EngrD 2190 – Lecture 33

Concept: Dimensional Analysis and Dynamic Scaling

Context: Designing a Dynamically Similar Model, cont'd

Defining Question: Why don't ants throw stones?

EngrD 2190
Chemical Process Design & Analysis

Peer Rating of Team Members - Second Team Assignments

Due TODAY

Name ________________________________ Team # ______

Please write the names of *all* your team members, *including yourself*, and rate the degree to which each member fulfilled his/her responsibilities for team-wide learning. Because the team's goal is to learn, questions are as important as answers, and critical analysis is as important as creative design. Use the following ratings:

Excellent	Consistently went above and beyond. Carried more than his/her fair share of the load. Prepared thought-provoking questions and/or educational explanations.
Very Good	Consistently fulfilled responsibilities for team-wide learning. Very well prepared and very cooperative.
Satisfactory	Often fulfilled responsibilities for team-wide learning. Sufficiently prepared and acceptably cooperative.
Ordinary	Usually fulfilled responsibilities for team-wide learning. Minimally prepared and narrowly cooperative.
Marginal	Sometimes failed to show up or contribute to the learning. Rarely prepared.
Unsatisfactory	Consistently failed to show up or contribute to the learning. Unprepared.
Superficial	Practically no participation.
No show	No participation.

These ratings should reflect each individual's level of participation, effort, and sense of responsibility, not his or her academic ability.

Name of team member **(include yourself)**	Rating
______________________	____________
______________________	____________
______________________	____________
______________________	____________

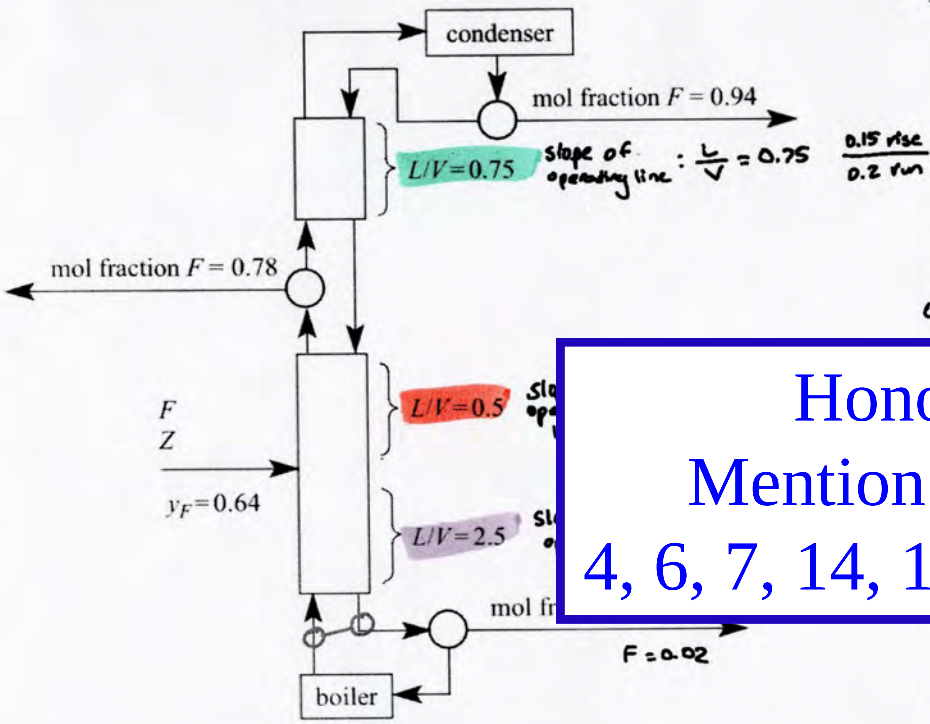
Include yourself

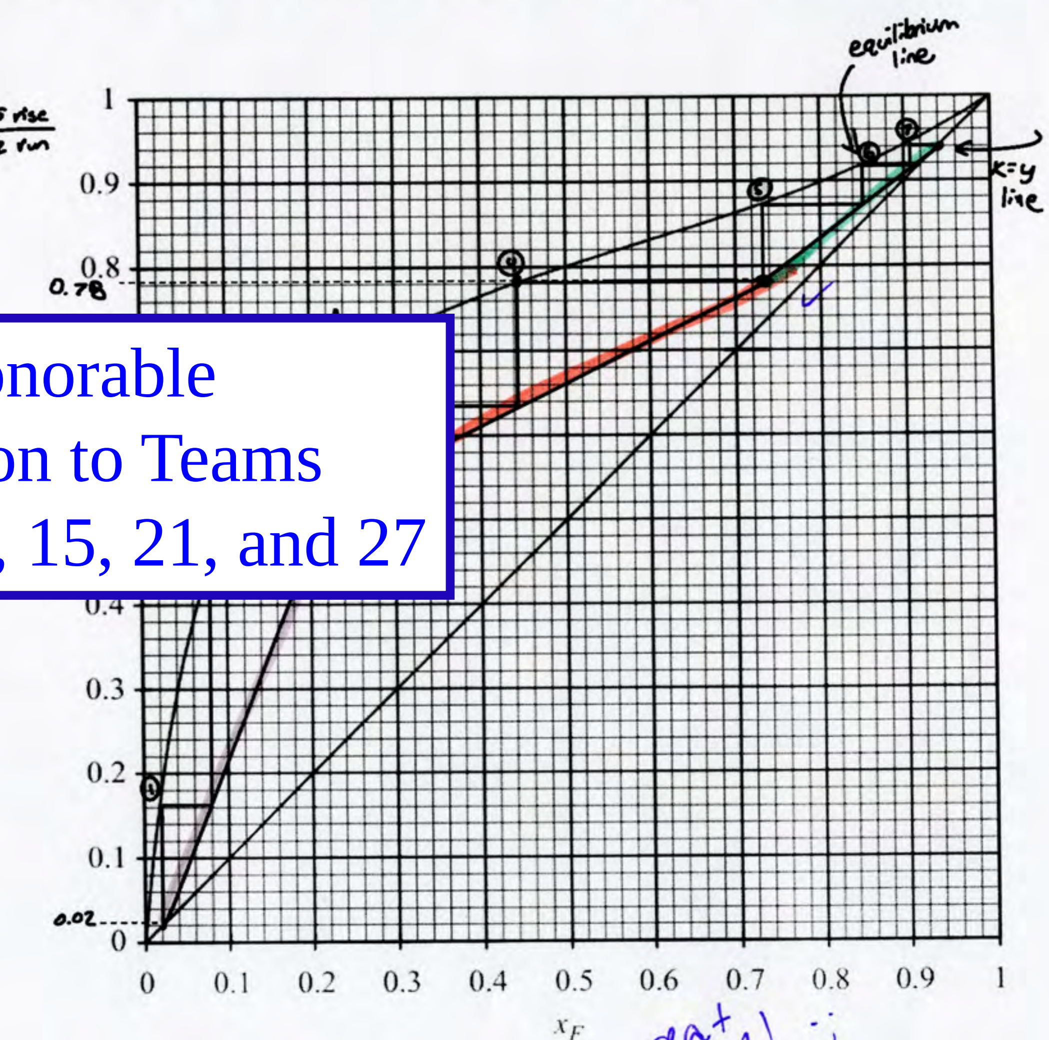
Please comment on your team and on the general concept of learning by working homework in teams. If your team was dysfunctional, was it a specific problem with team members or an inherent problem with the concept?

Homework 8 Excellence – Exercise 4.67 – Team 11

Week coordinator: Isabelle Bennie
Steffanie Jones
Anna Voronova

(B) The distillation column shown below provides an intermediate output stream. How many stages are needed? At what stage does the feed enter? Above what stage does the intermediate stream leave? **Hint:** Contrary to usual practice, begin stepping off stages at the top.





Honorable Mention to Teams 4, 6, 7, 14, 15, 21, and 27

- 7 stages are needed ✓
- feed enters at stage 3 ✓
- The intermediate stream leaves above 4 (between 4 & 5) ✓

great job! :)

Takeaways:

1. Stages are labeled using the equilibrium line
2. The slope of the operating line is $\frac{L}{V}$
3. When fractions leave the system the slope of the operating line changes.

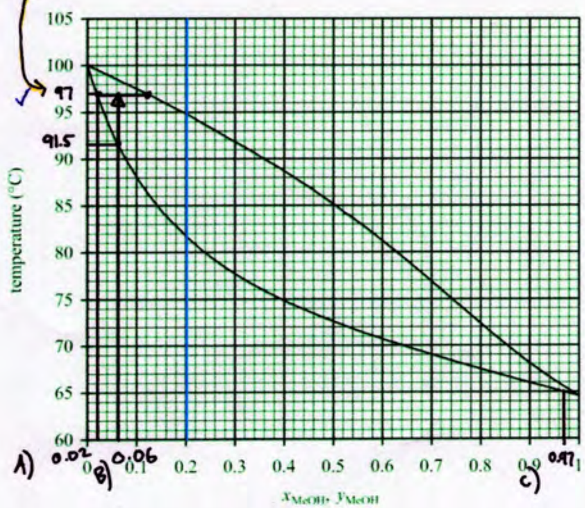
Homework 8 Excellence – Exercise 4.68 – Team 4

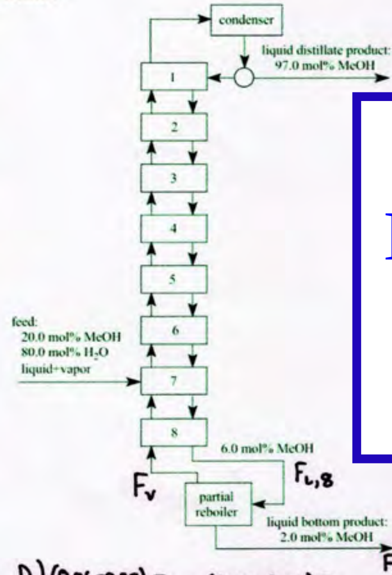
Honorable Mention to Teams 9, 12, 17, 18, 23, and 25

4.68: Andy Glesmann, Evan Brezicki, Bradley Reiff Team 4

4.68 The distillation column shown below differs from traditional columns: the reboiler is an additional equilibrium stage. The liquid from the bottom stage of the column (stage 8) is *partially* reboiled. The liquid and vapor streams that leave the reboiler are in equilibrium.

A) 97°C



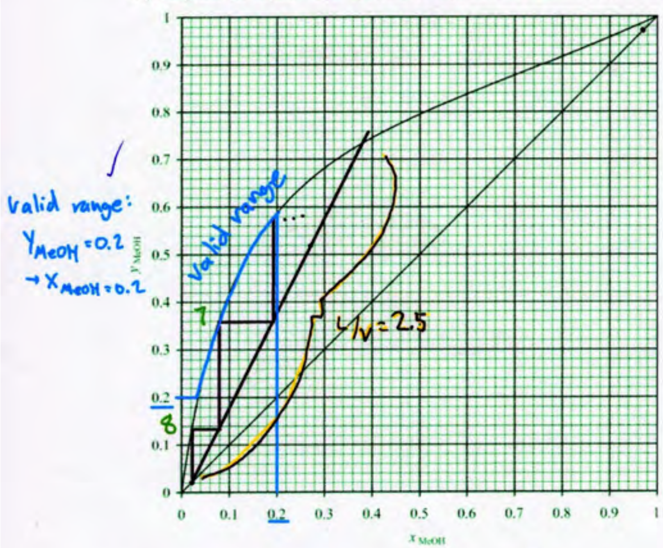


x–y diagram for liquid–vapor systems of methanol + H_2O mixtures at 1 atm.

D) $(0.06-0.02)F_L = (0.12-0.06)F_V$

$\Rightarrow \frac{F_L}{F_V} = 1.5$, $F_{L,8} = F_V + F_L$

$\Rightarrow \frac{F_{L,8}}{F_V} = \frac{F_L + F_V}{F_V} = 2.5$



Takeaways:
- Use eq Points to find T
- Reboilers can be an equilibrium stage
- Feed stream doesn't need to be in the middle

(B) What is the temperature in stage 8 (in °C)? 91.5°C

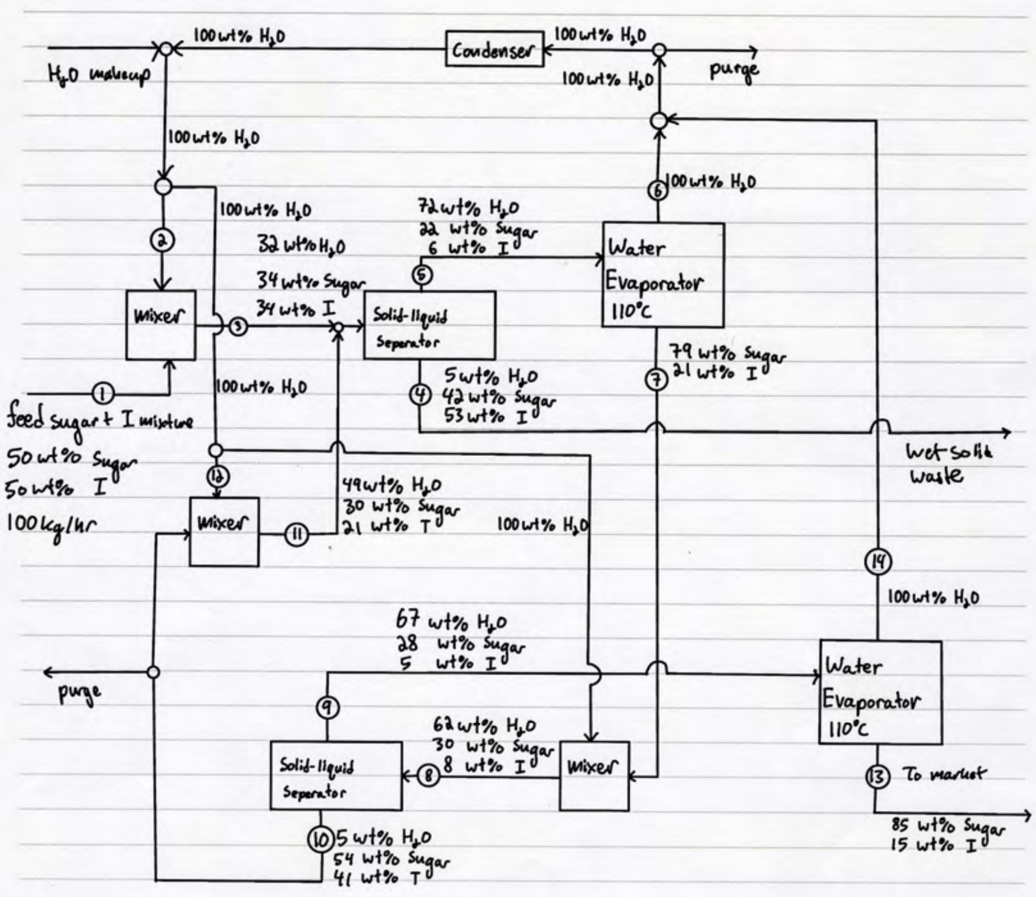
(C) What is the maximum temperature in the condenser (in °C)? 65°C

(D) What is the slope of the operating line for the stripping section (the stages below the feed stage)? 2.5

(E) Is stage 7 an appropriate feed stage for the specifications shown below? If not, which stage is appropriate? Stage 7 is in the valid range for the feed stream. So stage 7 is an appropriate feed stage.

