EngrD 2190 – Lecture 32

Concept: Dimensional Analysis and Dynamic Scaling

Context: Designing a Dynamically Similar Model

Defining Question: Restaurants display models of food items.
Why don’t fashion boutiques display clothes on dolls?

EngrD 2190
Chemical Process Design & Analysis

Peer Rating of Team Members - Second Team Assignments

Due Friday 11/21

Name ____ Team # ______

Please write the names of *all* your team members, *including yourself*, and rate the degree to which each member fulfilled his/her responsibilities for team-wide learning. Because the team's goal is to learn, questions are as important as answers, and critical analysis is as important as creative design. Use the following ratings:

Excellent	Consistently went above and beyond. Carried more than his/her fair share of the load. Prepared thought-provoking questions and/or educational explanations.
Very Good	Consistently fulfilled responsibilities for team-wide learning. Very well prepared and very cooperative.
Satisfactory	Often fulfilled responsibilities for team-wide learning. Sufficiently prepared and acceptably cooperative.
Ordinary	Usually fulfilled responsibilities for team-wide learning. Minimally prepared and narrowly cooperative.
Marginal	Sometimes failed to show up or contribute to the learning. Rarely prepared.
Unsatisfactory	Consistently failed to show up or contribute to the learning. Unprepared.
Superficial	Practically no participation.
No show	No participation.

These ratings should reflect each individual's level of participation, effort, and sense of responsibility, not his or her academic ability.

Include yourself

Name of team member **(include yourself)**	Rating

Please comment on your team and on the general concept of learning by working homework in teams. If your team was dysfunctional, was it a specific problem with team members or an inherent problem with the concept?

Prelim 3, Tuesday 11/25

- Prelim 3: Tuesday 11/25, 7:30-9:30 p.m., 128 and 245 Olin Hall.
  Graphical Modeling for Mass Balances - Part 2:
    - tie lines on H-(x,y) and ternary diagrams.
    - translating thermodynamic maps: T-(x,y) to x-y, for example.
    - operating lines for multistage absorbers and strippers.
    - operating lines for multistage distillation.
    - design using single-stage and countercurrent multi-stage units - may include use of T-(x,y), P-(x,y), H-(x,y) and ternary diagrams.

  Open notes and open exercise solutions.
  Bring a calculator and a ruler. Graphing calculators are allowed.

- Practice exercises: 4.37, 4.40, 4.47, 4.48, 4.58, 4.59, 4.69, 4.70, 4.98, 4.109, and 4.110. *Solutions are posted.*

Three Skills in Dimensional Analysis / Dynamic Scaling

1. Derive a set of Dimensionless Groups given a List of Parameters.

    Lectures 28, 29, and 30:

        Pendulums Swinging, People Walking, and Spheres Falling.

    Practice Exercises: 5.8, 5.13, 5.15, 5.20, and 5.22.

2. Use a Universal Correlation of Dimensionless Groups.

    Lecture 31 Monday.

    Practice Exercises: 5.28, 5.29, and 5.30.

3. Design a Dynamically Similar Model by Scaling.

    Lectures 32 (today) and 33.

    Practice Exercises: 5.33, 5.34, and 5.39.

Solutions to practice exercises are posted at the EngrD 2190 homepage.

Designing a Dynamically Similar Model

Assume the system is described by n Π groups: $\Pi_1, \Pi_2, \ldots, \Pi_n$

1. Set $(\Pi_i)_{\text{model}} = (\Pi_i)_{\text{actual}}$ for $i = 1$ to $n - 1$.
2. Use the model to measure Π_n.
3. Use Π_n to calculate parameters for the actual system.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

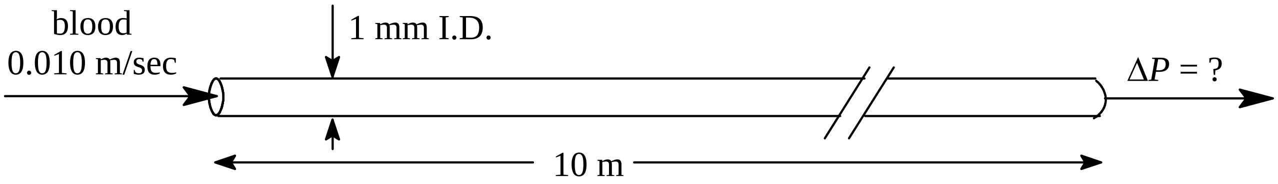


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions
pressure drop	ΔP	M/LT^2
pipe length	ℓ	L
pipe diameter	d	L
fluid velocity	v	L/T
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3

3 core variables: ΔP, ℓ, and μ.

our goal

$$\Pi_1 = \frac{\Delta P}{v^2 \rho_{fluid}} = \text{Eu, the Euler Number, } \frac{\text{friction force}}{\text{inertial force}}$$

use ×10 in model

$$\Pi_2 = \frac{\ell}{d} = \text{reduced pipe length, } \frac{\text{pipe length}}{\text{pipe diameter}}$$

$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}$$

need ×10 in model

Use to design model

Example 1: Design a model to predict the pressure drop in an artificial kidney.

