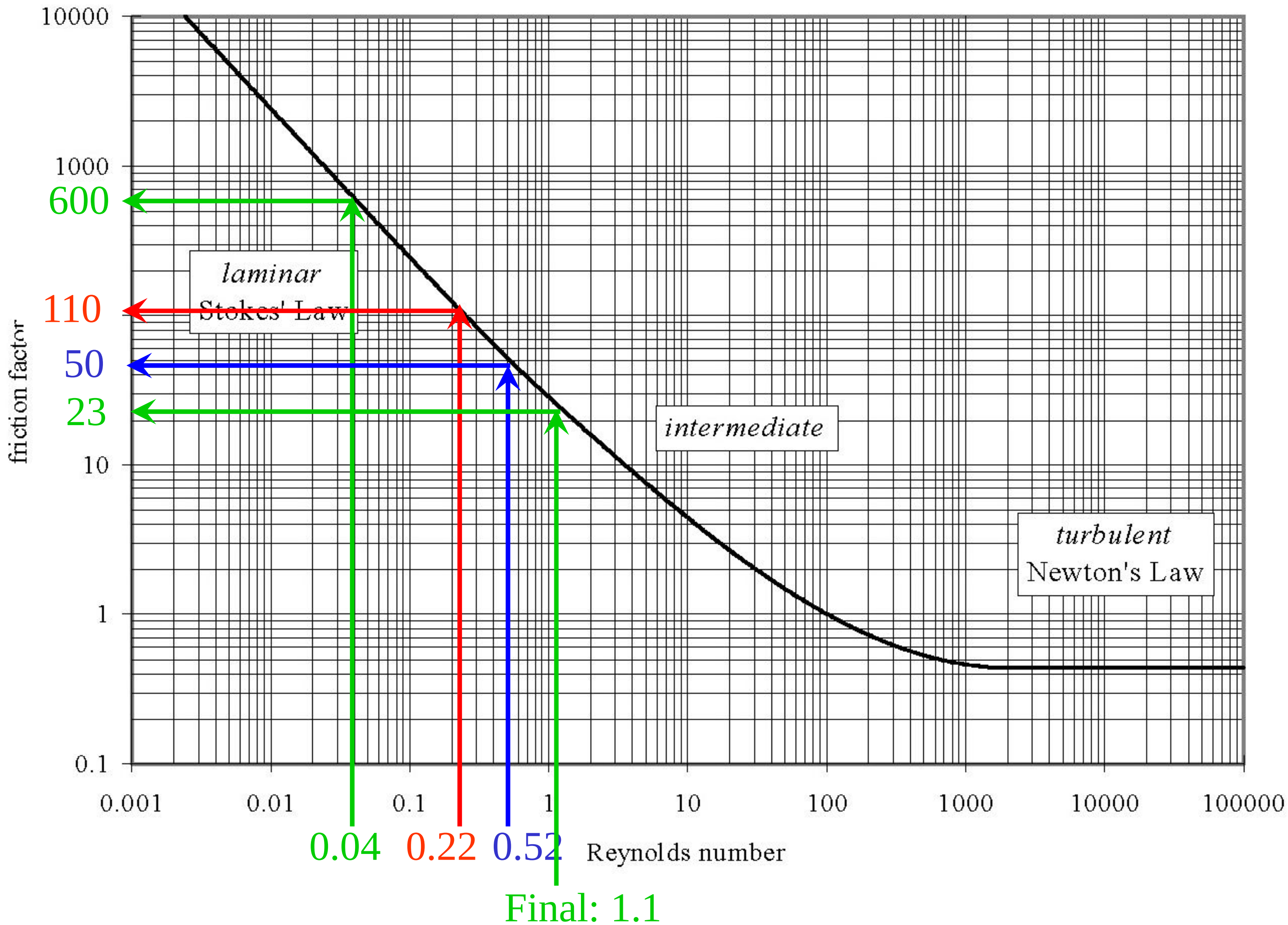
$$v = \left(\frac{4}{3}\right)^{1/2}\left(\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}\right)^{1/2}\left(\frac{gD}{\text{FF}}\right)^{1/2} = \left(\frac{4}{3}\right)^{1/2}\left(\frac{2600 - 1.3\ \text{kg/m}^3}{1.3\ \text{kg/m}^3}\right)^{1/2}\left(\frac{(9.8\ \text{m/sec}^2)(6 \times 10^{-5}\ \text{m})}{\text{FF}}\right)^{1/2}$$