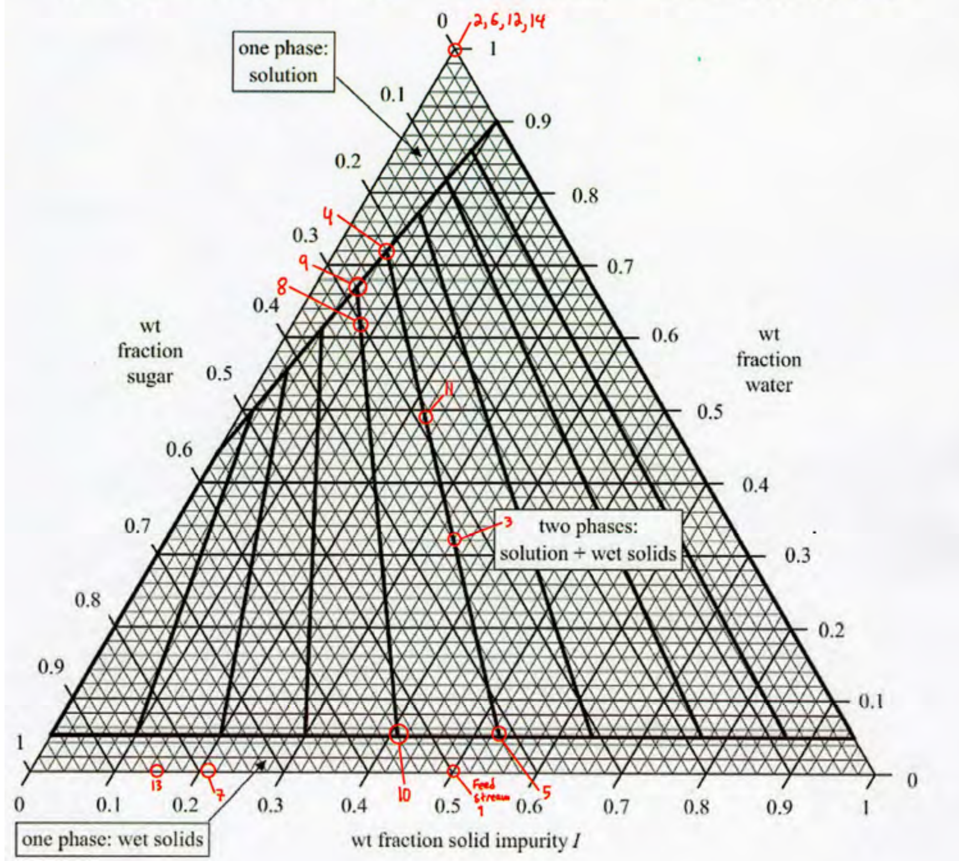
Homework 8 Excellence – Exercise 4.103 – Team 9

(D) Design a process to produce dry solids with less than 15. wt% impurity *I*. You may start with the process above and add units. Or you may start with the feed (stream 1) and design your own process. Number every stream that leaves a separator or an evaporator and indicate the stream on the ternary phase diagram.



We wish to decrease the amount of impurity *I* in a mixture of sugar and *I*. The process below capitalizes on the solubility differences to decrease the wt fraction of *I* in the mixture.

Ternary phase diagram for sugar + *I* (soluble solid) + water mixtures at 1 atm.

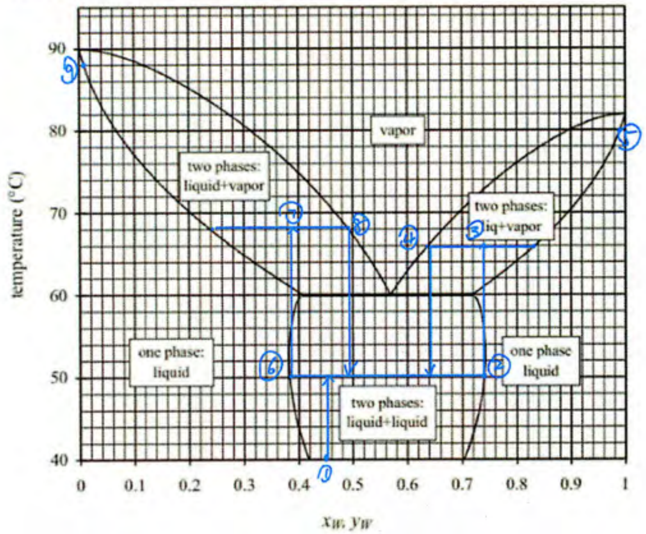


Honorable Mention to Teams 11, 18, 21, 23, and 24.

Homework 8 Excellence – Exercise 4.108 – Team 12

Team 12 — Youyou Xie yx645; Xindi Zhang xz985 ✓; Hank Xue zx344

4.108 Temperature–composition phase diagram for $W + R$ mixtures at 1 atm.



(A) at 50 °C, because at 50 °C we have the most W-rich and R-rich liquid at the ends of the tie line.

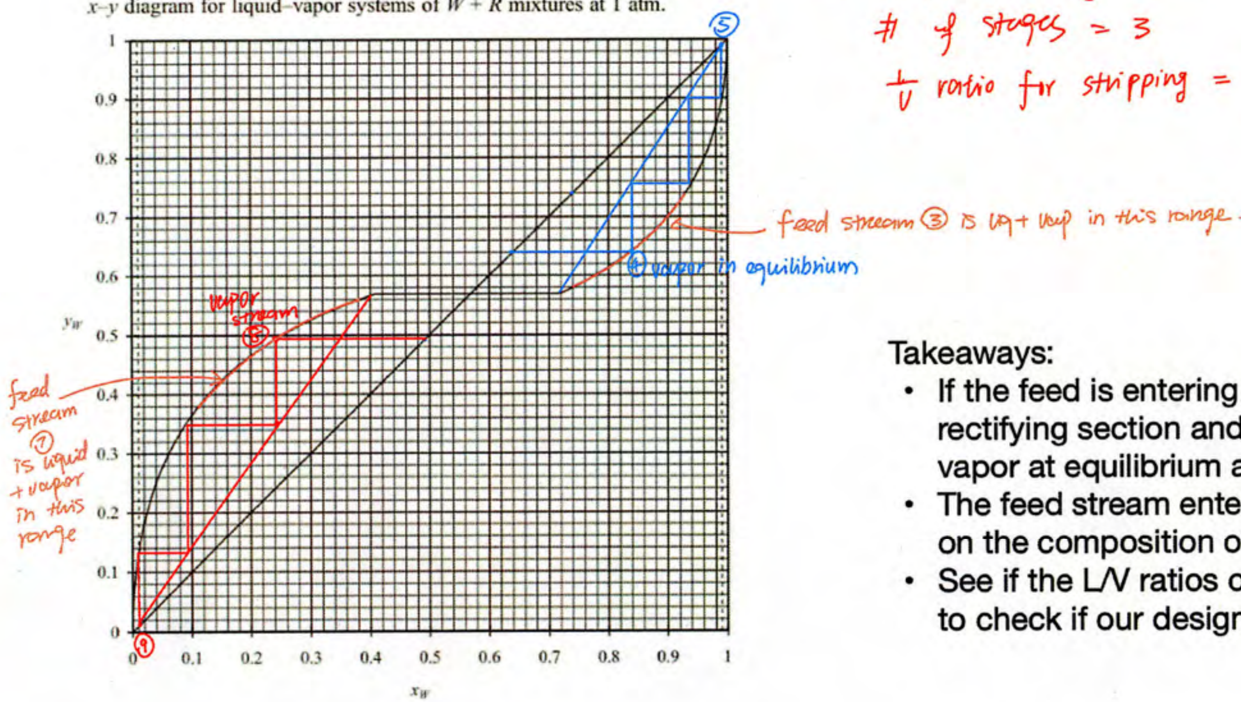
(B) on the graph

(C) temperature of heater 2 : 66 °
\# of stages = 3
L/V ratio for stripping = 0.99 − … / 0.99 − …

(d) temperature of heater 3 :
\# of stages = 3
L/V ratio for stripping = (0.57 − 0.01) / (0.4 − 0.01) = 1.44.

Honorable Mention to Teams 3, 4, 6, 11, 12, 14, 17, 19, 21, 23, and 30.

x–y diagram for liquid–vapor systems of $W + R$ mixtures at 1 atm.



Takeaways:

- If the feed is entering at the top stage, there isn't rectifying section and the distillate stream is just vapor at equilibrium at preheated temperature
- The feed stream enters at a range which depends on the composition out from the heater
- See if the L/V ratios of stripping is greater than 1 to check if our design makes sense

Prelim 3, Tuesday 11/25

- Prelim 3: Tuesday 11/25, 7:30-9:30 p.m., 128 and 245 Olin Hall.
  Graphical Modeling for Mass Balances - Part 2:
    tie lines on H-(x,y) and ternary diagrams.
    translating thermodynamic maps: T-(x,y) to x-y, for example.
    operating lines for multistage absorbers and strippers.
    operating lines for multistage distillation.
    design using single-stage and countercurrent multi-stage units -
      may include use of T-(x,y), P-(x,y), H-(x,y) and ternary diagrams.

  Open notes and open exercise solutions.
  Bring a calculator and a ruler. Graphing calculators are allowed.

- Practice exercises: 4.37, 4.40, 4.47, 4.48, 4.58, 4.59, 4.69, 4.70, 4.98, 4.109, and 4.110. *Solutions are posted.*

Lecture Monday 11/24 is cancelled

Lecture is cancelled to accommodate the dynamic scaling lab session.

There will be no dynamic scaling lab in Fall 2025.

Lecture Monday 12/1 is cancelled

Lecture is cancelled to accommodate the dynamic scaling lab session.

There will be no dynamic scaling lab in Fall 2025.

Three Skills in Dimensional Analysis / Dynamic Scaling

1. Derive a set of Dimensionless Groups given a List of Parameters.

Lectures 28, 29, and 30:

Pendulums Swinging, People Walking, and Spheres Falling.

Practice Exercises: 5.8, 5.13, 5.15, 5.20, and 5.22.

2. Use a Universal Correlation of Dimensionless Groups.

Lecture 31.

Practice Exercises: 5.28, 5.29, and 5.30.

3. Design a Dynamically Similar Model by Scaling.

Lectures 32 and 33 (today).

Practice Exercises: 5.33, 5.34, and 5.39.

Solutions to practice exercises are posted at the EngrD 2190 homepage.

Designing a Dynamically Similar Model by Scaling - *Recap*

Assume the system is described by n Π groups: $\Pi_1, \Pi_2, \ldots, \Pi_n$

1. Set $(\Pi_i)_{\text{model}} = (\Pi_i)_{\text{actual}}$ for $i = 1$ to $n-1$.
2. Use the model to measure Π_n.
3. Use Π_n to calculate parameters for the actual system.

If your model has a different length and/or mass dimension - your model is smaller or less massive - your model must also change something so there is a different time dimension.

Usually, use a different fluid for the model.

Fluid parameters: viscosity: $[\mu] = \dfrac{\text{M}}{\text{LT}}$

surface tension: $[\gamma] = \dfrac{\text{M}}{\text{T}^2}$

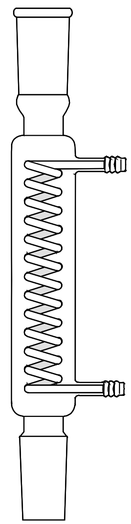
heat capacity: $[C_\text{P}] = \dfrac{\text{L}^2}{\text{T}^2\Theta}$

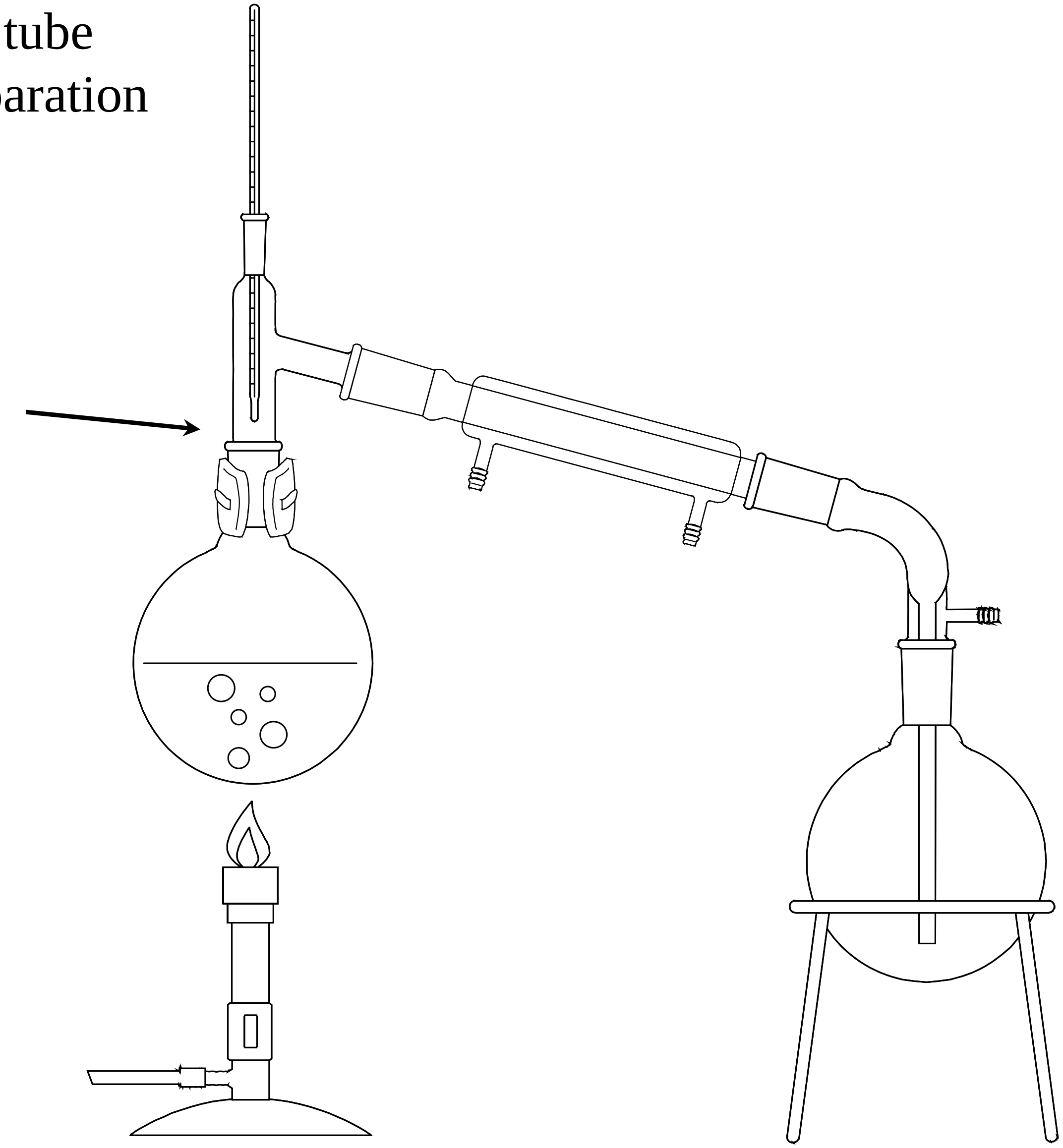
thermal conductivity: $[k] = \dfrac{\text{ML}}{\text{T}^3\Theta}$

Dynamic Scaling in Distillation

Insert condenser tube
for improved separation

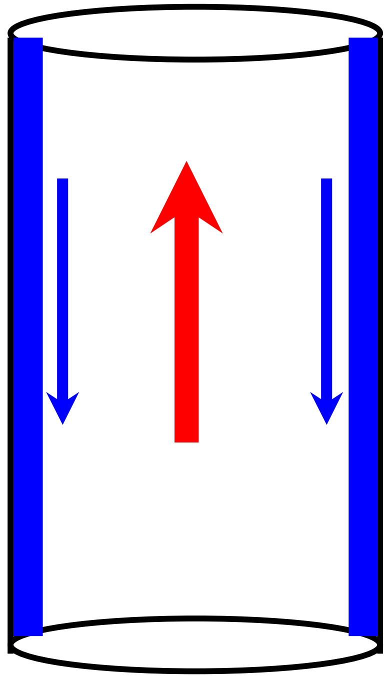
Low volatility component
condenses and liquid
returns to boiling flask.





This design works well
at the lab-bench scale.
why bother with equilibrium
trays in large distillation?

Dynamic Scaling in Distillation



Liquid film condenses on inside of tube and flows downward.

Vapor rises inside tube. Low volatility component is selectively absorbed at the vapor-liquid interface.

Rate of equilibration is limited by the interfacial area.