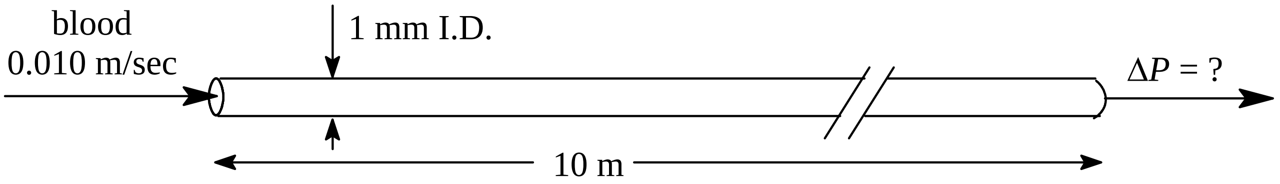


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions	artificial kidney	model	
pressure drop	ΔP	M/LT^2	?	(to be measured)	
pipe length	ℓ	L	10 m		Use Π_2 to calculate
pipe diameter	d	L	0.001 m	0.010 m	first guess
fluid velocity	v	L/T	0.010 m/s		Use Π_3 to calculate
fluid viscosity	μ	M/LT	0.0040 Pa·sec	0.050	vegetable oil
fluid density	ρ_{fluid}	M/L^3	1025 kg/m^3	930	

fluid	density (kg/m^3)	viscosity (Pa·s)
water	1000.	0.0010
blood	1025.	0.0040
vegetable oil	930.	0.050
glycerin	1250.	1.2

~×10 Use vegetable oil in the model.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

$$\Pi_1 = \frac{\Delta P}{v^2 \rho_{fluid}} = \text{Eu, the Euler Number, } \frac{\text{friction force}}{\text{inertial force}}$$

$$\Pi_2 = \frac{\ell}{d} = \text{reduced pipe length, } \frac{\text{pipe length}}{\text{pipe diameter}}$$

$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}$$

Use Π_2 to calculate the pipe length (l) in our model.

$$\left(\frac{\ell}{d}\right)_{\text{model}} = \left(\frac{\ell}{d}\right)_{\text{kidney}}$$

$$\ell_{\text{model}} = \left(\frac{\ell}{d}\right)_{\text{kidney}} d_{\text{model}} = \left(\frac{10\ \text{m}}{0.001\ \text{m}}\right) 0.01\ \text{m} = 100\ \text{m of tube}$$

Use Re (Π_3) to calculate the fluid velocity (v) in our model.

Re = 2.6

$$\left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{kidney}}$$

$$v_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{kidney}} \left(\frac{\mu}{\rho_{\text{fluid}} d}\right)_{\text{model}} = \left(\frac{1025 \times 0.01 \times 0.001}{0.004}\right)\left(\frac{0.05}{930 \times 0.01}\right)$$

$$v_{\text{model}} = 0.014\ \text{m/sec} = 1.4\ \text{cm/sec}$$ reasonable

If we used water in the model, $v_{\text{model}} = 0.03$ cm/sec. Too slow. If glycerin, $v_{\text{model}} = 25$ cm/sec. Too fast.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

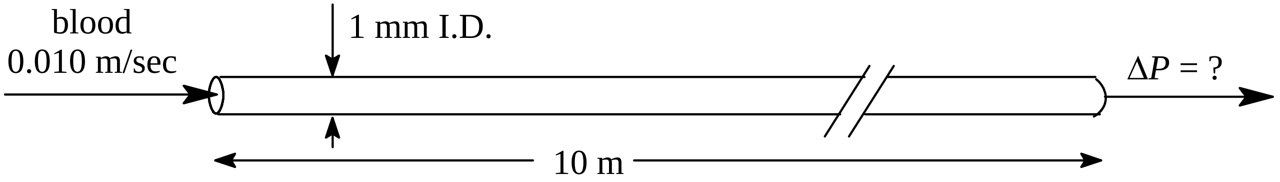


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions	artificial kidney	model
pressure drop	ΔP	M/LT^2	?	(to be measured)
pipe length	ℓ	L	10 m	100 m from Π_2
pipe diameter	d	L	0.001 m	0.010 m first guess
fluid velocity	v	L/T	0.010 m/s	0.014 m/sec from Re
fluid viscosity	μ	M/LT	0.0040 Pa·sec	0.050 } vegetable oil
fluid density	ρ_{fluid}	M/L^3	1025 kg/m³	930

We build the model and measure $\Delta P = 9.0 \times 10^5$ Pa = 8.9 atm = 130 psi

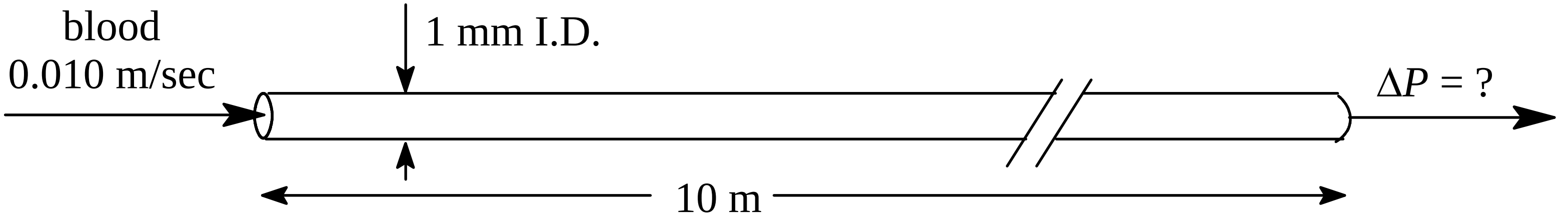
Use Π_1 to calculate ΔP in our artificial kidney.