$$v = \frac{1.25}{\text{FF}^{1/2}}$$

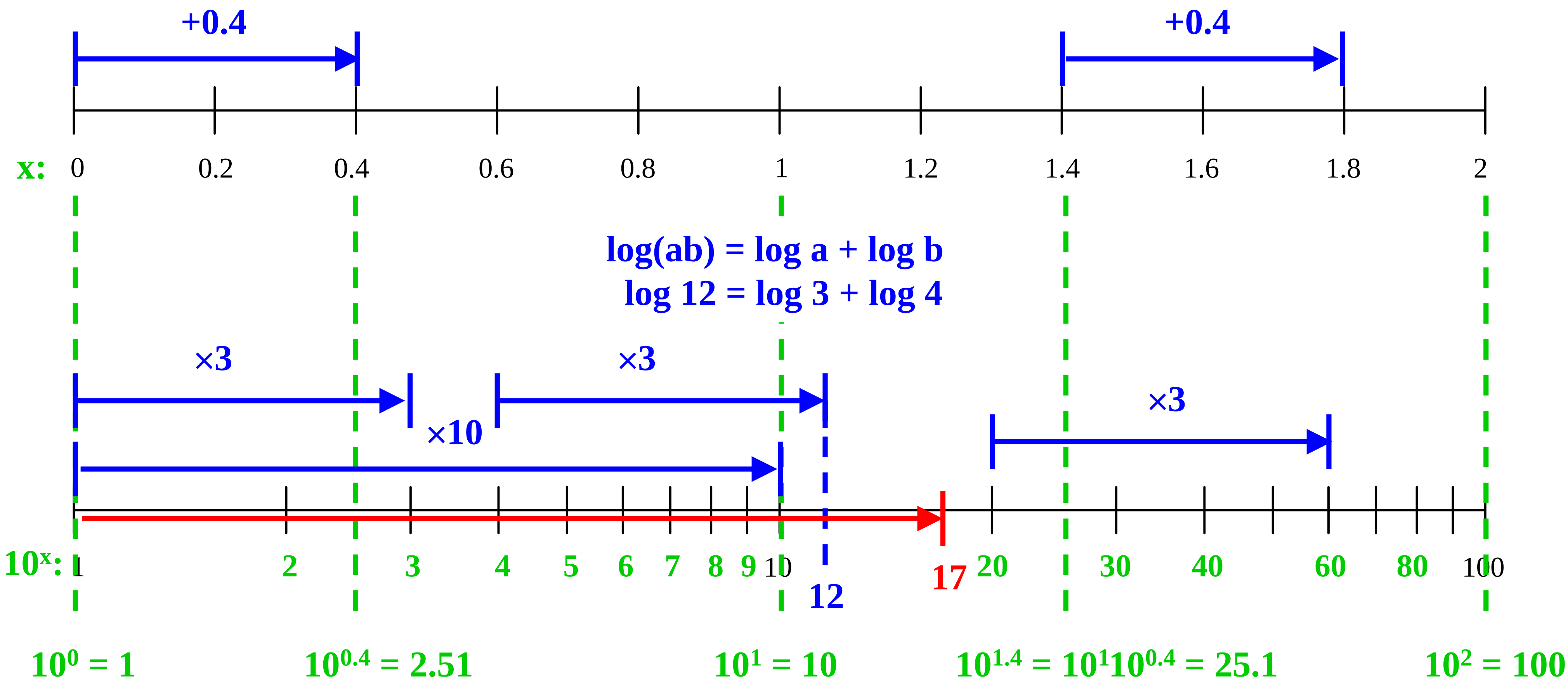
5. If initial v = calculated v, method has converged. If not, return to step 1.

Method 1 Iterations:

Iteration	v (m/sec) guess	calculate Re = 4.3v	friction factor (from graph)	$v = \frac{1.25}{\sqrt{FF}}$
1	0.01	0.043	600	0.05
2	0.05	0.22	110	0.12
3	0.12	0.52	50	0.18
n	0.26	1.1	23	0.26



How to plot a point on a log scale.



How to plot 17?

Plot 16 (= 4 × 4) and 18 (= 3 × 6)