For a cylindrical tube of height *h* and diameter *d* -

$$\frac{\text{interfacial area}}{\text{vapor volume}} = \frac{\cancel{h} \times \text{circumference}}{\cancel{h} \times \text{cross sectional area}} = \frac{\pi d}{\pi\left(\frac{d}{2}\right)^2} = \frac{4}{d}$$

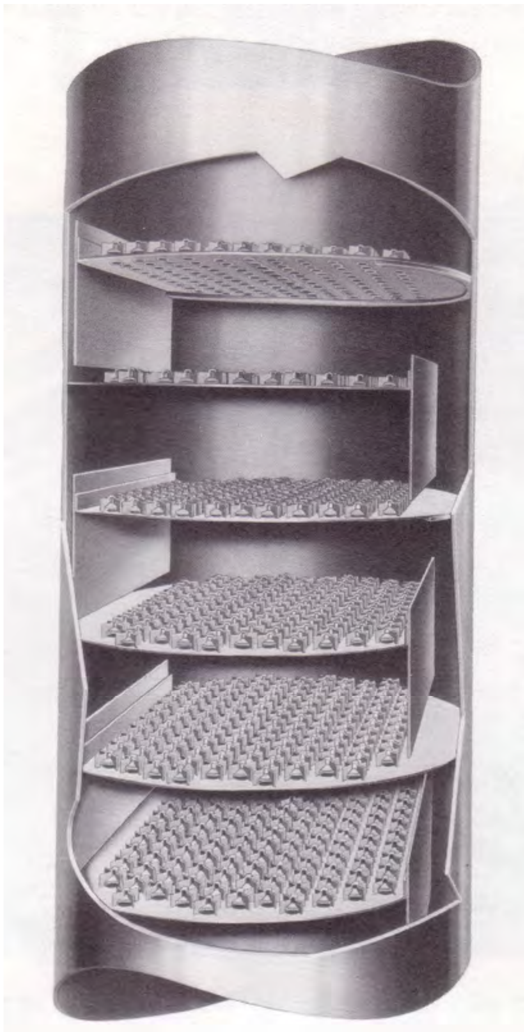
Ratio is proportional to $1/d$.

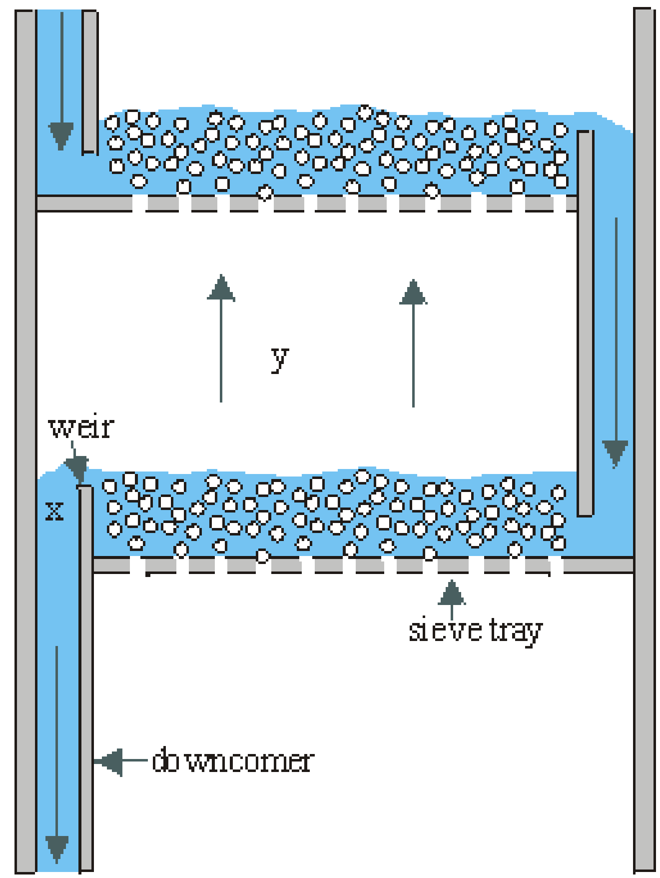
$$\text{for 1-cm diameter tube}: \frac{400\ \text{m}^2}{1\ \text{m}^3} \qquad \text{for 1-m diameter tube}: \frac{4\ \text{m}^2}{1\ \text{m}^3}$$

Dynamic Scaling in Distillation

How to maintain 1-cm as key length scale in large distillation columns?

Small vapor bubbles rising through liquid.





Dynamic Scaling in Distillation

How to maintain 1-cm as key length scale in large distillation columns?

Liquid films on small (1 cm) solid pieces.

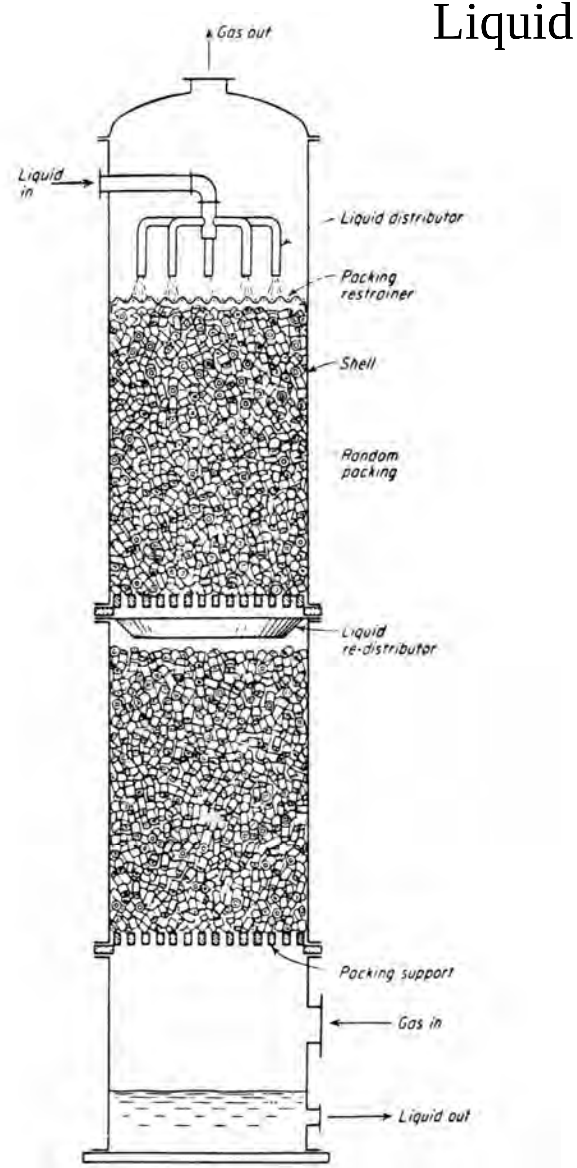
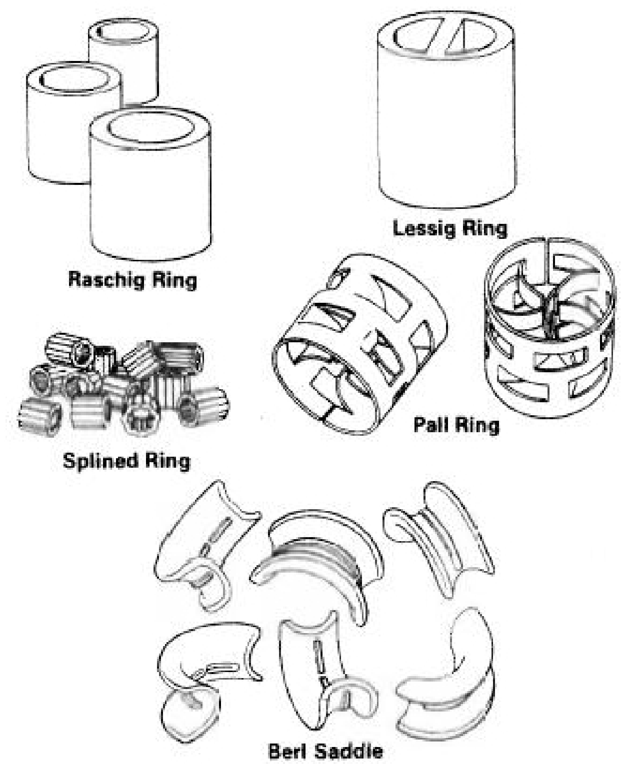


Figure 1-2 Gas absorber. (R. E. Treybal, *Mass Transfer Operations*, McGraw-Hill, New York, 1955, p. 134.)

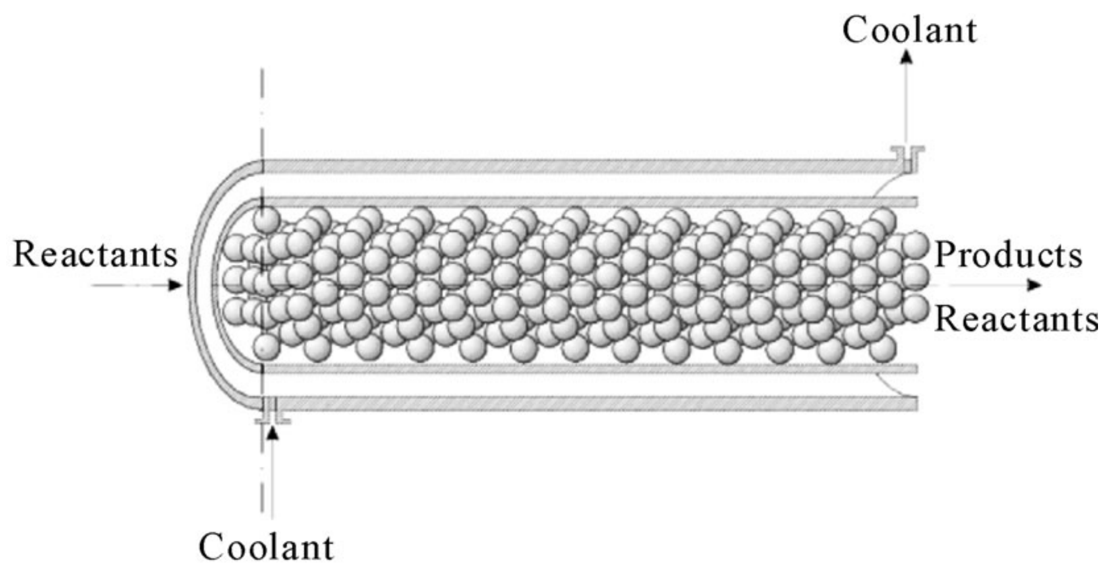


Flow through Packed Beds

Coffee makers

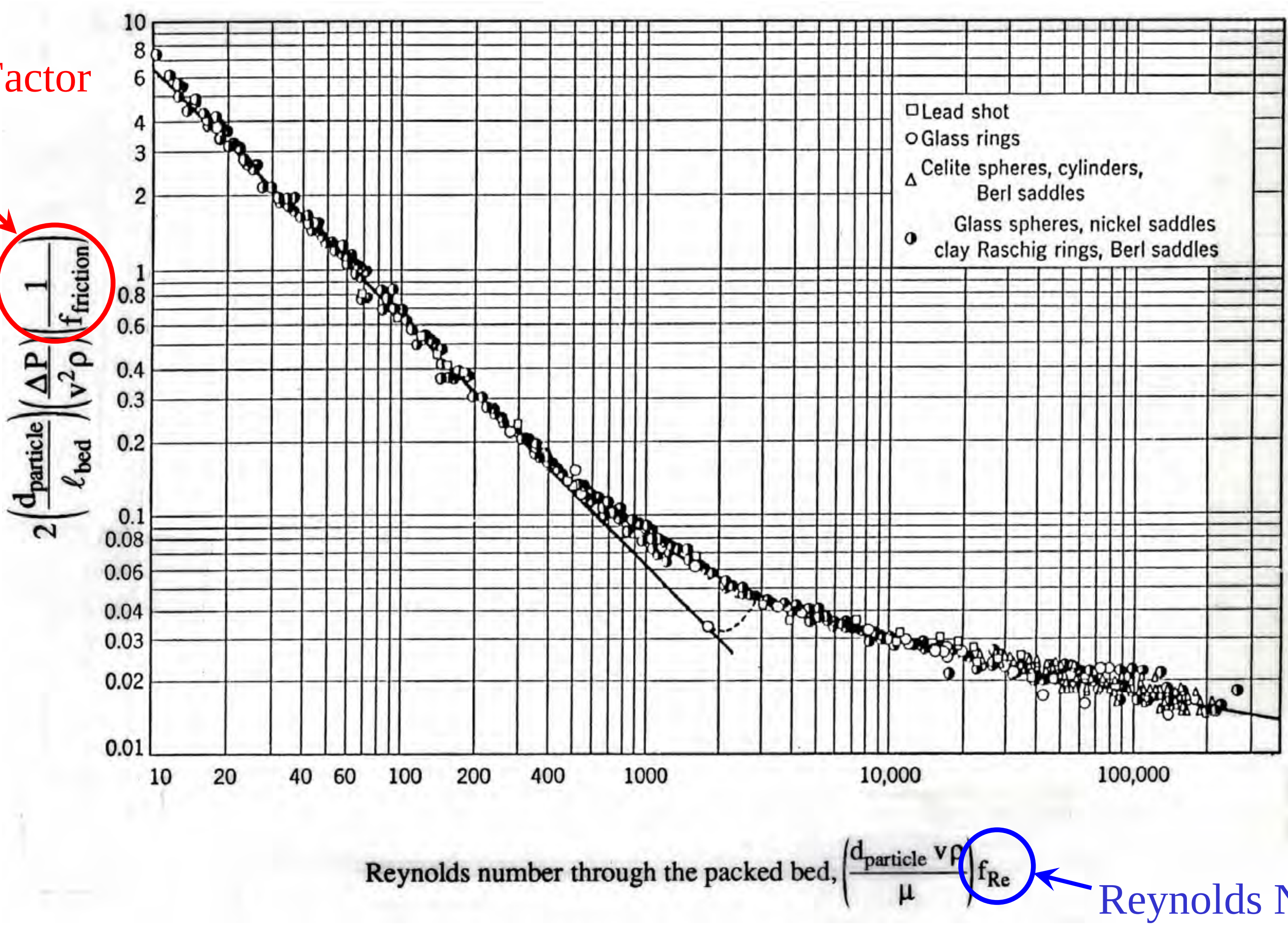
Water purifiers

Catalytic reactors

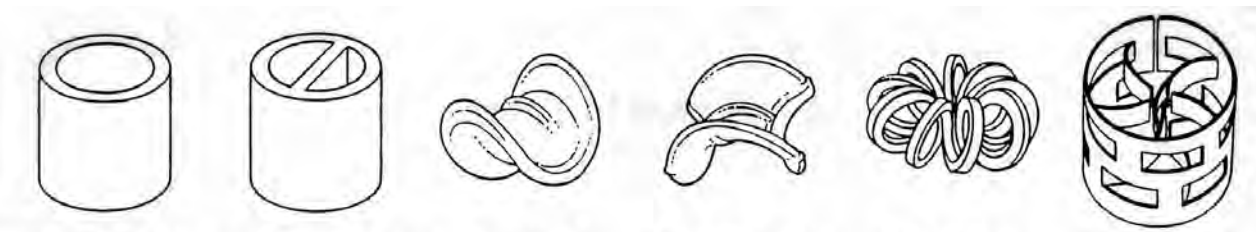


Flow through a Packed Bed – Pressure Drop?