$$\left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{kidney}} = \left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{model}}$$

$$(\Delta P)_{\text{kidney}} = \left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{model}} \left(v^2 \rho_{\text{fluid}}\right)_{\text{kidney}} = \left(\frac{9.0 \times 10^5}{(0.014)^2 \times 930}\right)\left((0.01)^2 \times 1025\right)$$

$$(\Delta P)_{\text{kidney}} = 5.1 \times 10^5 \text{ Pa} = 5.0 \text{ atm} = 74 \text{ psi}$$

Example 1: Design a model to predict the pressure drop in an artificial kidney.



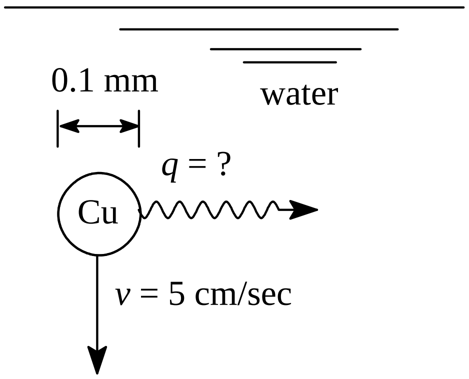
$$(\Delta P)_{\text{kidney}} = 9.0\times10^5 \text{ Pa} = 5.0 \text{ atm} = 74 \text{ psi}$$

Blood pressure is measured in units of mm Hg. 160/80 ~ 3.1 psi/1.5 psi.

Pressure drop is too large. What to do?

Use a hundred 100-cm tubes in parallel.
Total length = 10 m, ΔP = 0.74 psi

Example 2: Hot microspheres descending and cooling in a liquid.



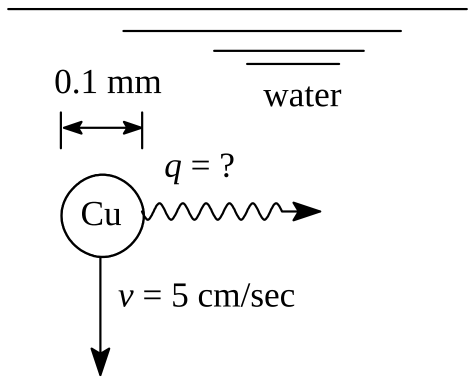
We need the sphere’s heat transfer coefficient, *h*.

$$h \equiv \frac{q}{(\text{area})(\Delta T)}$$

$$q = h(\text{sphere surface area})(T_{\text{sphere}} - T_{\text{fluid}})$$

Design a more convenient model with larger spheres.

Example 2: Hot microspheres descending and cooling in a liquid.



	parameter	symbol	dimensions
the sphere	diameter	d	L
the fluid	fluid viscosity	μ	M/LT
	fluid density	ρ_{fluid}	M/L^3
	fluid heat capacity	C_P	$L^2/\Theta T^2$
	fluid thermal conductivity	k	$ML/\Theta T^3$
dynamics	sphere velocity	v	L/T
	heat transfer coefficient	h	$M/\Theta T^3$

7 parameters – 4 dimensions = 3 core variables: h, C_P, and v.

our goal

$$\Pi_1 = \frac{hd}{k} = \text{Nu, the Nusselt Number, } \frac{\text{convective heat transfer}}{\text{conductive heat transfer}}$$

$$\left.\begin{aligned}\Pi_2 &= \frac{C_P\mu}{k} = \text{Pr, the Prandtl Number, } \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}\\ \Pi_3 &= \frac{\rho_{\text{fluid}} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}\end{aligned}\right\} \text{Use to design model}$$

The Prandtl number comprises fluid properties only.

Dynamic similarity requirement $(\text{Pr})_{\text{model}} = (\text{Pr})_{\text{actual}}$ demands model system and actual system use same fluid: water.

Example 2: Hot microspheres descending and cooling in a liquid.

	parameter	symbol	dimensions	micro spheres	model
the sphere	diameter	d	L	1×10^{-4} m	0.010 m
the fluid	fluid viscosity	μ	M/LT	10^{-3} Pa·sec	use Prandtl number; model must use water
	fluid density	ρ_{fluid}	M/L^3	1000 kg/m^3	
	fluid heat capacity	C_P	$\text{L}^2/\Theta\text{T}^2$	4200 J/(kg·K)	
	fluid thermal conductivity	k	$\text{ML}/\Theta\text{T}^3$	0.59 J/(m·sec·K)	
dynamics	sphere velocity	v	L/T	0.05 m/s	use Reynolds number
	heat transfer coefficient	h	$\text{M}/\Theta\text{T}^3$	?	measure

Calculate v_{model}.