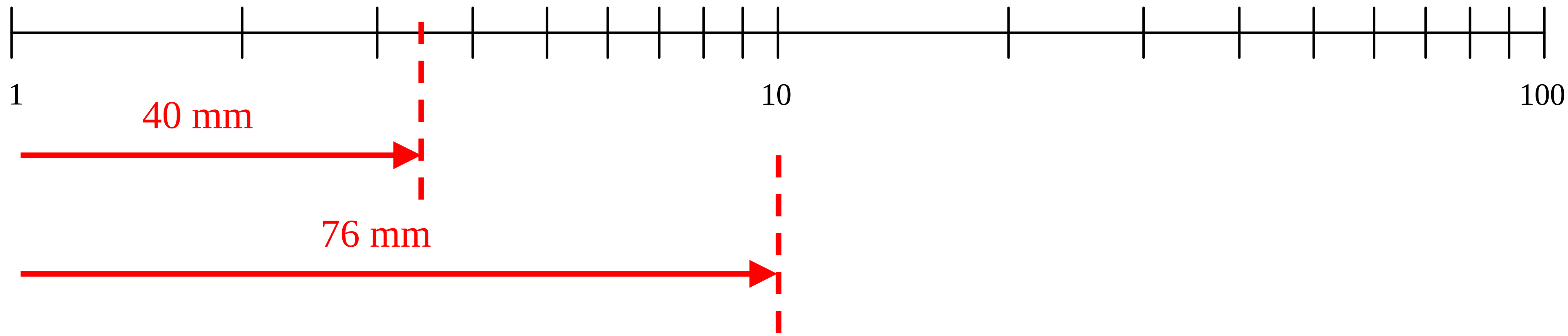
$$\log_{10} 10 = 76\,\text{mm}$$

$$\log_{10} 17 = x\,\text{mm}$$

$$\frac{\log_{10} 17}{\log_{10} 10} = \frac{x}{76\,\text{mm}}$$

$$x = (\log_{10} 17)(76\,\text{mm}) = 93.5\,\text{mm}$$

How to read a point on a log scale.



$$\log_{10} x = 40\,\text{mm}$$

$$\log_{10} 10 = 76\,\text{mm}$$

$$\frac{\log_{10} x}{\log_{10} 10} = \frac{40\,\text{mm}}{76\,\text{mm}}$$

$$x = 10^{\frac{40}{76}} = 3.4$$

Method 2: Derive a relation FF = f(Re) for the specific system.

Eliminate the unknown parameter, v, from the two dimensionless groups.

$$\mathrm{Re} = \frac{\rho_{\text{fluid}} v D}{\mu} \quad \Rightarrow \quad v = \frac{\mu}{\rho_{\text{fluid}} D}\mathrm{Re} \qquad (1)$$

$$\mathrm{FF} = \frac{4}{3}\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}} \bigg/ \frac{v^2}{gD} \quad \Rightarrow \quad v = \left[\frac{4}{3}\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}\frac{gD}{\mathrm{FF}}\right]^{1/2} \qquad (2)$$

Set eqn (1) equal to eqn (2) and solve for FF.

$$\frac{\mu}{\rho_{\text{fluid}} D}\mathrm{Re} = \left[\frac{4}{3}\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}\frac{gD}{\mathrm{FF}}\right]^{1/2}$$

$$\mathrm{FF} = \frac{4}{3}\frac{\rho_{\text{fluid}}(\rho_{\text{sphere}} - \rho_{\text{fluid}})gD^3}{\mu^2}\frac{1}{\mathrm{Re}^2}$$

Substitute parameters for the specific system.

$$\mathrm{FF} = \frac{4}{3}\frac{(1.3\ \mathrm{kg/m^3})(2600\ \mathrm{kg/m^3} - 1.3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/sec^2})(6\times 10^{-5}\ \mathrm{m})^3}{(1.8\times 10^{-5}\ \mathrm{Pa\cdot sec})^2}\frac{1}{\mathrm{Re}^2}$$