Friction Factor
Factor

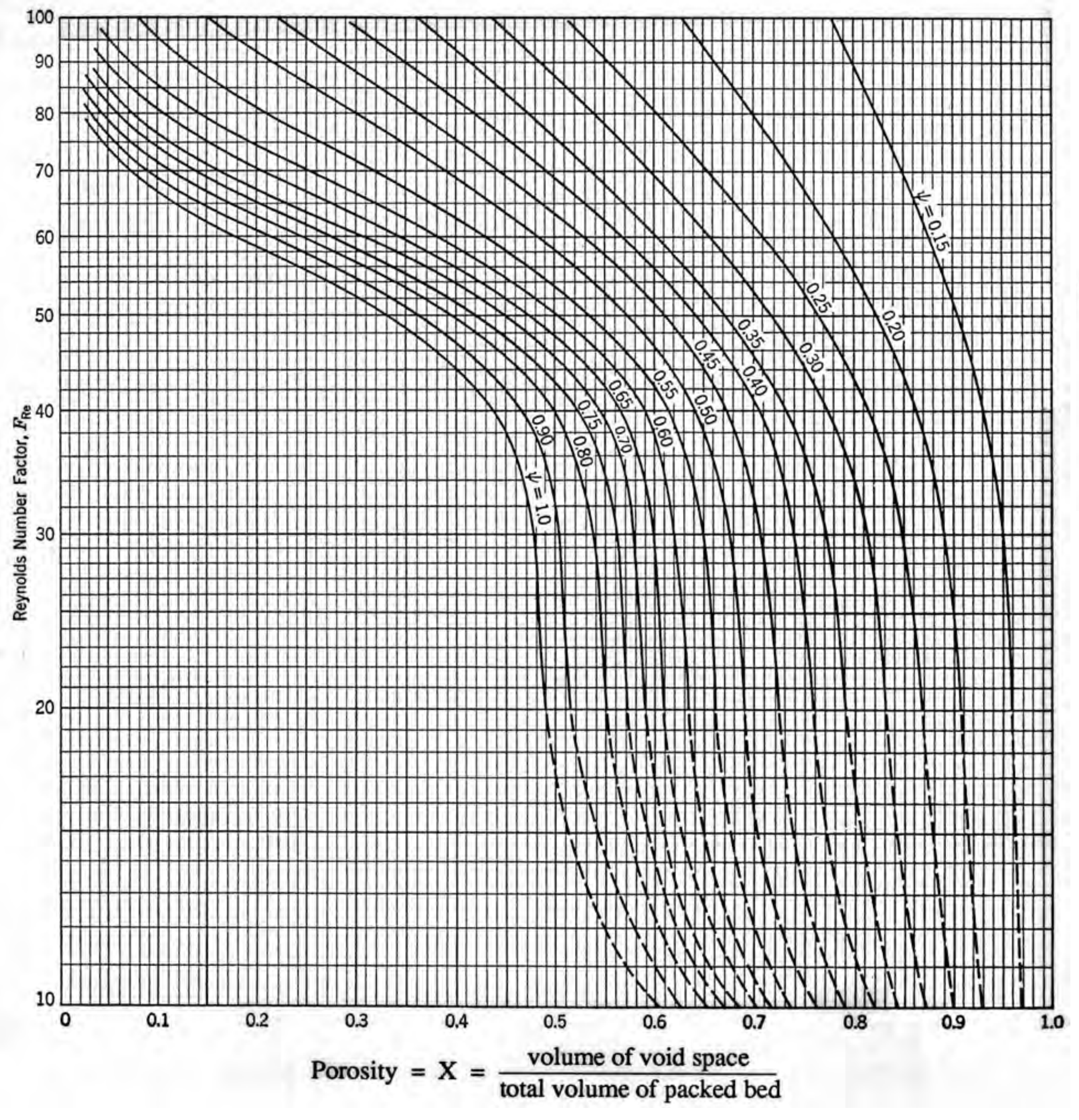


Reynolds Number Factor



(a) Raschig ring (b) Lessing ring (c) Berl saddle (d) Intalox saddle (e) Tellerette (f) Pall ring

The Reynolds Number Factor



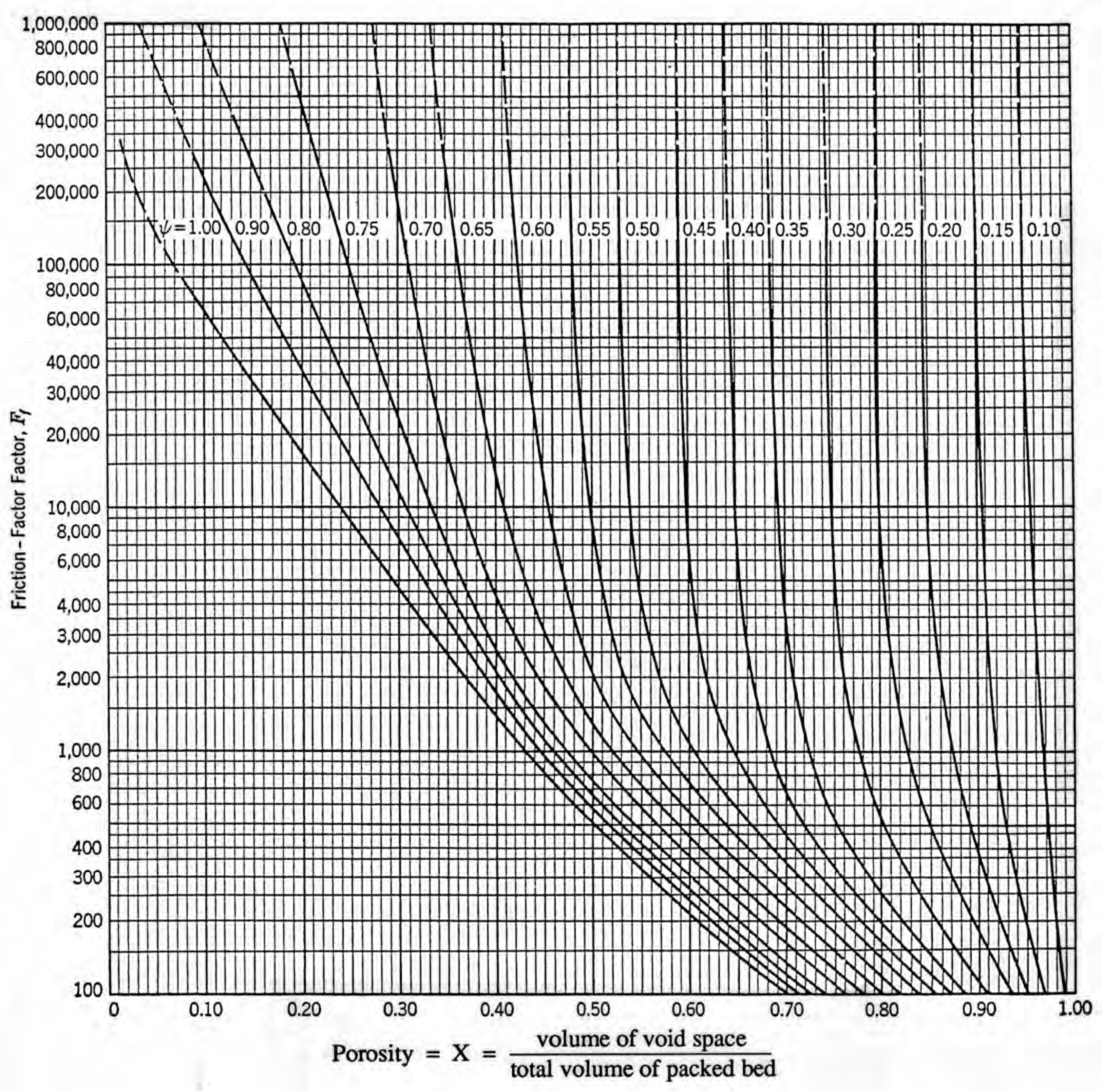
ψ = sphericity = $\dfrac{\text{surface area of a sphere of volume V}}{\text{surface area of actual particle with volume V}}$

for spheres, $\psi = 1$

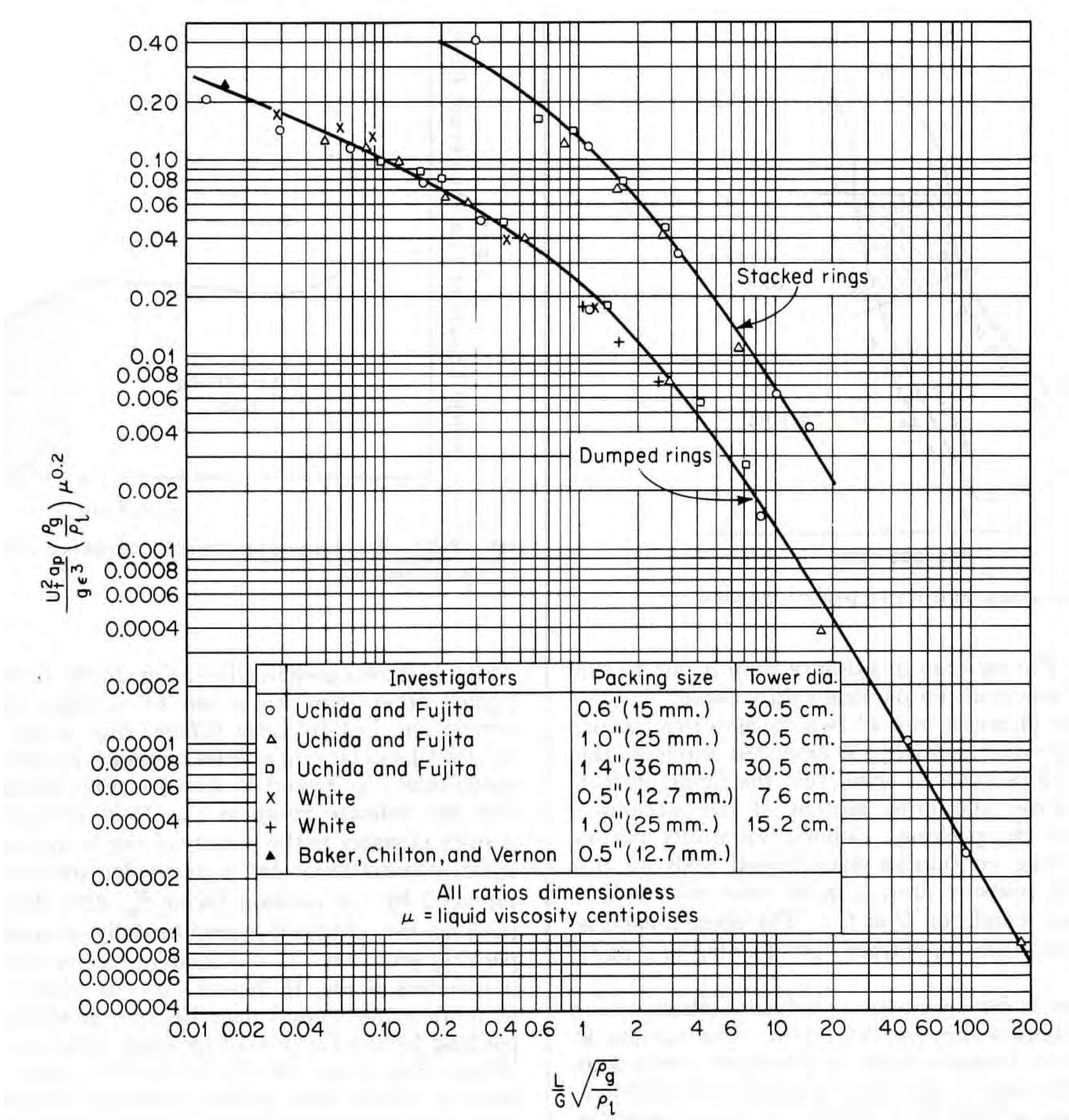
for cylinders ($\ell = 10 \times r$), $\psi = 0.76$

for cylinders ($\ell = 1000 \times r$), $\psi = 0.17$

The Friction Factor Factor



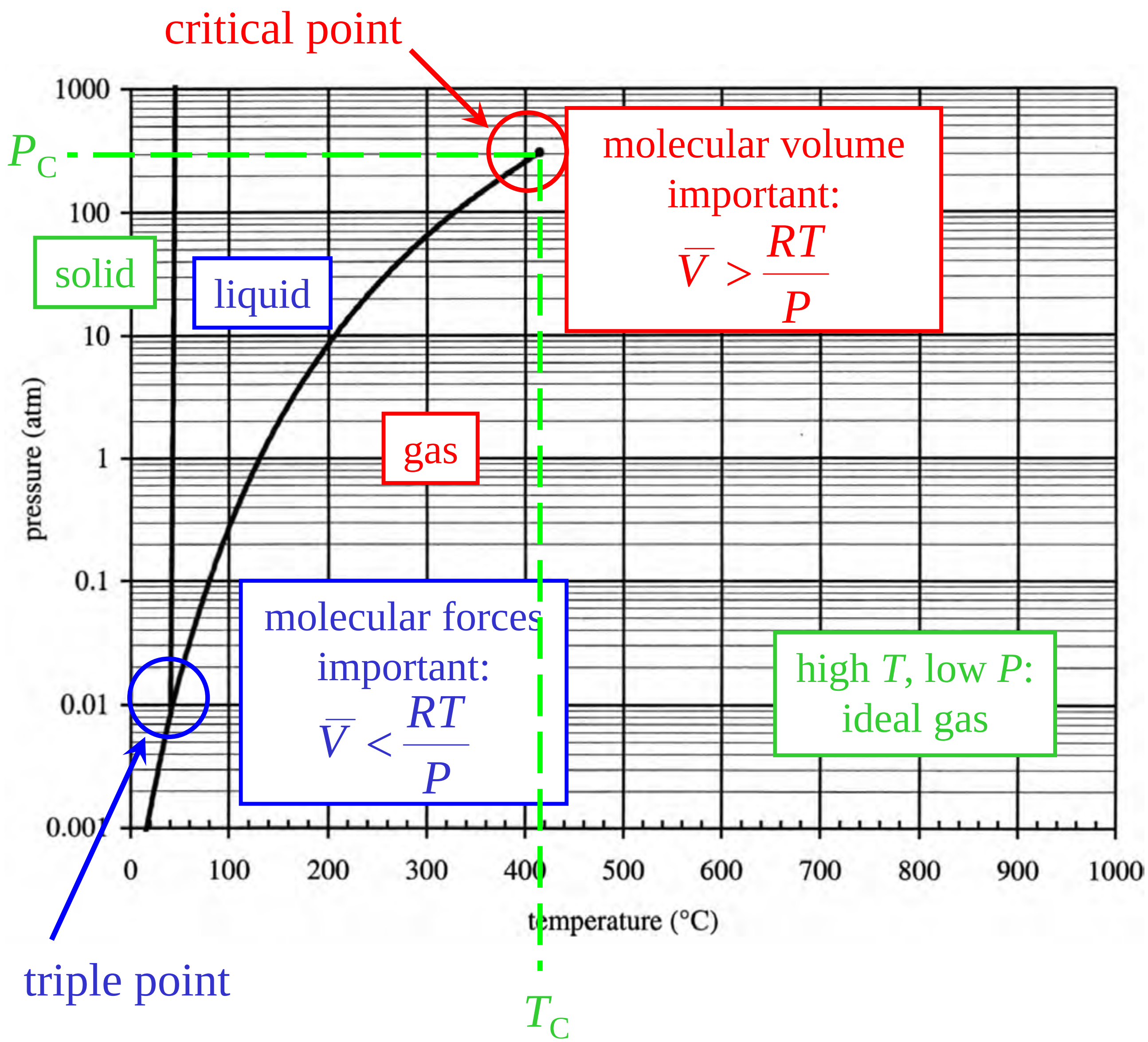
Designing Distillation Columns - Flooding & Weeping



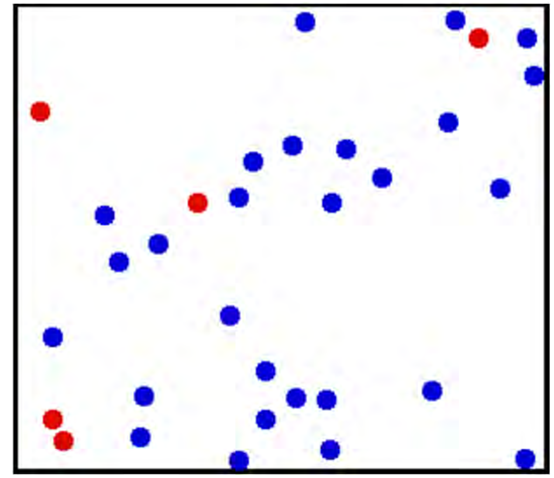
Dimensionless correlation of flooding limits for packed columns.
Sherwood et al. Indus. Eng. Chem. **30** *768* (1938).