$$\text{Re}_{\text{model}} = \text{Re}_{\text{microsphere}}$$

$$\left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{microsphere}}$$

$$v_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{microsphere}} \left(\frac{\mu}{\rho_{\text{fluid}} d}\right)_{\text{model}}$$

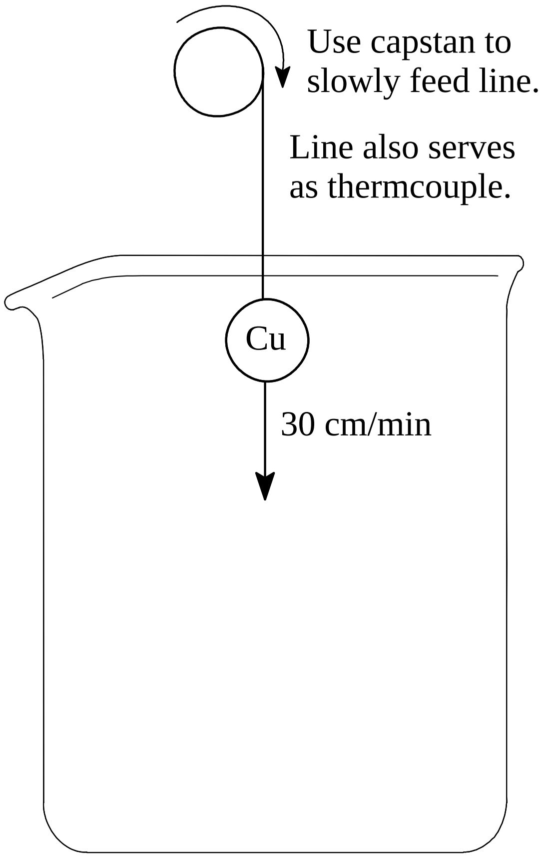
$$v_{\text{model}} = (vd)_{\text{microsphere}} \left(\frac{1}{d}\right)_{\text{model}} = \frac{0.05\ \text{m/sec} \times 10^{-4}\ \text{m}}{0.01\ \text{m}} = 5 \times 10^{-4}\ \text{m/sec}$$

v_{model} = 0.5 mm/sec = 30 cm/min

Too slow for a free-falling 1-cm copper sphere in water.

Example 2: Hot microspheres descending and cooling in a liquid.

$v_{\text{model}} = 0.5\text{ mm/sec} = 30\text{ cm/min}$



Use capstan to slowly feed line.

Line also serves as thermcouple.

30 cm/min

Conduct experiments, measure $h_{\text{model}} = 240\text{ J/(m}^2\cdot\text{sec}\cdot\text{K)}$

Calculate the Nusselt Number -

$$\text{Nu} = \frac{hd}{k} = \frac{(240\text{ J/(m}^2\cdot\text{sec}\cdot\text{K))(0.01 m)}}{0.59\text{ J/(m}\cdot\text{sec}\cdot\text{K)}} = 4.1$$

Calculate $h_{\text{microsphere}}$

$$\text{Nu}_{\text{microsphere}} = \text{Nu}_{\text{model}}$$

$$\left(\frac{hd}{k}\right)_{\text{microsphere}} = 4.1$$

$$h_{\text{microsphere}} = 4.1\left(\frac{k}{d}\right)_{\text{microsphere}}$$

$$h_{\text{microsphere}} = 4.1\left(\frac{0.59\text{ J/(m}\cdot\text{sec}\cdot\text{K)}}{0.0001\text{ m}}\right) = 2.4\times10^4\text{ J/(m}^2\cdot\text{sec}\cdot\text{K)}$$

Designing a Dynamically Similar Model by Scaling - *Recap*

Assume the system is described by n Π groups: $\Pi_1, \Pi_2, \ldots, \Pi_n$

1. Set $(\Pi_i)_{\text{model}} = (\Pi_i)_{\text{actual}}$ for $i = 1$ to $n-1$.
2. Use the model to measure Π_n.
3. Use Π_n to calculate parameters for the actual system.

If your model has a different length and/or mass dimension - your model is smaller or less massive - your model must also change something so there is a different time dimension.

Usually, use a different fluid for the model.