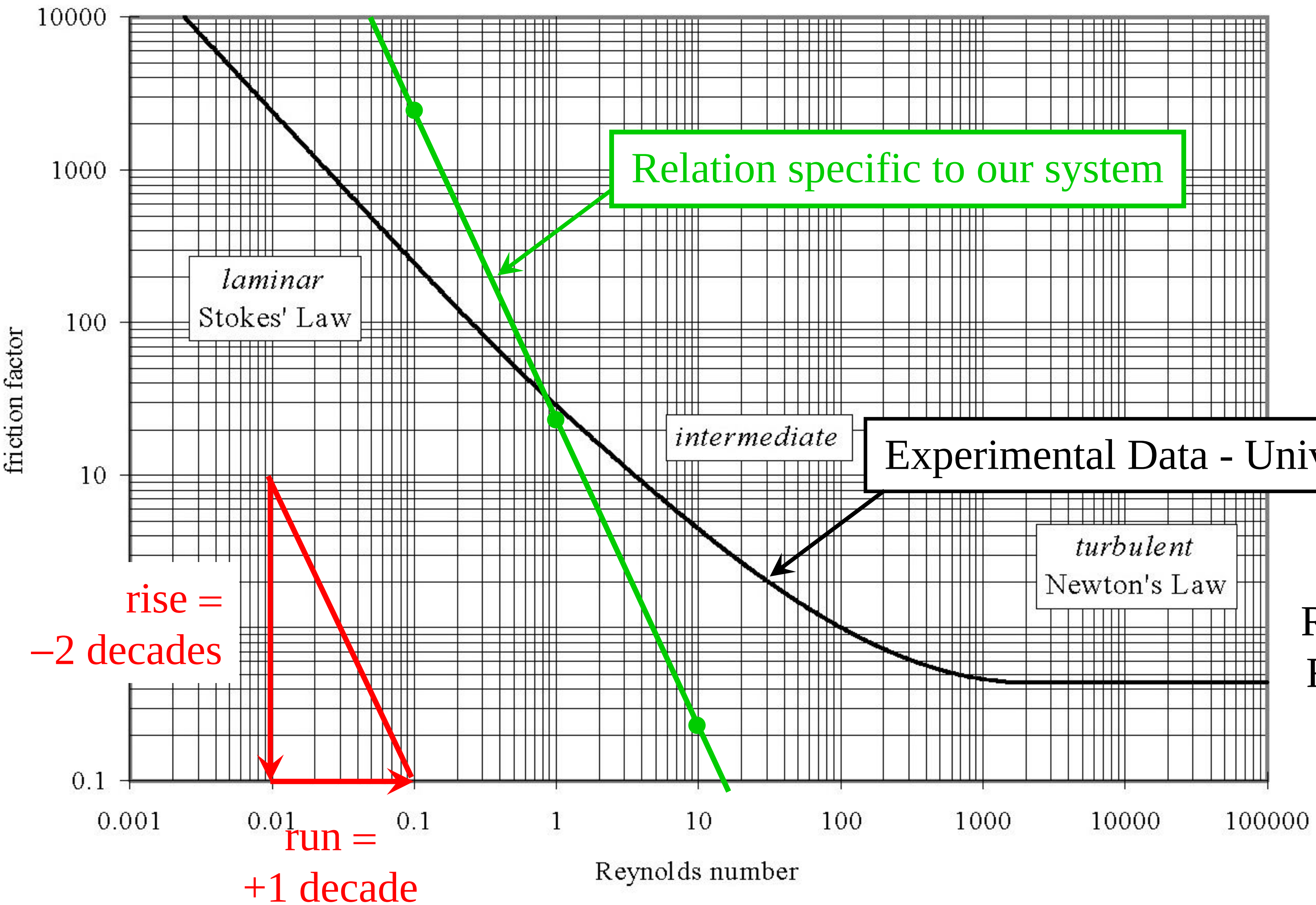
$$\mathrm{FF} = \frac{22}{\mathrm{Re}^2}$$

Method 2: Plot the relation FF = f(Re) for the specific system.

$$\mathrm{FF} = \frac{22}{\mathrm{Re}^2} \quad \Rightarrow \quad \underbrace{\log_{10}\mathrm{FF}}_{y} = -2\,\underbrace{\log_{10}\mathrm{Re}}_{x} + \log_{10}22$$

straight line with slope –2

$\log_{10}$FF at $\log_{10}$Re = 0 (at Re =1)



