1. Mathematical models are required for parts (A) and (B). If one applies the Conservation of Mass, one must unambiguously define system borders and state assumptions. In this case, valid assumptions are steady state and no chemical reaction. These assumptions could be summarized by writing “rate in = rate out”.

If one uses non-standard nomenclature (standard is $F \equiv$ mass flow rate in kg/min, $T \equiv$ total) one must define their nomenclature.

If one invokes a process specification, one must state the specification and then translate to equations.

\( \text{(A)} \) Nomenclature: Pulp flow rate in stream \( i \equiv F_{\text{pulp},i} \)

Draw system borders around every unit but the grinder and write a total mass balance.

$$ F_{T,2} = F_{T,3} + F_{T,5} + F_{T,6} \quad \text{(1)} $$

The total flow rates of streams 2 and 3 are given: 100. and 71.5 kg/min. Substitute into equation (1) and solve for $F_{T,6}$.

$$ 100. = 71.5 + F_{T,5} + F_{T,6} $$

$$ F_{T,6} = 28.5 - F_{T,5} \quad \text{(2)} $$

Use the same system borders and write a mass balance on pulp.

$$ F_{\text{pulp},2} = F_{\text{pulp},3} + F_{\text{pulp},5} + F_{\text{pulp},6} \quad \text{(3)} $$

Use the pulp wt%’s in streams 2, 3, 5, and 6 to write equations in terms of total flow rates.

$$ F_{\text{pulp},2} = 0.088F_{T,2} = 8.8 \text{ kg/min} \quad \text{(4)} $$

$$ F_{\text{pulp},3} = 0 \quad \text{(5)} $$

$$ F_{\text{pulp},5} = 0.043F_{T,5} \quad \text{(6)} $$

$$ F_{\text{pulp},6} = 0.67F_{T,6} \quad \text{(7)} $$

Substitute equations (4)-(7) into equation 3 and solve for $F_{T,6}$.

$$ 8.8 = 0 + 0.043F_{T,5} + 0.67F_{T,6} $$

$$ F_{T,6} = \frac{8.8}{0.67} - \frac{0.043}{0.67}F_{T,5} = 13.13 - 0.0642F_{T,5} \quad \text{(8)} $$

Set equation 2 equal to equation 8 and solve for $F_{T,5}$.

$$ 28.5 - F_{T,5} = 13.13 - 0.0642F_{T,5} $$

$$ (1 - 0.0642)F_{T,5} = 28.5 - 13.13 $$

$$ F_{T,5} = 16.4 \text{ kg/min} \quad \text{(9)} $$

The flow rate “low pulp” orange juice is 16.4 kg/min.
(B) Repeat the derivation above, but do not substitute \( F_{T,3} = 71.5 \text{ kg/min} \). Thus equation 2 is

\[
F_{T,6} = 100 - F_{T,3} - F_{T,5}
\]

Set equation 2\( a \) equal to equation 8 and solve for \( F_{T,5} \).

\[
100 - F_{T,3} - F_{T,5} = 13.13 - 0.0642F_{T,5}
\]

\[
(1-0.0642)F_{T,5} = 100 -13.13 - F_{T,3}
\]

\[
F_{T,5} = 92.8 - 1.07F_{T,3}
\]

Check equation 10 in the case that \( F_{T,3} = 71.5 \text{ kg/min} \).

\[
F_{T,5} = 92.8 - 1.07(71.5) = 16.3 \text{ kg/min}
\]

The formula for the flow rate of “low pulp” orange juice is

“low pulp” orange juice = 92.8 – 1.07 × (“no pulp” orange juice).

2. Conveniently, mols are conserved in all reactions. Start with a mass balance around reactor 1. All the \( A \) is consumed and \( \text{mol } X = \text{mol } Y \). This yields the output from reactor 1, which also yields the output from the first separator.

Draw borders around the lower recycle loop. \( B \) enters at 20 mol/min and leaves at 0.2 mol/min. So 19.8 (≈20.) mol/min of \( B \) is consumed. Thus \( P \) is produced at 40 mol/min. Use the equation for the reactor 2 output. From the purge stream we know the ratio \( B: X \) is 0.2:20 = 1:100. Thus for the reactor 2 output, \( \text{mol } X = 100 \times (\text{mol } B) \). Substitute into the equation and solve for \( \text{mol } B \).

\[
\frac{(\text{mol } P)^2}{(\text{mol } B)(\text{mol } X)} = \frac{40^2}{(\text{mol } B)100(\text{mol } B)} = \frac{16}{(\text{mol } B)^2} = 4
\]

\[
(\text{mol } B)^2 = \frac{16}{4} = 4
\]

\[
\text{mol } B = 2
\]

Thus \( \text{mol } X = 100 \times (\text{mol } B) = 200 \). Use the reactor 2 output to complete the mass balance around the recycle loop.
3. The process below produces pure $X$ and an inseparable mixture of $D$, $I$, and $Y$. Obligatory purges on recycle streams have been omitted in these initial designs.

The output of pure $X$ is only half the maximum that could obtain from 100 mol $D$ because half of reactant $D$ is lost in the mixture with $Y$. We can shift the equilibrium toward products $X$ and $Y$ by increasing the concentration of reactant $C$ in the recycle loop, as shown below. The flow rates are more precise than the estimates requested in the exercise statement.

One can also recycle the $D+I+Y$ mixture to decrease the $D:Y$ ratio in the purge, and thus decrease the $D$ lost in the $D+I+Y$ mixture from a splitter on the recycle. However, this does not improve either the output or purity of the $D+I+Y$ mixture purged. Again, the flow rates are more precise than the estimates requested in the exercise statement.

Product outputs (and revenue) can be improved by decreasing the recycle flow of $Y$. Because $D$ and $Y$ are inseparable, we must convert $Y$ to $Z$ and then separate $Z$ from $D$. Add a second reactor to convert $Y$ to $Z$. Recycle the $D+Y$ mixture through the second reactor to increase the $D:Y$ ratio in the stream recycled to the first
The process above yields the maximum revenue from product $X$: 99 mol/min at 100 mol% purity. The other revenue is also increased: 99 mol/min of pure $Z$ has more value than 91 mol/min of 90 mol% $Y$. However, we can increase revenue further by converting the pure $Z$ to pure $Y$. Feed the $Z$ output from the process above to a third reactor, as shown in the process addendum below.