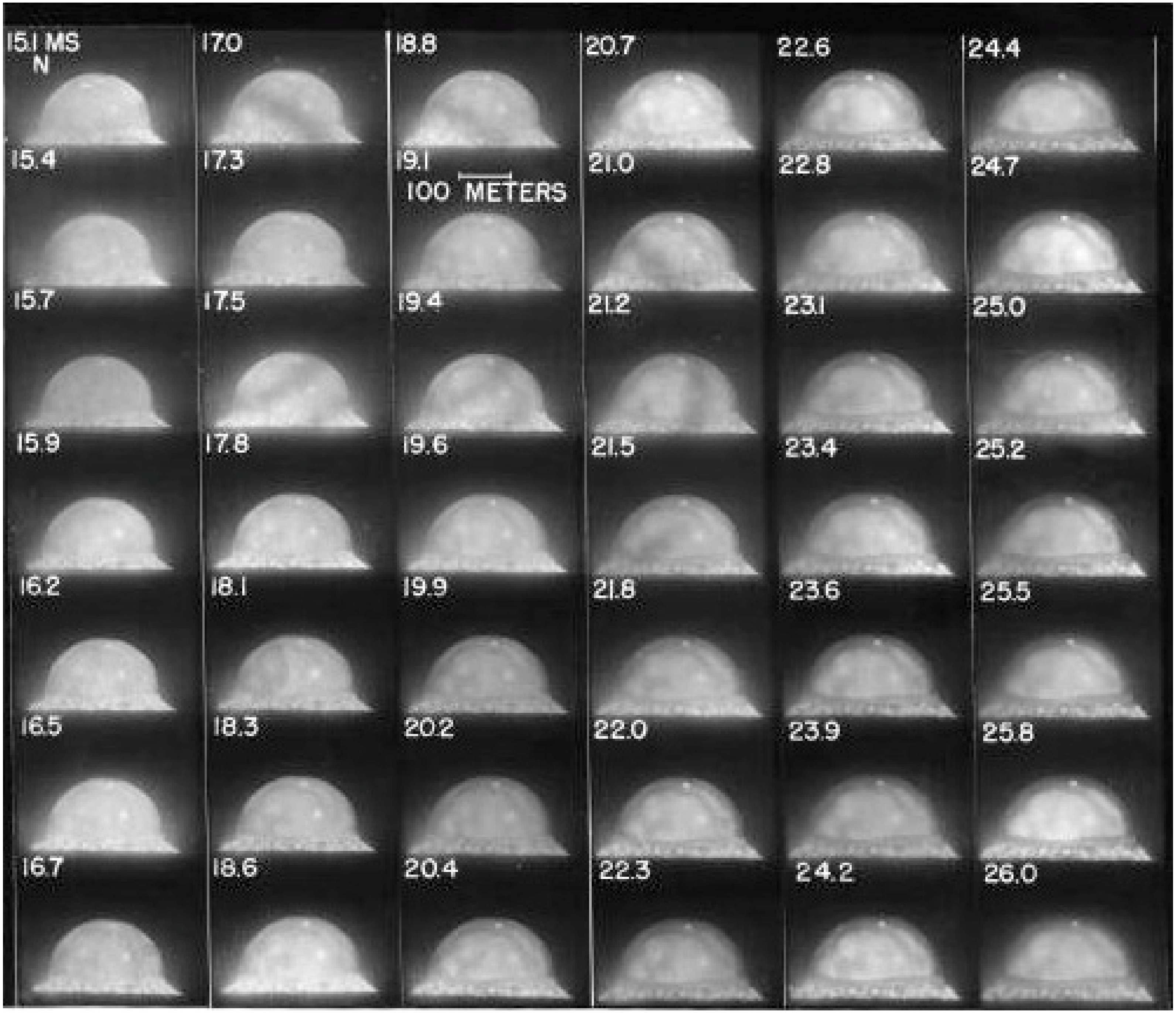
Fluid parameters: viscosity: $[\mu] = \dfrac{\mathrm{M}}{\mathrm{LT}}$

surface tension: $[\gamma] = \dfrac{\mathrm{M}}{\mathrm{T}^2}$

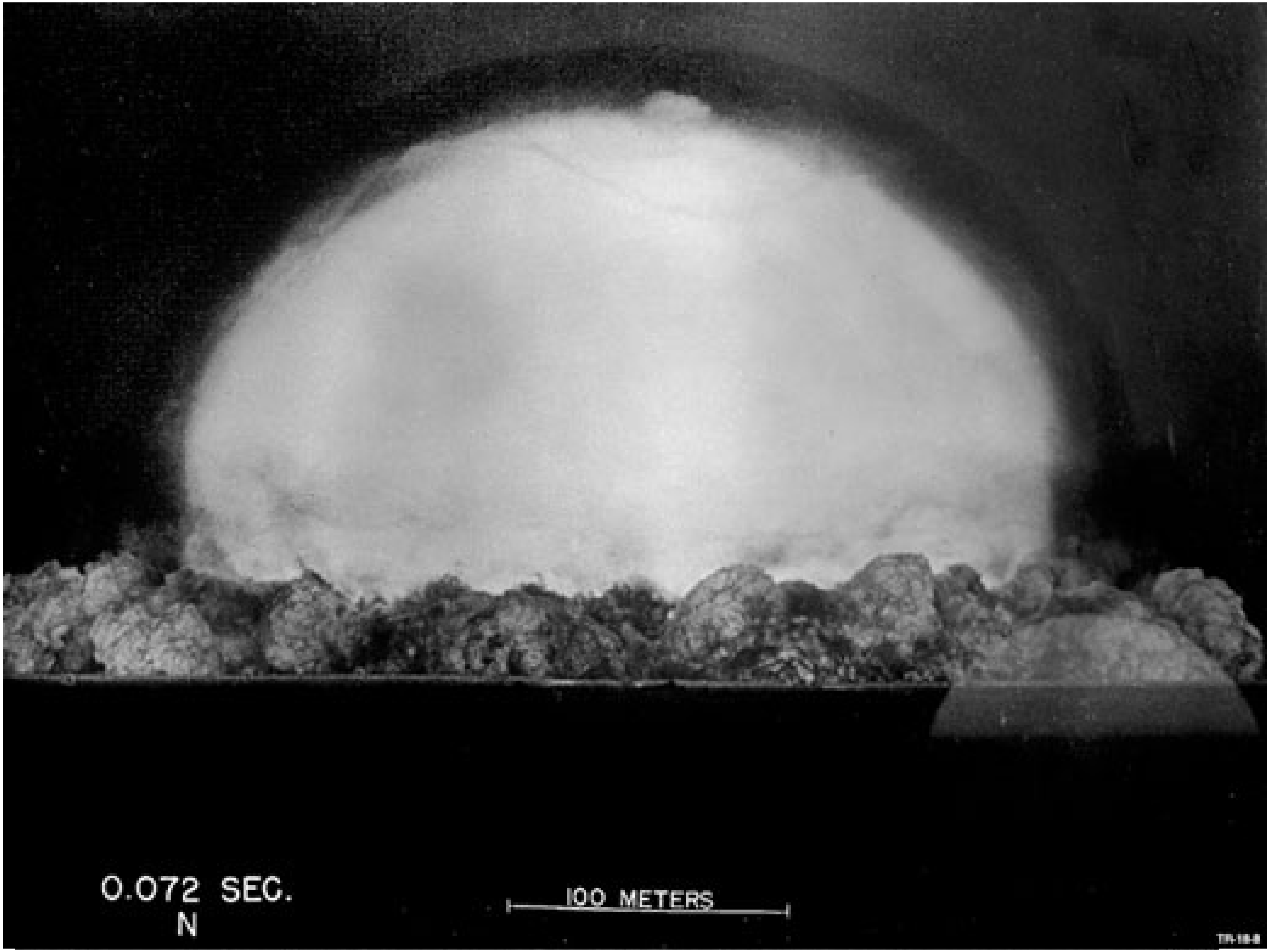
heat capacity: $[C_{\mathrm{P}}] = \dfrac{\mathrm{L}^2}{\mathrm{T}^2\Theta}$