or plot points

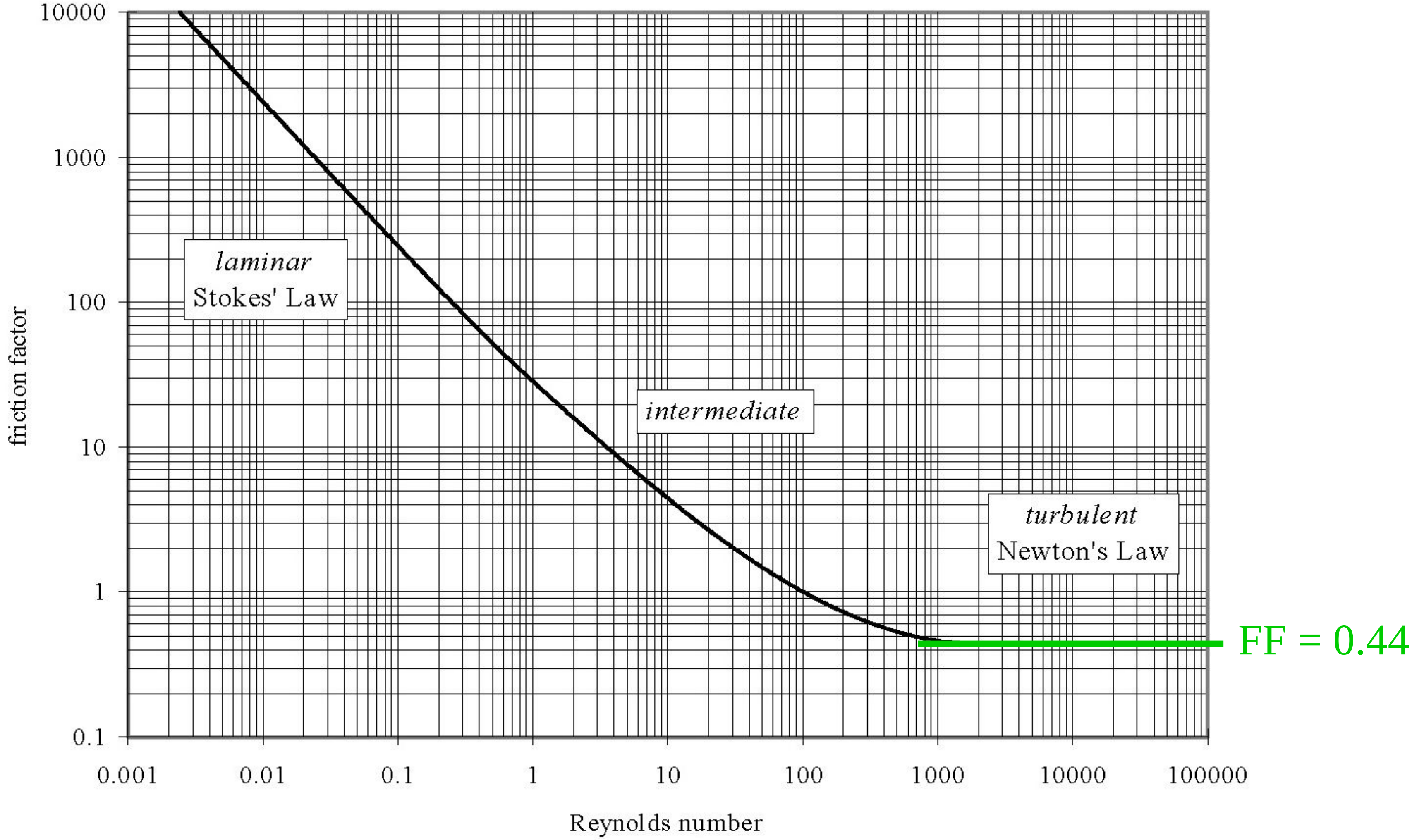
Re	FF
0.1	2200
1	22
10	0.22

Intersection is at
Re = 0.9 (v = 0.21)
FF = 30 (v = 0.23)

Method 3: Convert the Dimensionless Correlation to Equations.

Two regions of the dimensionless correlation for the terminal velocity of a sphere are straightforward to convert to equations.

For Re > 1000 (turbulent flow) the correlation is horizontal: FF = 0.44.



Method 3: Convert the Dimensionless Correlation to Equations.

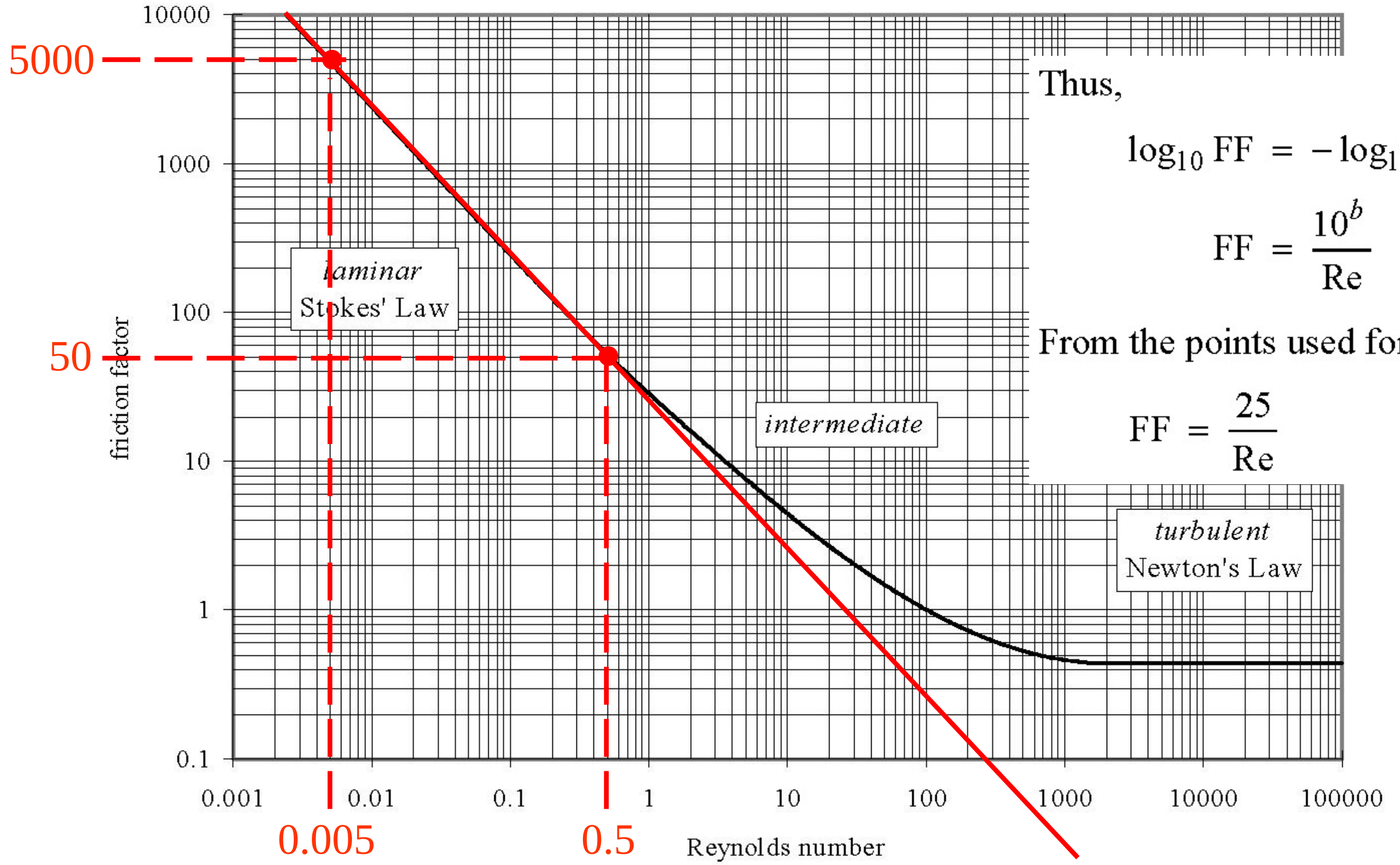
For Re < 1 (laminar flow) the correlation is a straight line on a log-log plot.