Dynamic Similarity of Gases - Dimensional Analysis

All gases have a similar pressure-temperature phase map



What defines a gas?



parameters

pressure, P
temperature, T
molar volume, $\overline{V}$
gas constant, R

} sufficient for an ideal gas

~~molecular volume~~
~~intermolecular forces~~

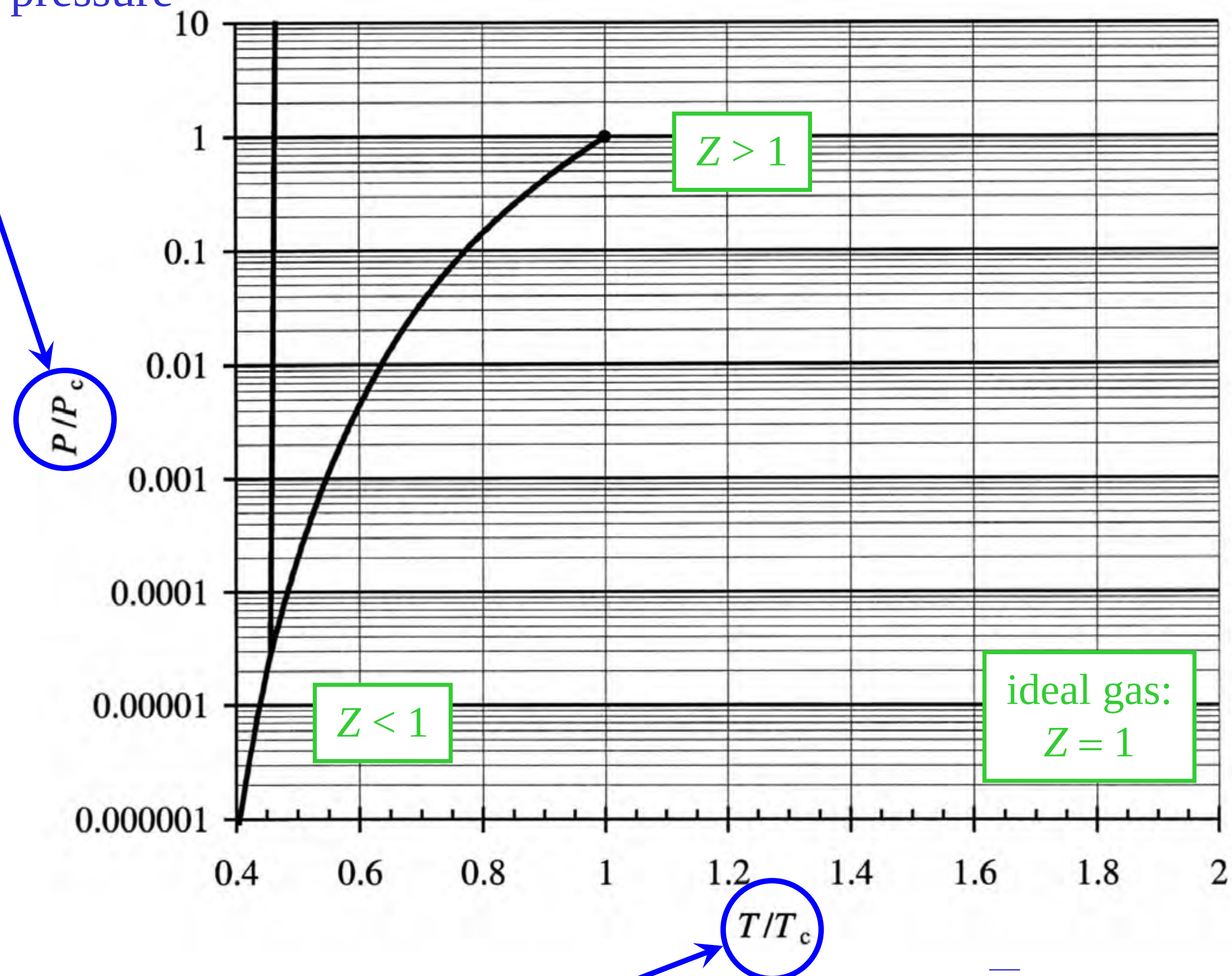
too difficult to measure

critical pressure, P_C
critical temperature, T_C

Dynamic Similarity of Gases - Dimensional Analysis

Three Dimensionless Groups

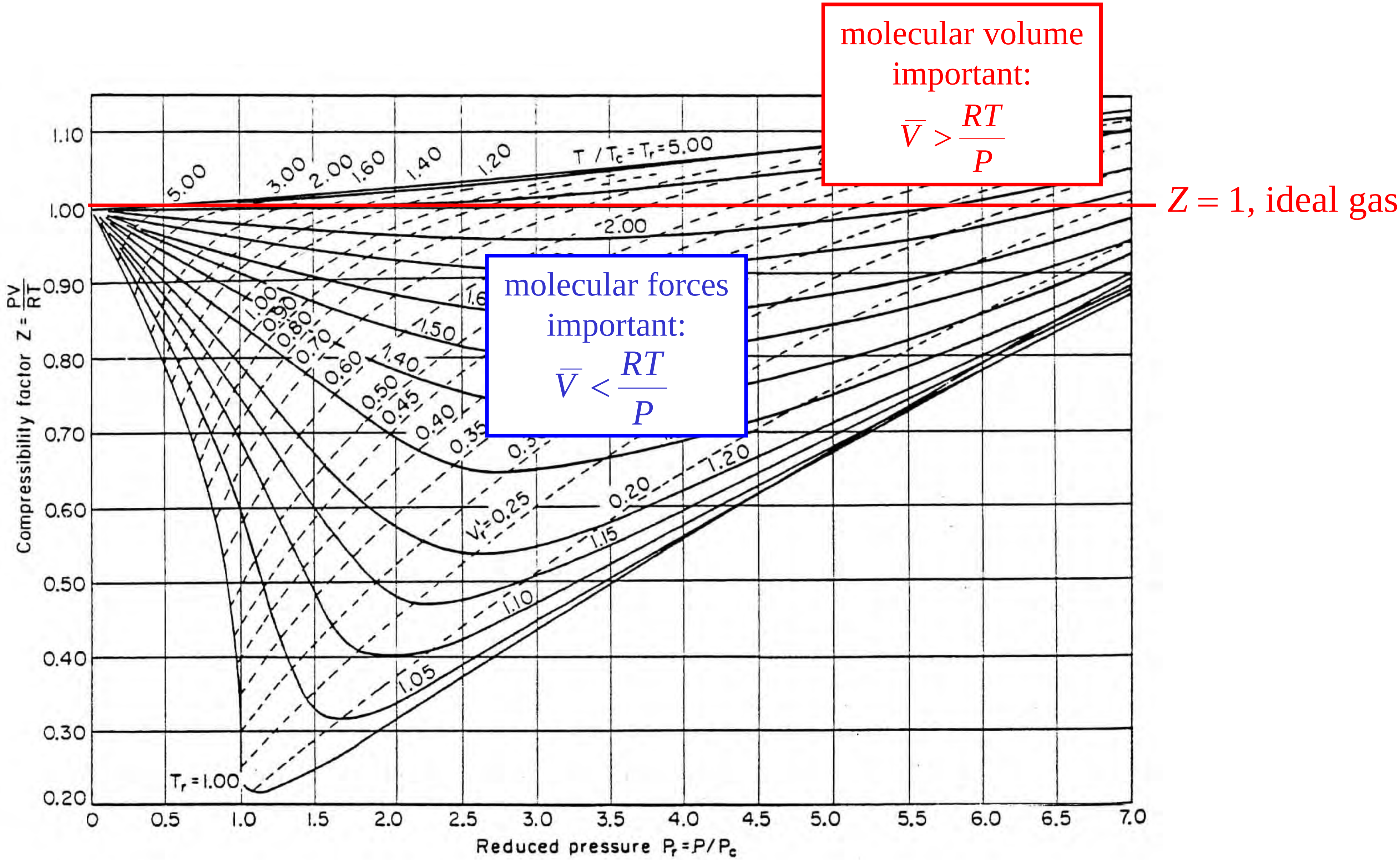
Π_2: reduced pressure



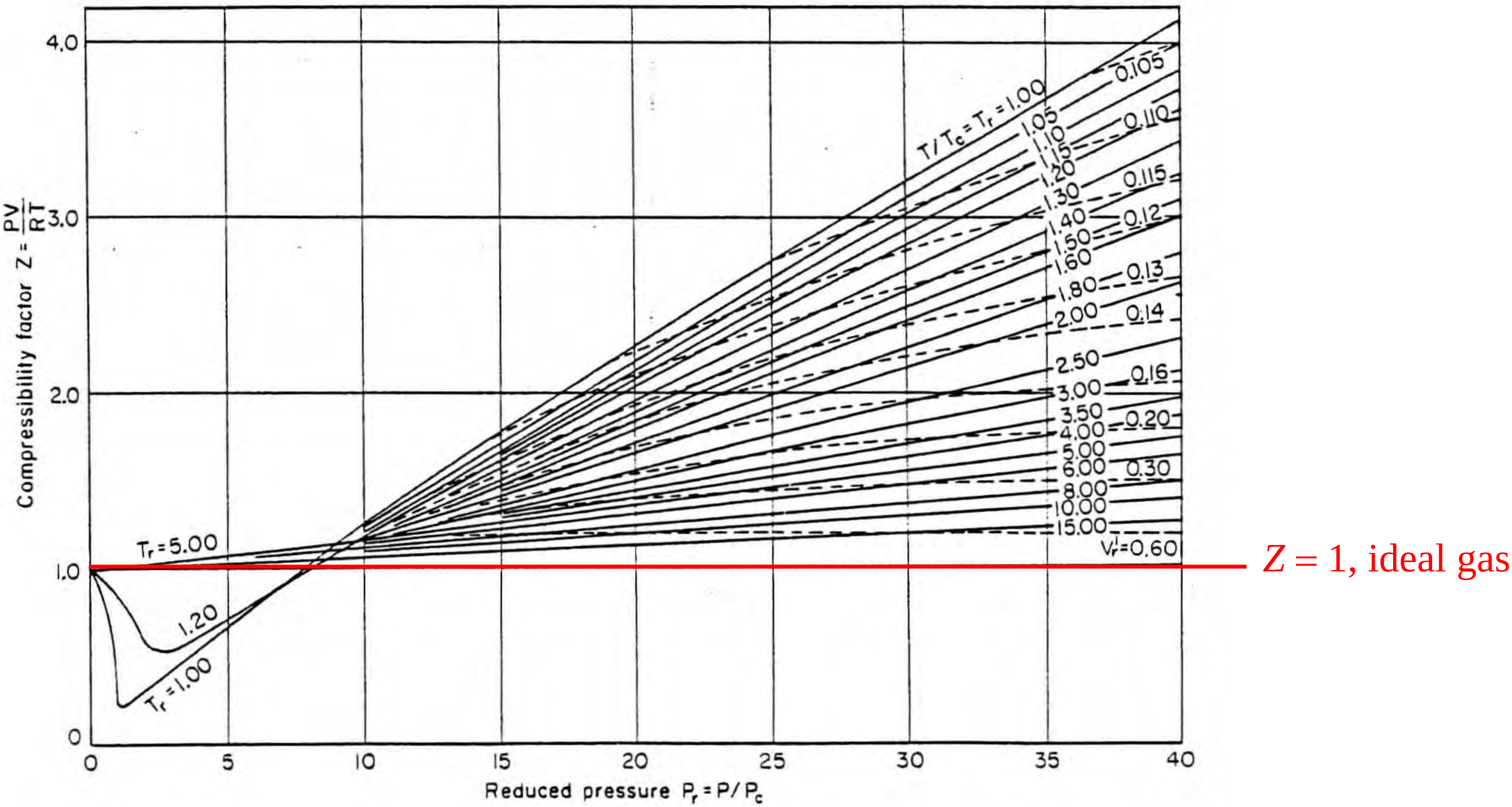
Π_1: reduced temperature

$$\Pi_3 = \frac{P\bar{V}}{RT} \equiv Z \quad \text{compressibility}$$

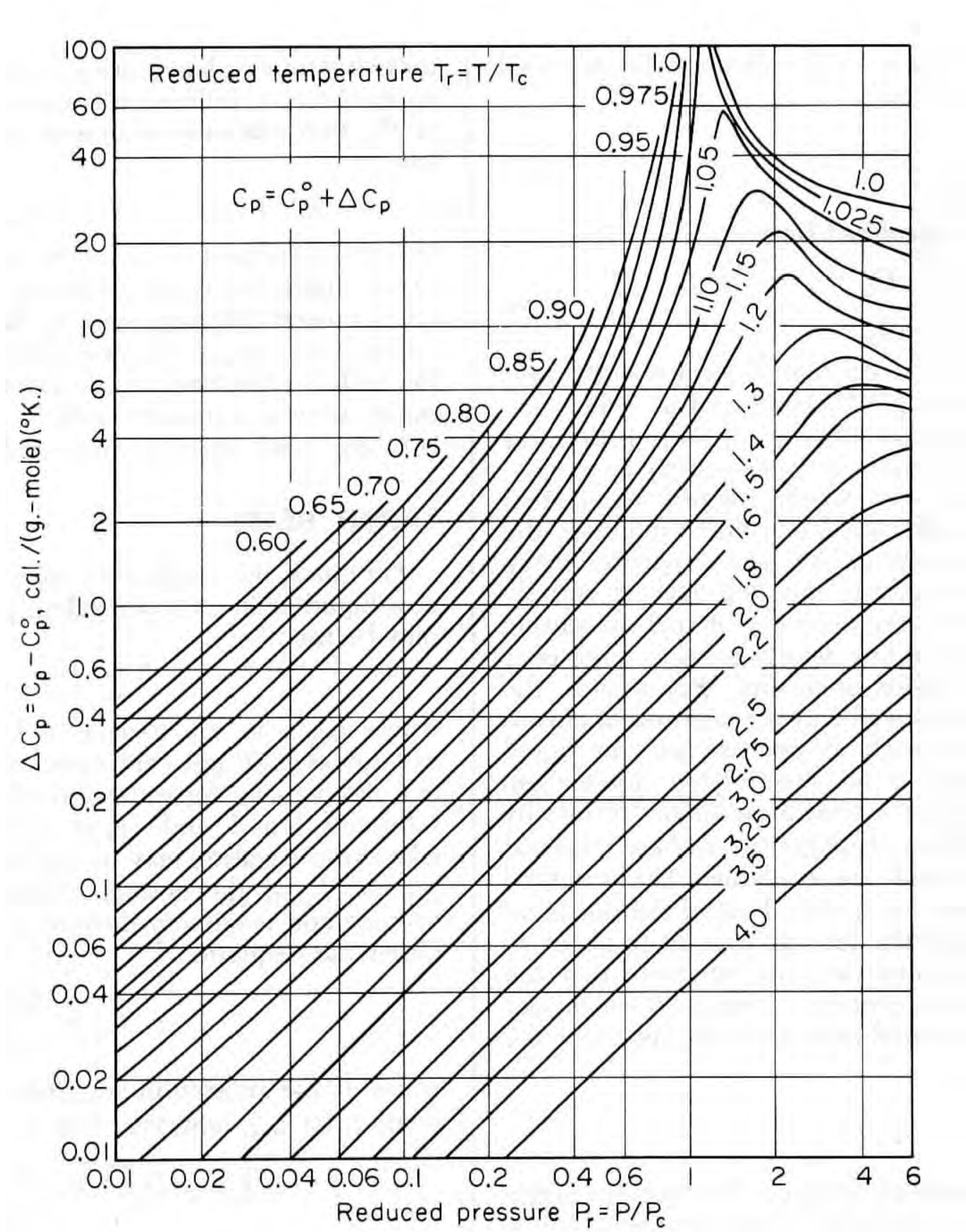
Dynamic Similarity of Gases - Dimensional Analysis