thermal conductivity: $[k] = \dfrac{\mathrm{ML}}{\mathrm{T}^3\Theta}$

Example 3: A Universal Correlation for an Explosive Shock Wave.



Example 3: A Universal Correlation for an Explosive Shock Wave.



The formation of a blast wave by a very intense explosion.

II. The atomic explosion of 1945

By Sir Geoffrey Taylor, F.R.S.

(*Received* 10 *November* 1949)

[Plates 7 to 9]

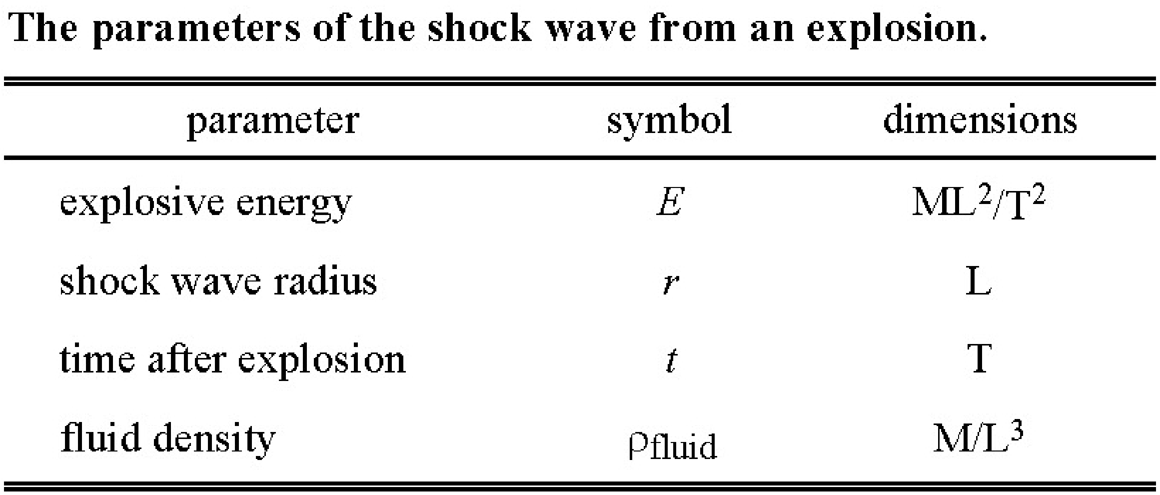
Photographs by J. E. Mack of the first atomic explosion in New Mexico were measured, and the radius, R, of the luminous globe or 'ball of fire' which spread out from the centre was determined for a large range of values of t, the time measured from the start of the explosion. The relationship predicted in part I, namely, that $R^{\frac{5}{2}}$ would be proportional to t, is surprisingly accurately verified over a range from $R=20$ to 185 m. The value of $R^{\frac{5}{2}}t^{-1}$ so found was used in conjunction with the formulae of part I to estimate the energy E which was generated in the explosion. The amount of this estimate depends on what value is assumed for γ, the ratio of the specific heats of air.

Two estimates are given in terms of the number of tons of the chemical explosive T.N.T. which would release the same energy. The first is probably the more accurate and is 16,800 tons. The second, which is 23,700 tons, probably overestimates the energy, but is included to show the amount of error which might be expected if the effect of radiation were neglected and that of high temperature on the specific heat of air were taken into account. Reasons are given for believing that these two effects neutralize one another.