$$y = mx + b$$

$$\log_{10} \mathrm{FF} = m \log_{10} \mathrm{Re} + b$$

Calculate the slope. Use the points (0.005, 5000) and (0.5, 50).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\log_{10} 5000 - \log_{10} 50}{\log_{10} 0.005 - \log_{10} 0.5} = \frac{\log_{10} 100}{\log_{10} 0.01} = \frac{2}{-2} = -1$$



Thus,

$$\log_{10} \mathrm{FF} = -\log_{10} \mathrm{Re} + b$$

$$\mathrm{FF} = \frac{10^b}{\mathrm{Re}}$$

From the points used for the slope, $10^b = 25$

$$\mathrm{FF} = \frac{25}{\mathrm{Re}}$$

Method 3: Convert the Dimensionless Correlation to Equations.

$$\mathrm{FF} = \frac{25}{\mathrm{Re}}$$

Substitute numerical values to obtain an expression for *v*.

$$\frac{4}{3}\frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}} \Bigg/ \frac{v^2}{gD} = \frac{25}{\dfrac{\rho_{\text{fluid}} v D}{\mu}}$$

$$v = \frac{4}{75}\frac{gD^2}{\mu}(\rho_{\text{sphere}} - \rho_{\text{fluid}}) \qquad (3)$$

$$v = \frac{4}{75}\frac{(9.8\ \text{m/sec}^2)(6\times 10^{-5}\ \text{m})^2}{1.8\times 10^{-5}\ \text{Pa}\cdot\text{sec}}(2600\ \text{kg/m}^3 - 1.3\ \text{kg/m}^3)$$

$$v = 0.27\ \text{m/sec}$$

This is the same result obtained with the previous two methods.

Note that eqn (3) is the similar to Stokes' law for laminar flow and $\rho_{\text{sphere}} \gg \rho_{\text{fluid}}$:

$$v = \frac{1}{18}\frac{gD^2}{\mu}\rho_{\text{sphere}} \qquad \text{Note: } \frac{1}{18} = \frac{4}{72} \approx \frac{4}{75}$$

Which derives from first principles.

Method 4: Recast the dimensional analysis with v as a core variable.

Use buoyancy, velocity, and viscosity as core variables.

$$\text{buoyancy:} \quad \frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}}$$

$$\text{velocity:} \quad \frac{v^2}{gD} = \text{Fr} = \text{Froude Number}$$

$$\text{viscosity:} \quad \frac{D^3 \rho_{\text{fluid}}^2 g}{\mu^2} = \text{Ga} = \text{Galileo Number}$$