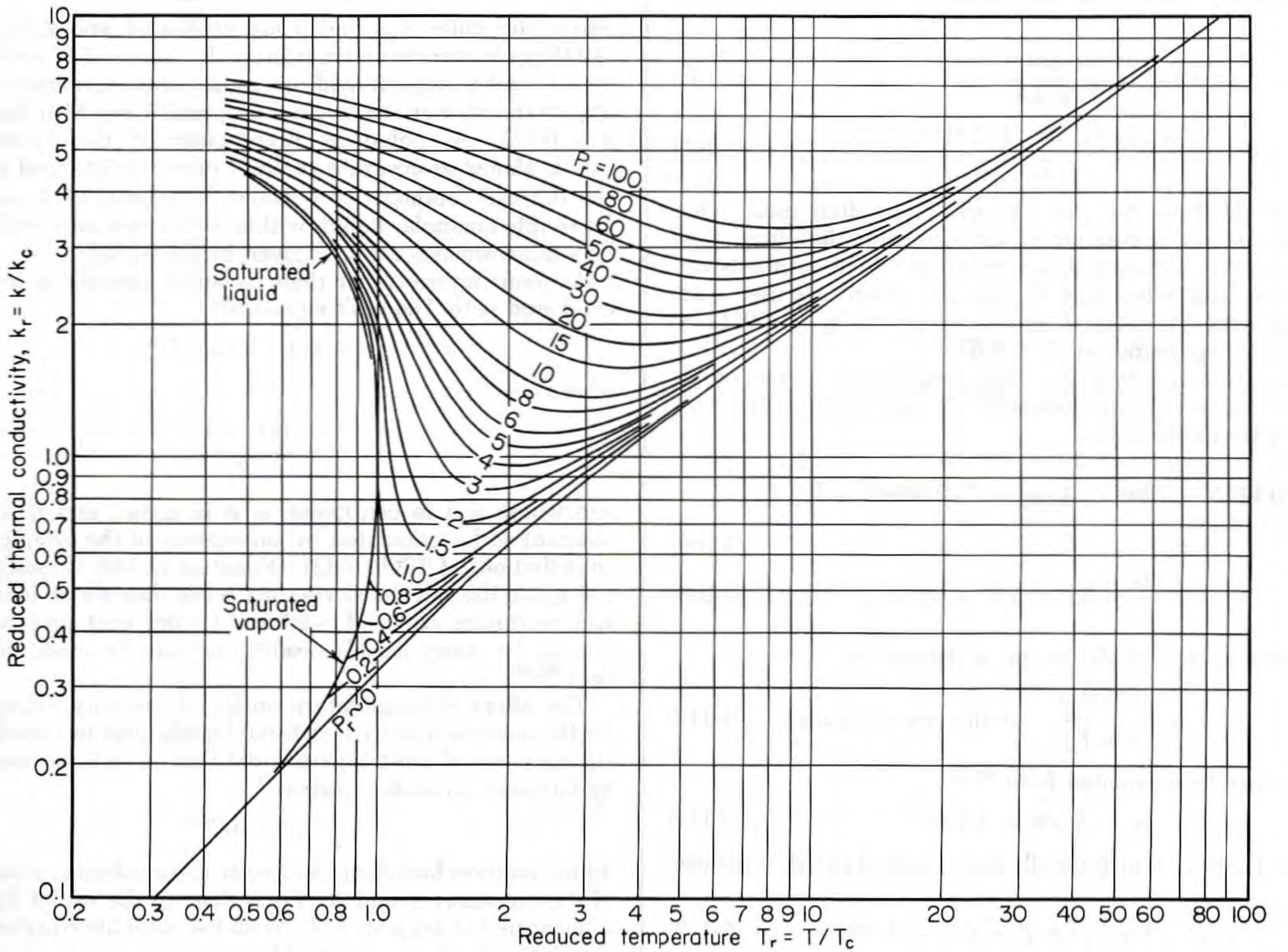
Dynamic Similarity of Gases - Dimensional Analysis



Dynamic Similarity of Gases - Dimensional Analysis



Dynamic Similarity of Gases - Dimensional Analysis



Dimensional Analysis and Dynamic Scaling

Why don’t ants throw stones?

Insect Defenses - Armor

Insect Defenses - Chemical Spray

Insect Defenses - Flight

Insect Defenses - Fight

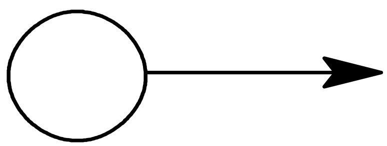
Dynamic Similarity: Why don't ants throw stones?

An ant throwing a stone in air is like you throwing <what object?> in <what fluid?>?

Calculate the ratio of the projectile's inertia vs the projectile's frictional losses.

Calculate the projectile's Reynolds number.

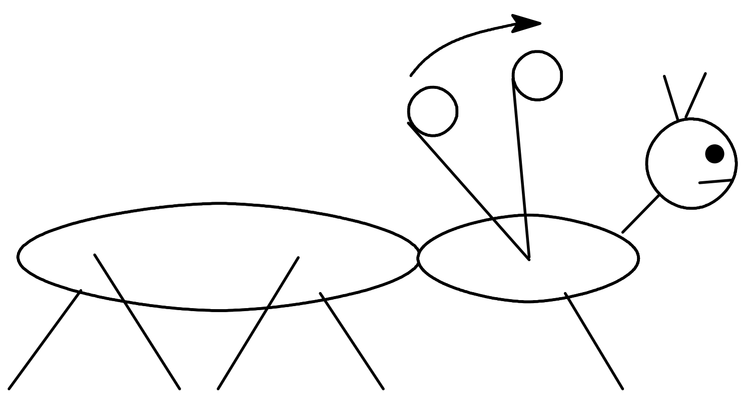
projectile properties: diameter, density

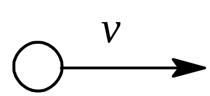


dynamics: velocity

$$\mathrm{Re} = \frac{\text{inertial force of projectile}}{\text{viscous force of fluid}} = \frac{\rho_{\text{projectile}} v d}{\mu_{\text{fluid}}}$$

fluid properties: viscosity, density





projectile: a grain of sand
d = 1 mm = 0.001 m
$\rho_{\text{sand}} = 2600\ \text{kg/m}^3$

fluid: air $\mu = 1.8 \times 10^{-5}$ Pa·sec

dynamics: v = 2 m/sec
(estimate assumes ant's
arm length is 3 mm.)

Dynamic Similarity: Why don't ants throw stones?

Calculate the Reynolds number of the ant's projectile

$$\text{Re for sand grain in air} = \frac{(2600\,\text{kg/m}^3)(2\,\text{m/sec})(0.001\,\text{m})}{1.8\times10^{-5}\,\text{Pa}\cdot\text{sec}} = \boxed{3\times10^5}$$

Calculate the Reynolds number of your projectile: a 10-cm stone, 50 mph = 20 m/sec.

$$\text{Re for stone in air} = \frac{(2600\,\text{kg/m}^3)(20\,\text{m/sec})(0.1\,\text{m})}{1.8\times10^{-5}\,\text{Pa}\cdot\text{sec}} = \boxed{3\times10^8}$$

An ant throwing a stone is not dynamically similar to a person throwing a stone.

$$\text{Re for stone in water} = \frac{(2600\,\text{kg/m}^3)(2\,\text{m/sec})(0.1\,\text{m})}{0.001\,\text{Pa}\cdot\text{sec}} = \boxed{5\times10^5}$$ Dynamically similar!

$$\text{Re for balloon in air} = \frac{(1.4\,\text{kg/m}^3)(20\,\text{m/sec})(0.2\,\text{m})}{1.8\times10^{-5}\,\text{Pa}\cdot\text{sec}} = \boxed{3\times10^5}$$ Dynamically similar!

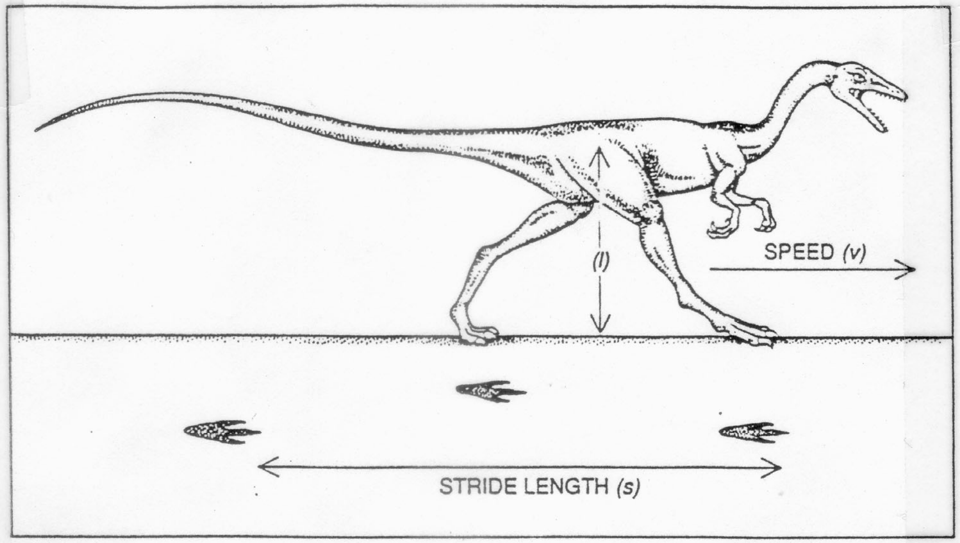
A 20-cm balloon stops after 2-3 m. A 1-mm grain of sand will stop after 20-30 mm.

Dimensional Analysis and Dynamic Scaling

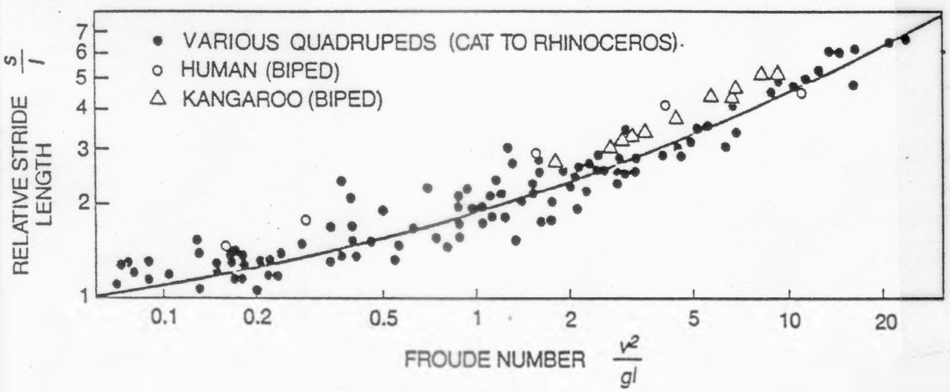
Dynamics of Dinosaurs

Walking, Running, and Flying

How Dinosaurs Ran

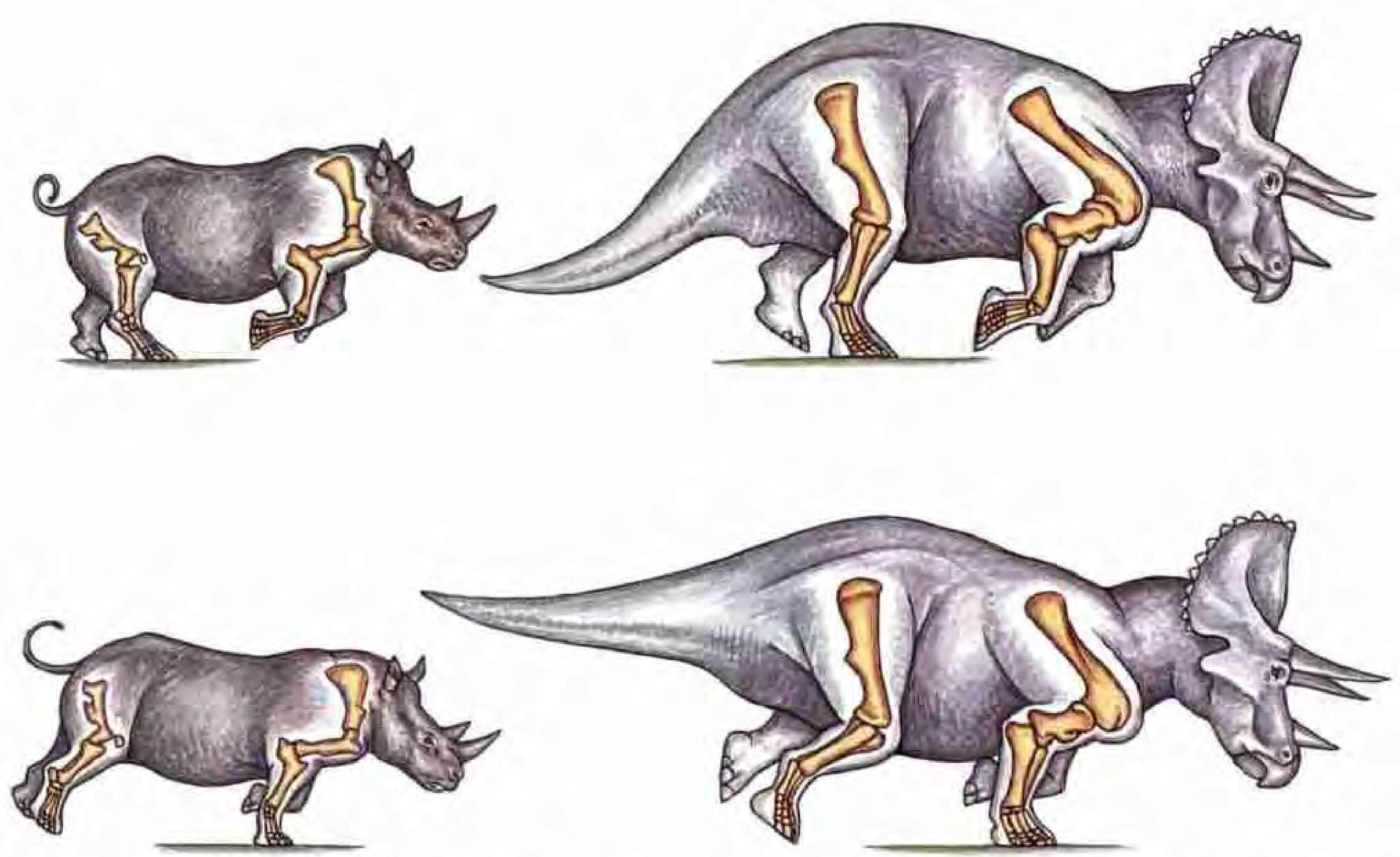


STRIDE LENGTH is the distance between two successive prints from the same foot. *Compsognathus*, a carnivore the size of a contemporary chicken, is depicted here.



FROUDE NUMBERS for kangaroos, humans and quadrupeds, such as rhinoceroses, are plotted against the animals' relative stride lengths. The numbers increase logarithmically—so that the difference between Froude numbers 0.1 and 20 is clear.

Triceratops Galloped



TRICERATOPS may have moved like the White rhinoceros, a modern, horned herbivore. The White rhinoceros here, which was sketched from a film, is galloping at seven meters per second—which is about the speed of a fast human run.

“How Dinosaurs Ran” R. N. Alexander, Scientific American, April 1991.