After the explosion a hemispherical volume of very hot gas is left behind and Mack's photographs were used to measure the velocity of rise of the glowing centre of the heated volume. This velocity was found to be 35 m./sec.

Until the hot air suffers turbulent mixing with the surrounding cold air it may be expected to rise like a large bubble in water. The radius of the 'equivalent bubble' is calculated and found to be 293 m. The vertical velocity of a bubble of this radius is $\frac{2}{3}\sqrt{(g\,29300)}$ or 35·7 m./sec. The agreement with the measured value, 35 m./sec., is better than the nature of the measurements permits one to expect.

Example 3: A Universal Correlation for an Explosive Shock Wave.



The parameters of the shock wave from an explosion.

parameter	symbol	dimensions
explosive energy	E	ML^2/T^2
shock wave radius	r	L
time after explosion	t	T
fluid density	ρ_{fluid}	M/L^3

4 parameters – 3 dimensions
= 1 Π group

$$\Pi = E^a r^b t^c \rho^d$$

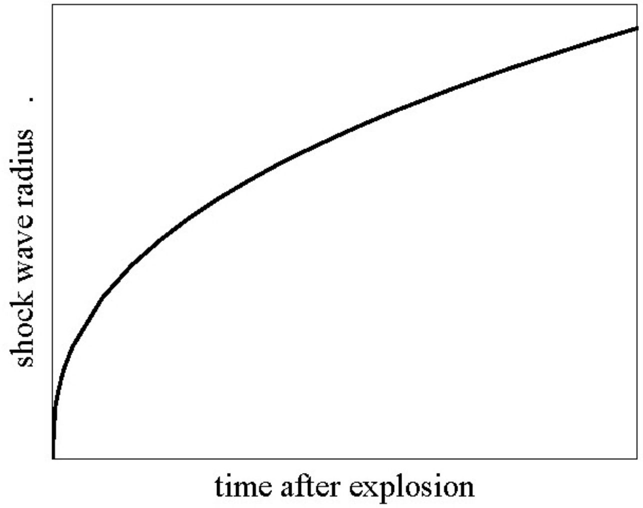
$$[\Pi] = \left[E^a r^b t^c \rho^d\right] = \left(\frac{\mathrm{M\,L}^2}{\mathrm{T}^2}\right)^a \mathrm{L}^b \mathrm{T}^c \left(\frac{\mathrm{M}}{\mathrm{L}^3}\right)^d = \mathrm{M}^{a+d}\mathrm{L}^{2a+b-3d}\mathrm{T}^{-2a+c}$$

Set $a = 1$

$$\Pi = \left(\frac{Et^2}{\rho r^5}\right)^{1/5}$$

From experiments, Π = 1

Thus $r = \left(\frac{Et^2}{\rho}\right)^{1/5}$



From the declassified photos, Geoffrey Taylor calculated the energy of the Trinity explosion:

E = 28 kilotons of TNT

Dimensional Analysis and Dynamic Scaling

Trucks, Power Shovels, and Barbie Dolls

Dynamic Similarity in Trucks

Dynamic Similarity in Trucks

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Dynamic Similarity in Trucks

Height to Load Bed

Taylor Dunn ET-3000

Terex TR-100

$$3.5\text{ ft} \xrightarrow{\times 4.3} 15\text{ ft}$$

Dynamic Similarity in Trucks

Vehicle Weight

Taylor Dunn ET-3000

Terex TR-100

$$0.95 \text{ ton} \xrightarrow{\times (4.3)^3} 76 \text{ ton}$$

Dynamic Similarity in Trucks

Load Capacity

Taylor Dunn ET-3000

Terex TR-100

$$3000\text{ lb} = 1.5\text{ ton} \xrightarrow{\times(4.1)^3} 100\text{ ton}$$

Dynamic Similarity in Trucks

Engine Power

Taylor Dunn ET-3000

Terex TR-100

$$15 \text{ hp} \xrightarrow{\times(4.1)^3} 1050 \text{ hp}$$

Dynamic Similarity in Trucks

Tires

Taylor Dunn ET-3000

Terex TR-100

20.5×8×10
20 inches = 1.7 ft

$$\xrightarrow{\times(4.3)^{3/2}}$$

59/80R63
14 ft

How Tires Carry the Load

Tire Contact Area Scales as Height Squared

How to Scale Tire Height?

Weight scales as $(\text{Height})^3$

Increase Tire Height as $(\text{Height})^1$:

$\Rightarrow$ Tire Area increases as $(\text{Height})^2$

Not Enough Tire Area

Increase Tire Height as $(\text{Height})^{3/2}$:

$\Rightarrow$ Tire Area increases as $(\text{Height}^{3/2})^2 = (\text{Height})^3$

Enough Tire Area!