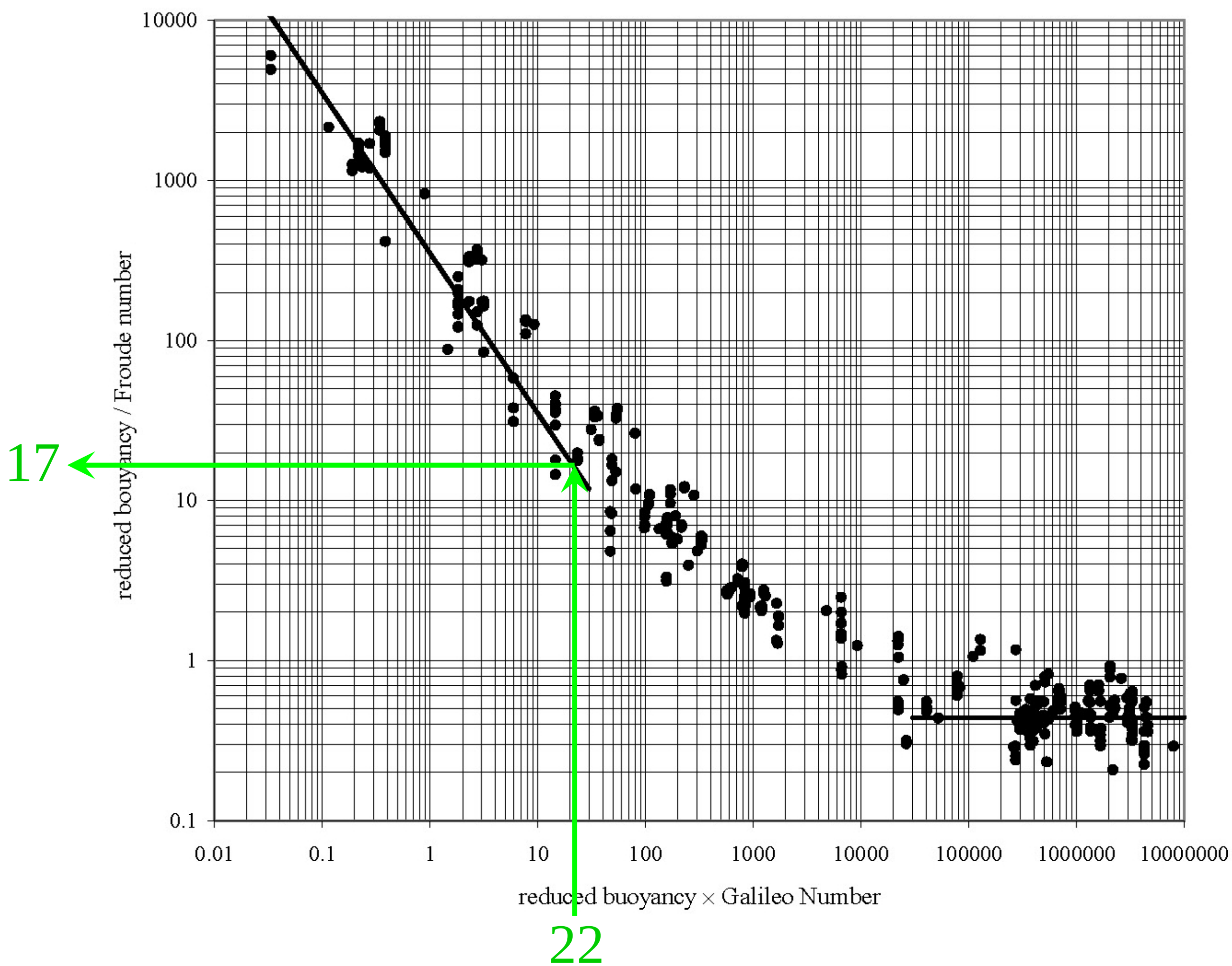
$$\text{Note: Ga} = \frac{\text{gravitational effects}}{\text{viscous effects}} = \frac{\text{Re}^2}{\text{Fr}}$$

Method 4: Recast the dimensional analysis with *v* as a core variable.

Calculate (reduced buoyancy) × Ga for the specific system.

$$(\text{reduced buoyancy}) \times \text{Ga} = \frac{\rho_{\text{sphere}} - \rho_{\text{fluid}}}{\rho_{\text{fluid}}} \frac{D^3 \rho_{\text{fluid}}^2 g}{\mu^2}$$

$$= \frac{2600 - 1.3\ \text{kg/m}^3}{1.3\ \text{kg/m}^3} \frac{(6 \times 10^{-5}\ \text{m})^3 (1.3\ \text{kg/m}^3)^2 (9.8\ \text{m/sec}^2)}{(1.8 \times 10^{-5}\ \text{Pa} \cdot \text{sec})^2} = 22$$



Read from the graph: (reduced buoyancy) / Fr = 17. Calculate v = 0.27 m/sec.

Summary of the four methods to obtain the value of a non-core variable from a dimensionless correlation.

Method 1: Guess 'n' check.

Slow to converge. No convergence for some cases: Re >1000

Method 2: Derive a relation $\Pi_1 = f(\Pi_2)$ for the specific system.

Lots of algebra.