How Dinosaurs Flew

Pteranodon

7 m wingspan, 20-45 kg

Late Cretaceous: 84-65 Mya

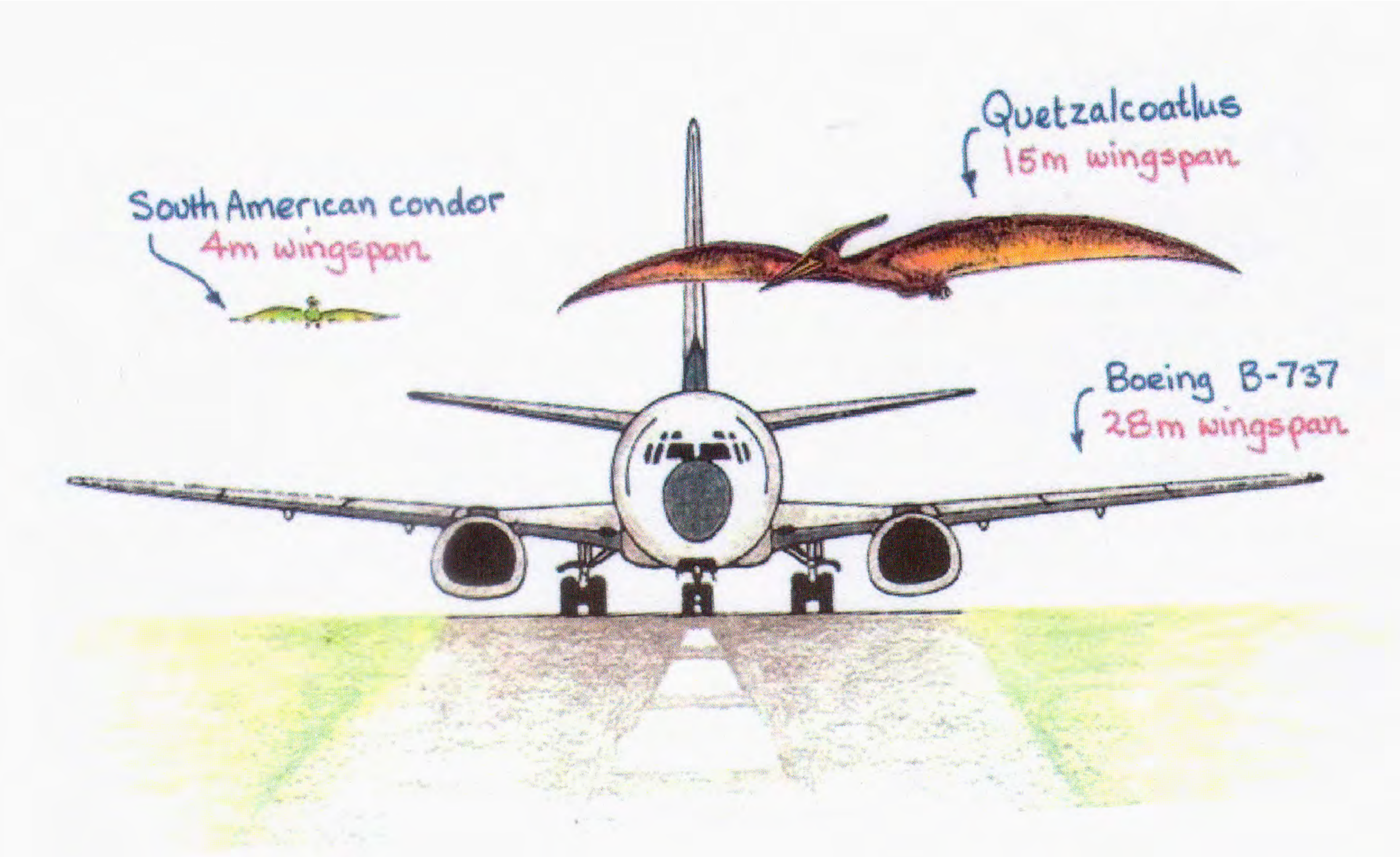
How Dinosaurs Flew

Quetzalcoatlus

15 m wingspan, 85-100 kg

Late Cretaceous: 84-65 Mya

Quetzalcoatlus



Octave Levenspiel, Oregon State University: levenspiel.com/dinosaurs/

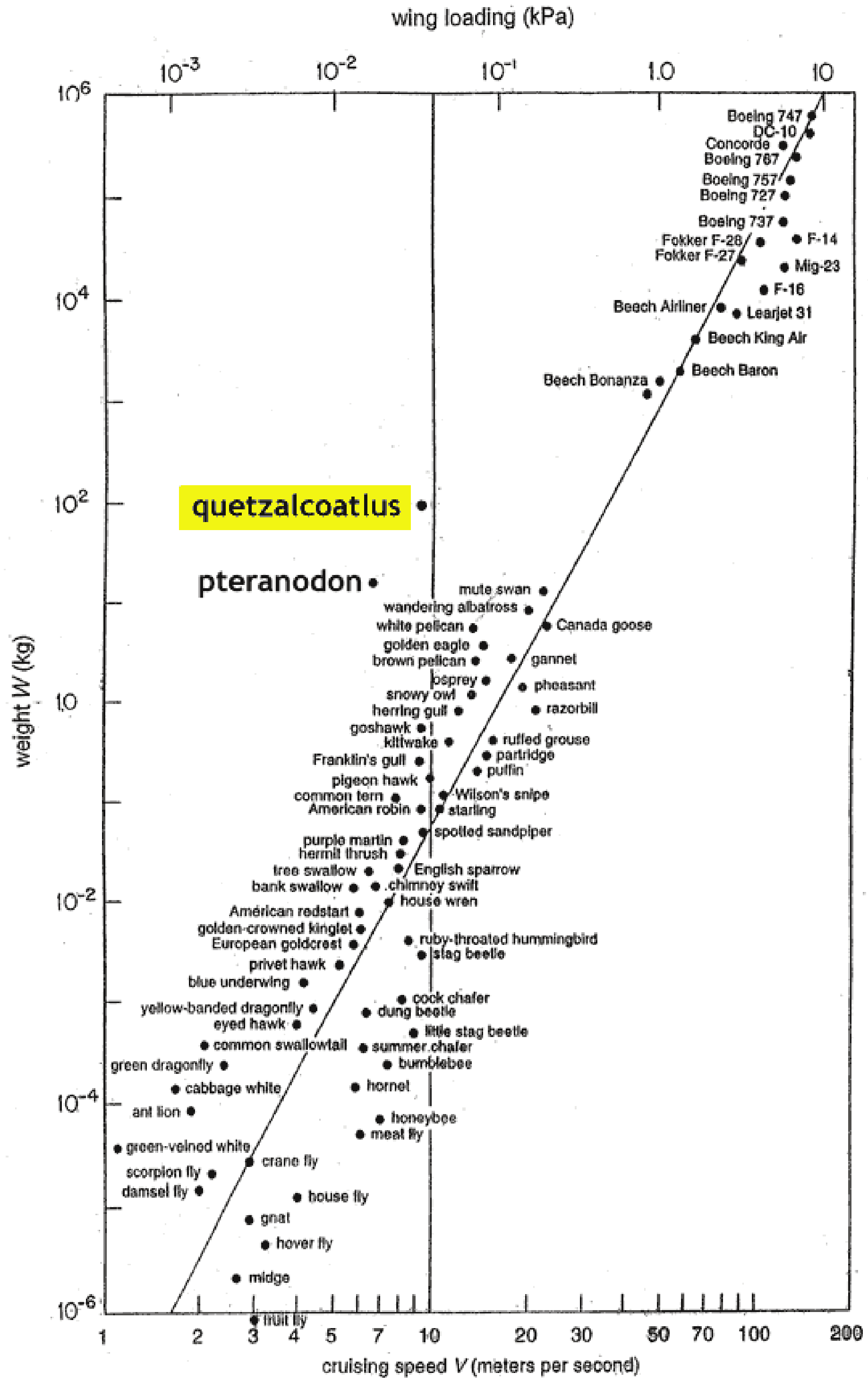
Quetzalcoatlus

Octave Levenspiel, Oregon State University: levenspiel.com/dinosaurs/

Quetzalcoatlus

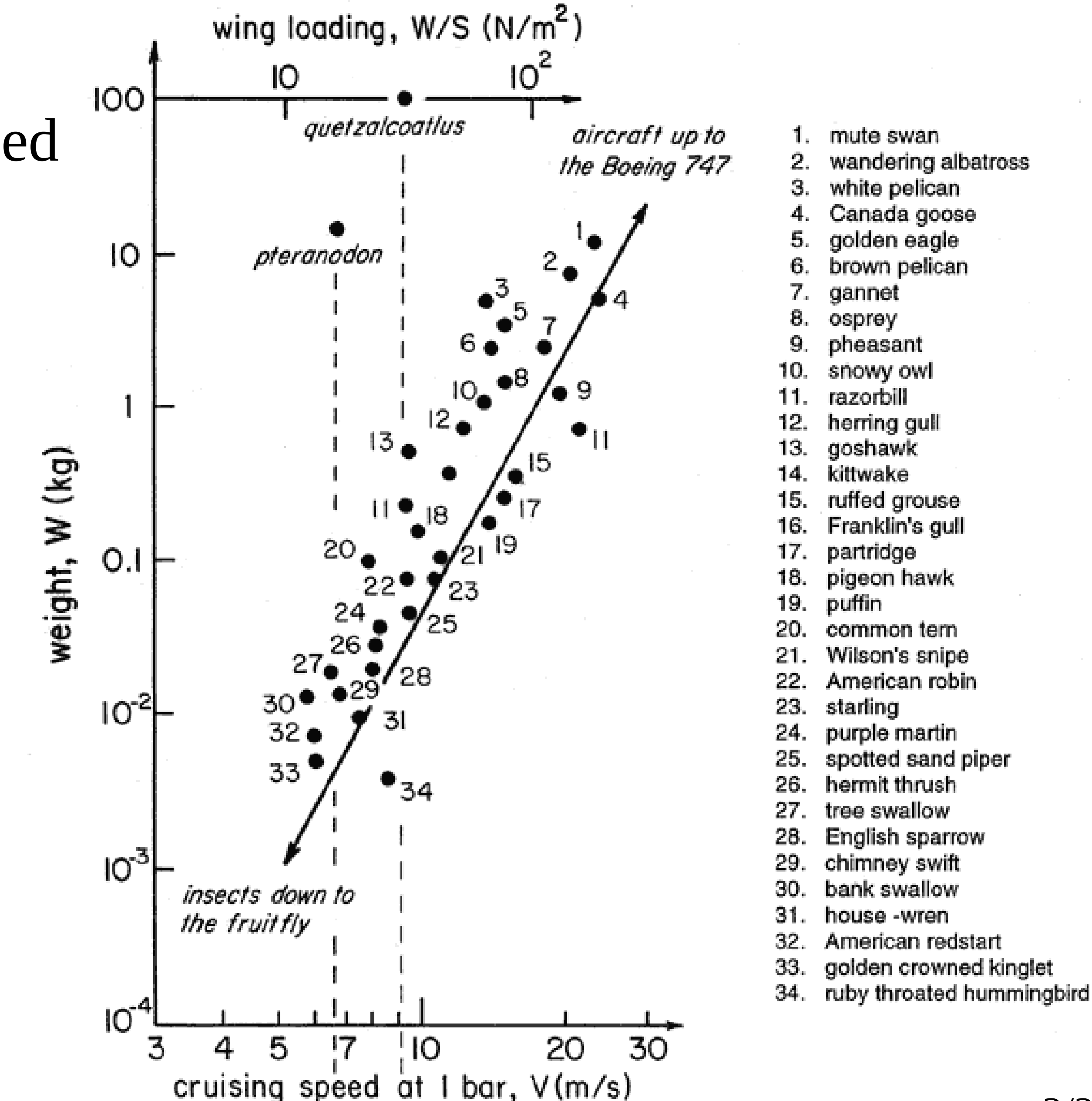
Octave Levenspiel, Oregon State University: levenspiel.com/dinosaurs/

Weight vs. Flying Speed



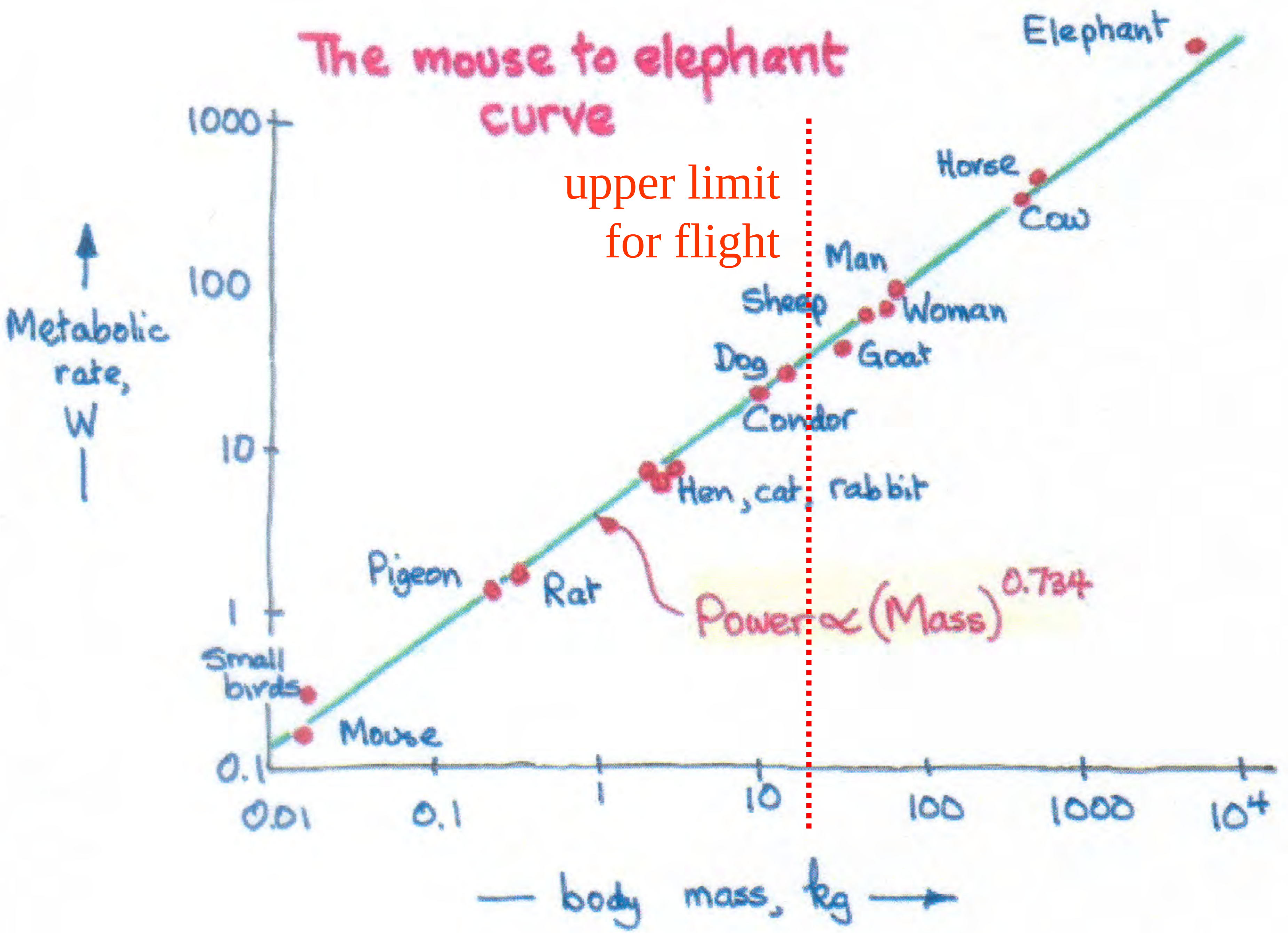
Octave Levenspiel, Oregon State University: levenspiel.com/dinosaurs/

Weight vs. Flying Speed



power needed is proportional to $$\frac{(\text{mass})^{3/2}}{(\text{wing area})^{1/2}(\text{air density})^{1/2}}$$