Dynamic Similarity in Trucks
Tires

Taylor Dunn ET-3000

Terex TR-100

20.5×8×10
20 inches = 1.4 ft

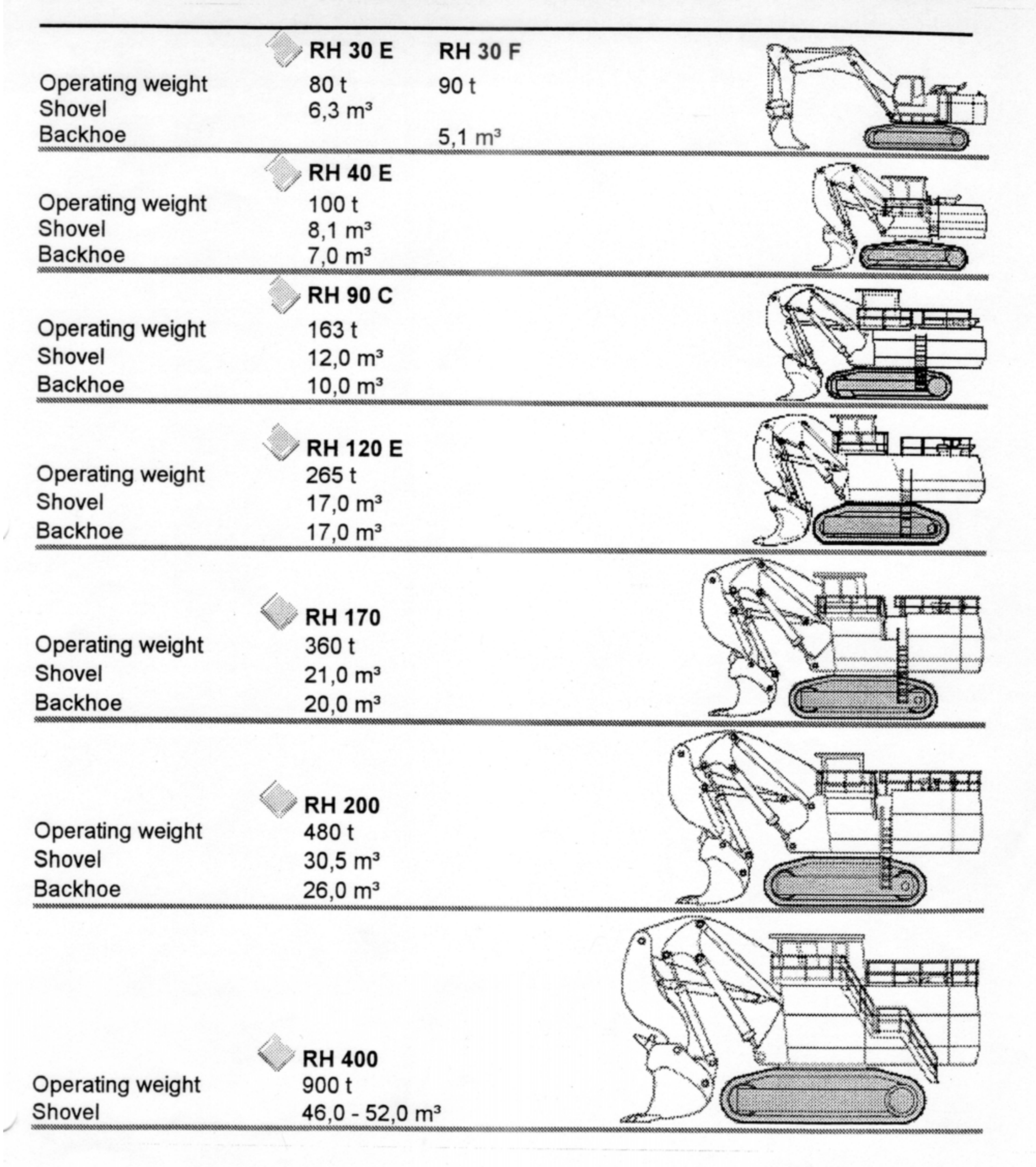
$$\xrightarrow{\times(4.3)^{3/2}}$$

59/80R63
14 ft

Dynamic Similarity in Power Shovels

Dynamic Similarity in Power Shovels

Dynamic Similarity in Power Shovels



	RH 30 E	RH 30 F
Operating weight	80 t	90 t
Shovel	6,3 m^3	
Backhoe		5,1 m^3

	RH 40 E
Operating weight	100 t
Shovel	8,1 m^3
Backhoe	7,0 m^3

	RH 90 C
Operating weight	163 t
Shovel	12,0 m^3
Backhoe	10,0 m^3

	RH 120 E
Operating weight	265 t
Shovel	17,0 m^3
Backhoe	17,0 m^3

	RH 170
Operating weight	360 t
Shovel	21,0 m^3
Backhoe	20,0 m^3

	RH 200
Operating weight	480 t
Shovel	30,5 m^3
Backhoe	26,0 m^3

	RH 400
Operating weight	900 t
Shovel	46,0 - 52,0 m^3

Restaurants display menu items.

Why don't fashion boutiques display clothes on dolls?

Dynamic Scaling in Fashion Dolls

Dynamic Scaling in Fashion Dolls

Dynamic Scaling in Fashion Dolls

Why does this garment display poorly?

What scaling was ignored?

Thread count!

The cloth used for this garment is the same cloth used at full scale.

The scaled equivalent is burlap at full scale, which has a thread count of about 10-12 per inch.

Burlap garments hang poorly.