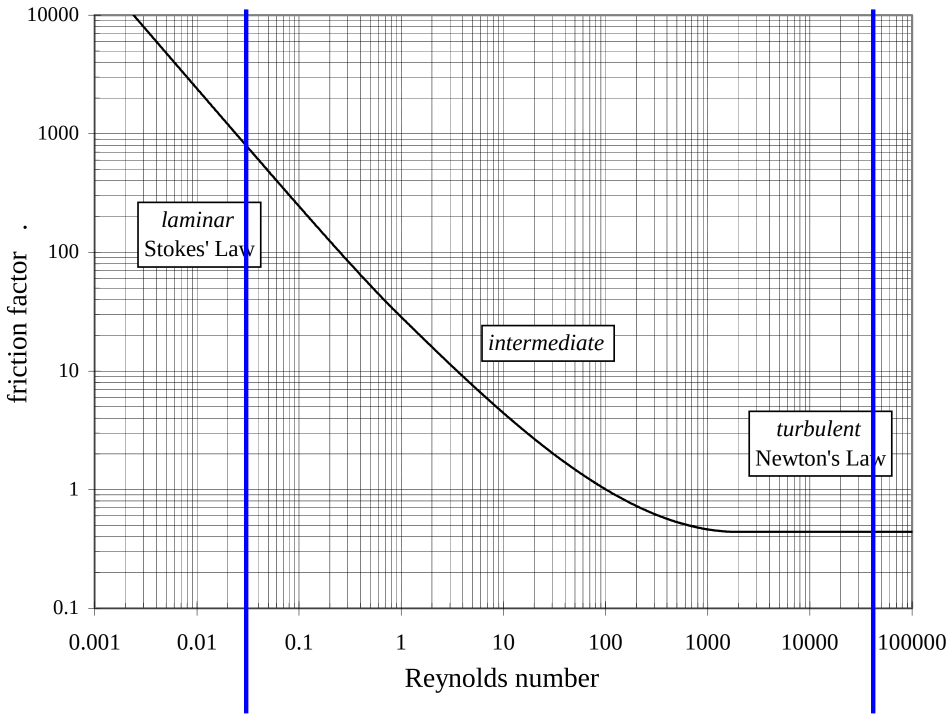
Method 3: Convert the dimensionless correlation to equations.

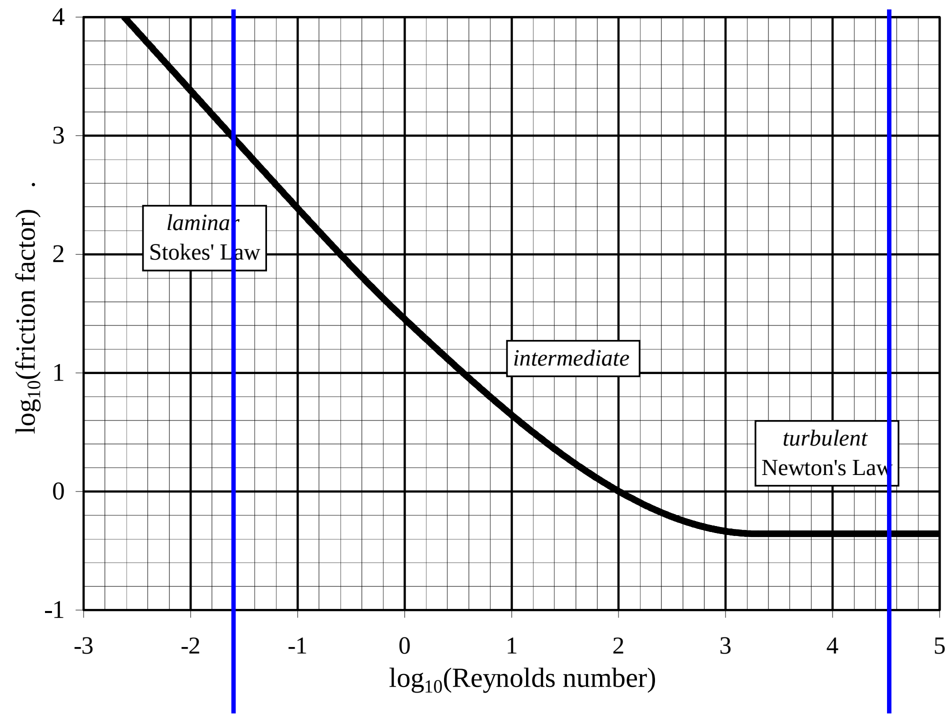
Useful for many readings. Large investment for one reading.

Method 4: Recast the dimensionless analysis with *v* as a core variable.

Useful for many readings. Not practical for one reading.

A Final Note – Log Scales





Re = 0.03

Re = 40,000

$\log_{10}(\text{Re}) = -1.6$

$\text{Re} = 10^{-1.6}$

$= 0.025$

$\log_{10}(\text{Re}) = 4.5$

$\text{Re} = 10^{4.5}$

$= 32,000$

friction factor = f(Re)

~~$\log_{10}$(friction factor) = $f(\log_{10}(\text{Re}))$~~

friction factor = f(Re)

Read Appendix D: Log-Log Graph Paper