Metabolic Rate vs. Body Mass



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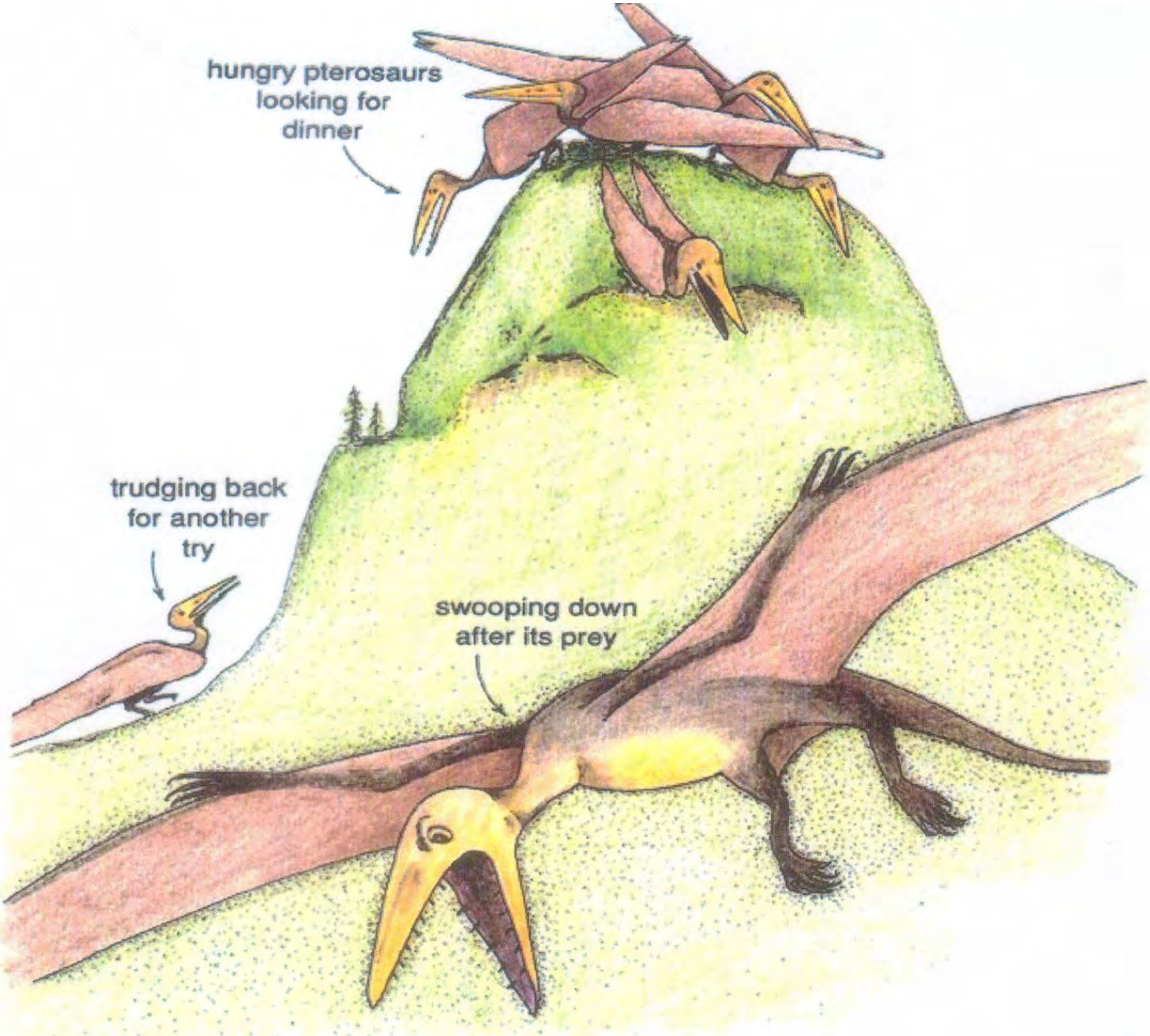
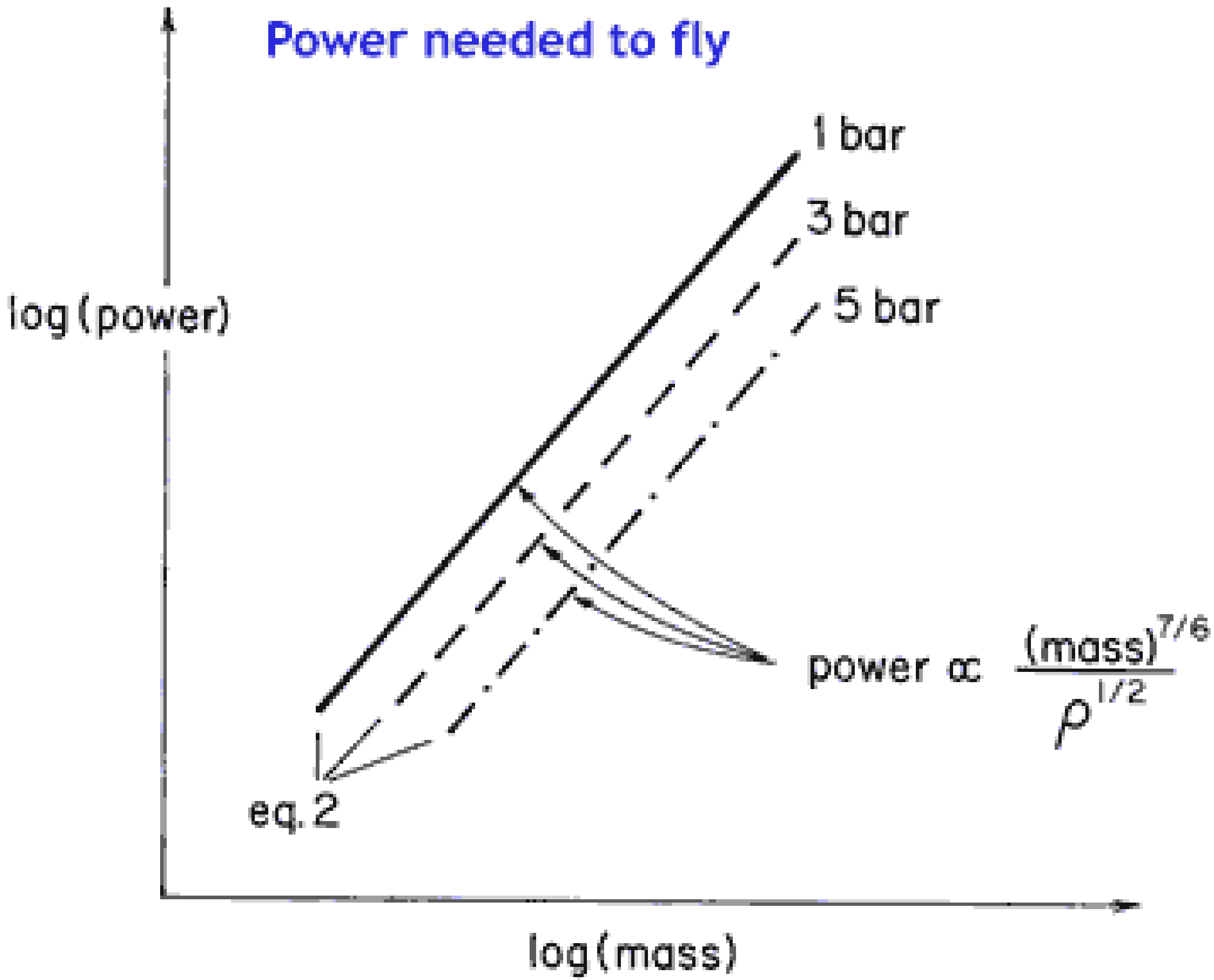

hungry pterosaurs looking for dinner
trudging back for another try
swooping down after its prey

Higher Pressure Explains How Quetzalcoatlus Flew.



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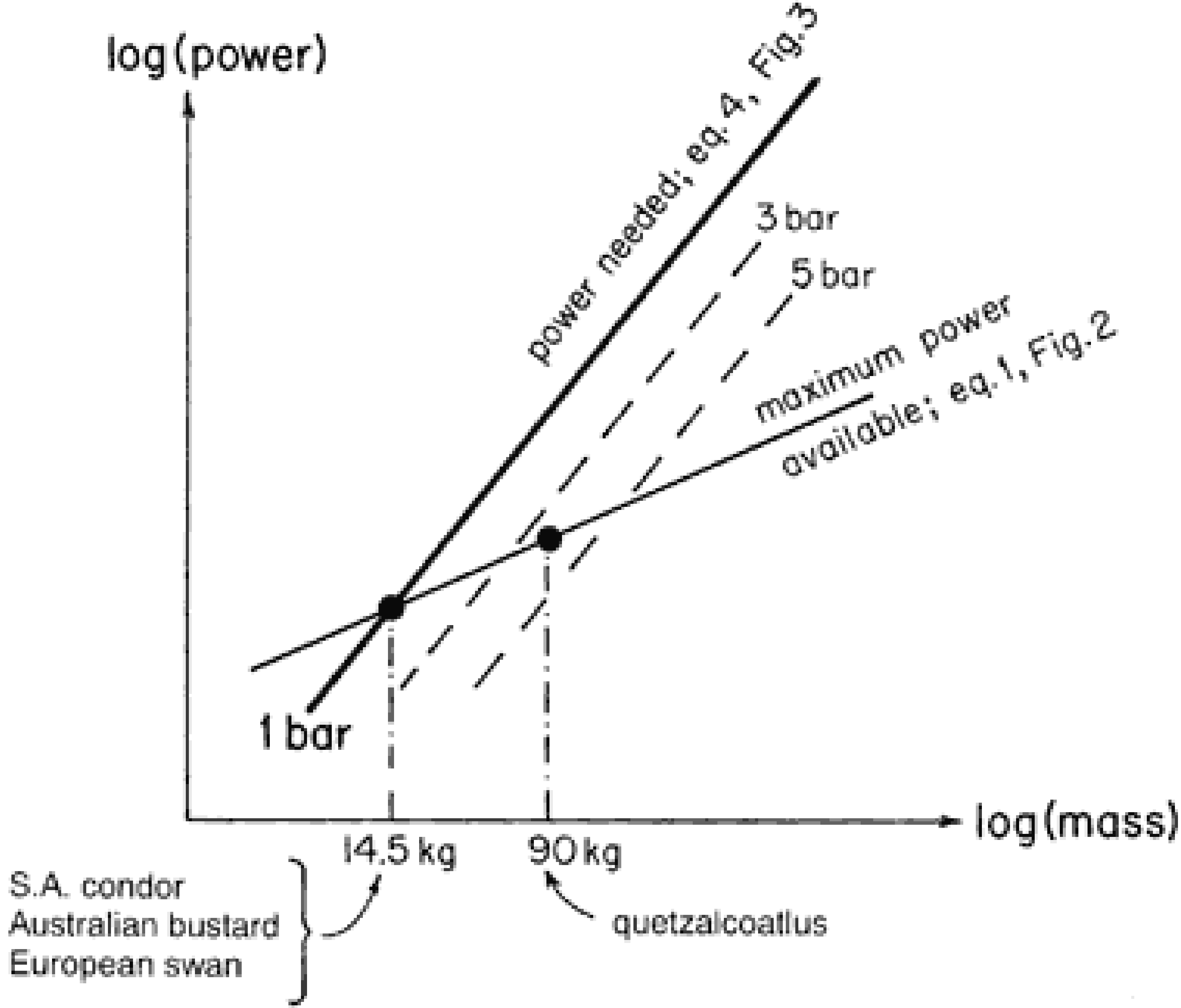
Higher Pressure Explains How Quetzalcoatlus Flew.



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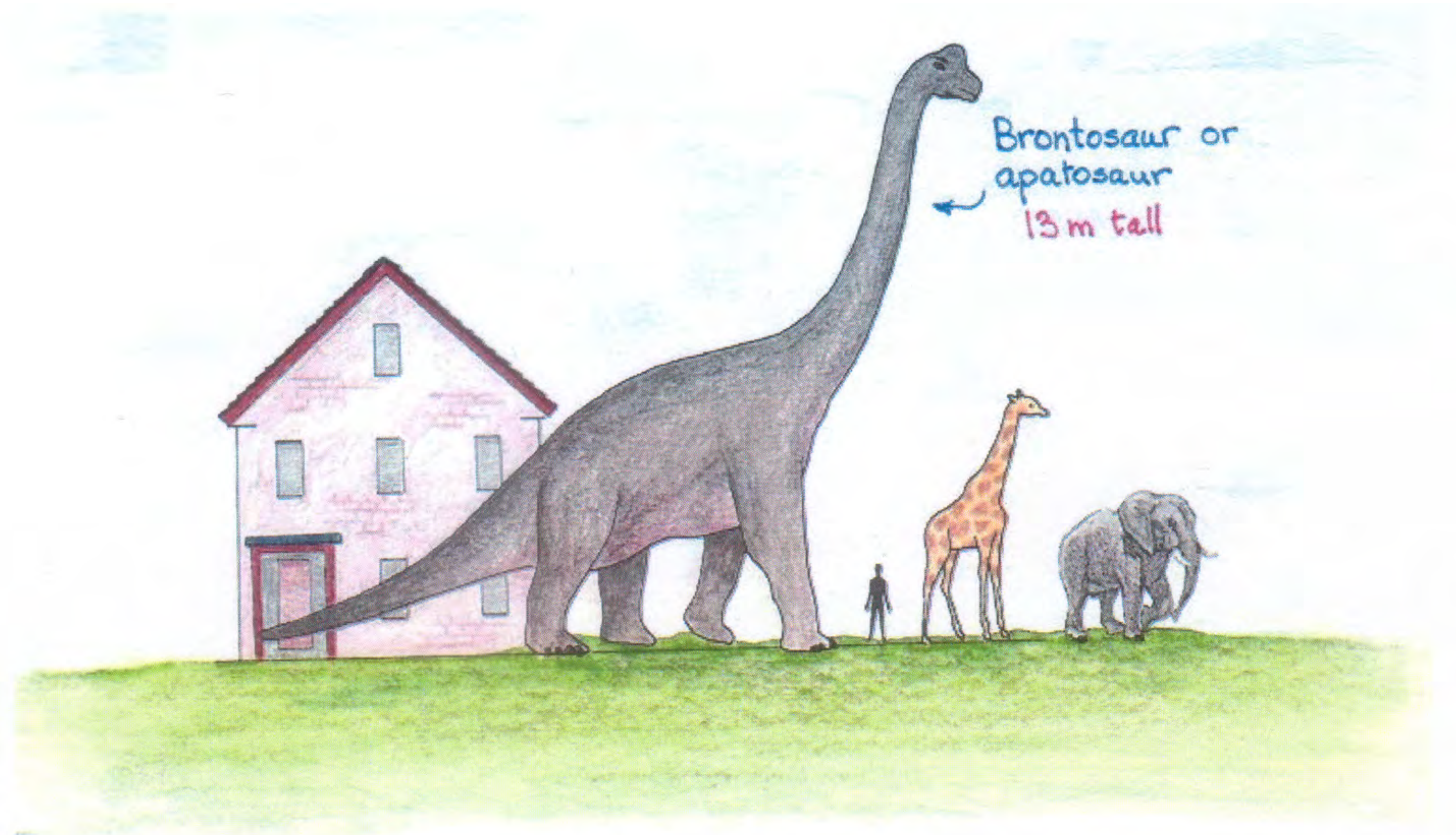
Higher Pressure Resolves the Problem of Pumping Blood to the Brain of a Brontosaur



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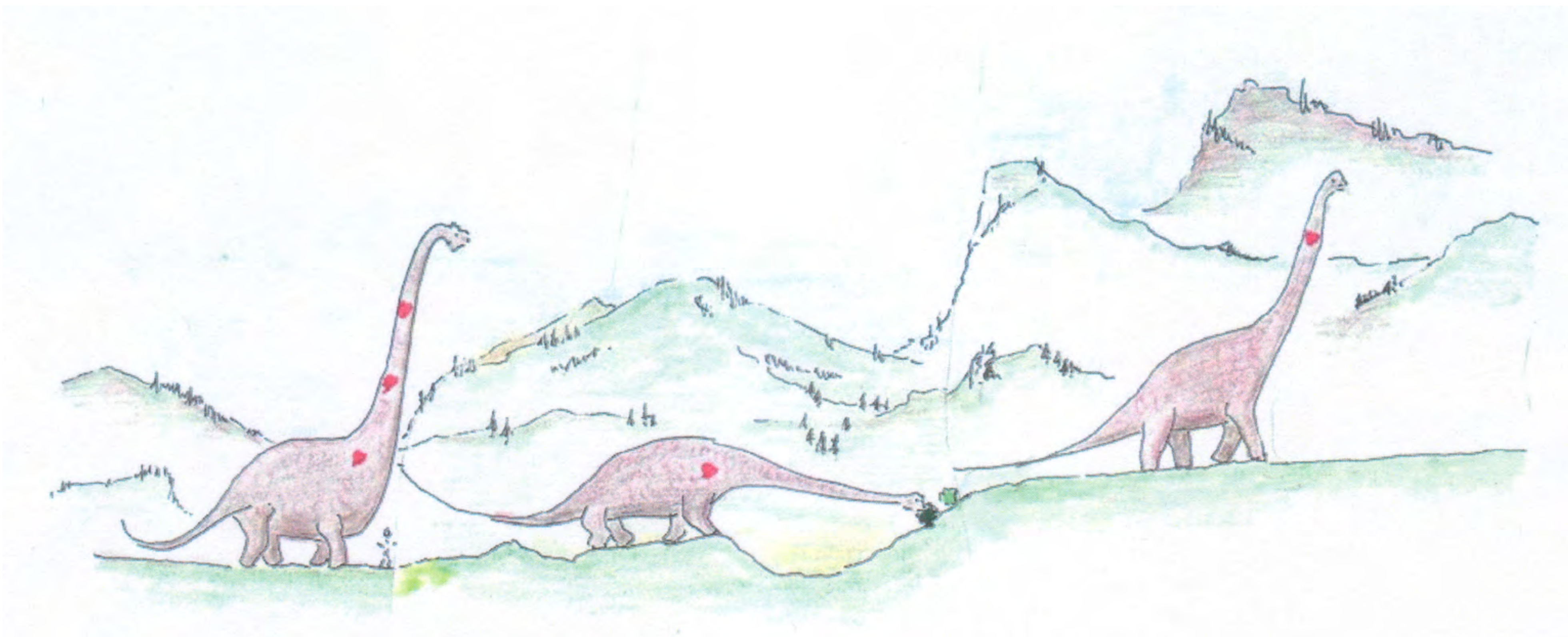

Higher Pressure Resolves the Problem of Pumping Blood to the Brain of a Brontosaur