1. Mathematical models are required for parts (A) and (B). If one applies the Conservation of Energy, one must unambiguously define system borders and state assumptions. In this case, valid assumptions were steady state and no chemical reaction. These assumptions could be summarized by writing \( \text{“rate of energy in = rate of energy out”} \). If one uses non-standard nomenclature (standard is \( F \equiv \text{mass flow rate in kg/hr, } q \equiv \text{energy flow rate in kJ/hr, } T \equiv \text{total} \) one must define their nomenclature. If one invokes a process specification, one must state the specification and then translate to equations.

(A) Nomenclature: 
- \( H_2O \) (steam or water) flow rate in stream \( i \equiv F_{H2O,i} \)
- oatmeal flow rate in stream \( i \equiv F_{oatmeal,i} \)
- Energy flow rate of stream \( i \equiv q_i \)

Strategy: If we knew the composition of stream 8 - the wt fraction of water - we could calculate the energy needed to produce stream 9; evaporate the water at 100\(^{\circ}\)C and heat the steam to 220\(^{\circ}\)C. Calculating the wt fraction of water in stream 8 would be arduous. We would need to calculate how much water evaporates in the porridge boiler and how much steam condenses in the heat exchanger.

Instead consider an energy balance on the overall process. Although we do not know the flow rates of water in stream 6 (hot porridge) and stream 12 (hot water purge), we know that the total of water in streams 6 and 12 equals the total water entering the process: 117 g/min in stream 1.

Draw system borders around the entire process and write a mass balance on \( H_2O \).

\[
F_{H2O,1} + F_{H2O,3} = F_{H2O,6} + F_{H2O,12} \quad (1)
\]

The flowsheet specifications provide that there is no \( H_2O \) in stream 3, dry oatmeal.

\[
F_{H2O,3} = 0 \quad (2)
\]

Substitute equation 2 into equation 1 to obtain an equation for the qualitative reasoning above; the water in stream 1 leaves via streams 6 and 12.

\[
F_{H2O,1} + 0 = F_{H2O,6} + F_{H2O,12}
\]

\[
F_{H2O,1} = F_{H2O,6} + F_{H2O,12} \quad (3)
\]

Again with system borders around the entire process, write a mass balance on oatmeal.

\[
F_{oatmeal,1} + F_{oatmeal,3} = F_{oatmeal,6} + F_{oatmeal,12} \quad (4)
\]

The flowsheet specifications provide that there is no oatmeal in streams 1 and 12.

\[
F_{oatmeal,1} = 0 \quad (5)
\]

\[
F_{oatmeal,12} = 0 \quad (6)
\]

Substitute equations 5 and 6 into equation 4 to obtain another equation for the qualitative reasoning above; the oatmeal in stream 3 leaves via stream 6.

\[
0 + F_{oatmeal,3} = F_{oatmeal,6} + 0
\]

\[
F_{oatmeal,3} = F_{oatmeal,6} \quad (7)
\]

Continuing with system borders around the entire process, write an energy balance.

rate in = rate out

\[
q_1 + q_3 + q_{heater} = q_6 + q_{12}
\]

\[
q_{heater} = q_6 + q_{12} - (q_1 + q_3) \quad (8)
\]

Divide into two energy changes: both water and oatmeal enter at 20\(^{\circ}\)C and exit at 100\(^{\circ}\)C.
\[ q_{\text{heater}} = \Delta q_{H_2O,1 \rightarrow 6,12} + \Delta q_{\text{oatmeal,3 \rightarrow 6}} \]  

\[ \Delta q_{H_2O,1 \rightarrow 6+12} = (q_{H_2O,6} + q_{H_2O,12}) - q_{H_2O,1} \] 

\[ \Delta q_{H_2O,1 \rightarrow 6+12} = F_{H_2O,1} \frac{C_p,\text{water}}{100 - 20^\circ C} \] 

\[ = \frac{117 \text{ g water}}{\text{min}} \left( \frac{4.17 \text{ J}}{\text{g water}(^\circ C)} \right)(100 - 20^\circ C) = 39,030 \text{ J/min (10)} \]

\[ \Delta q_{\text{oatmeal,3 \rightarrow 6}} = q_{\text{oatmeal,6}} - q_{\text{oatmeal,3}} \] 

\[ \Delta q_{\text{oatmeal,3 \rightarrow 6}} = F_{\text{oatmeal,3}} \frac{C_p,\text{oatmeal}}{100 - 20^\circ C} \] 

\[ = \frac{78.1 \text{ g oatmeal}}{\text{min}} \left( \frac{1.37 \text{ J}}{\text{g oatmeal}(^\circ C)} \right)(100 - 20^\circ C) = 8,560 \text{ J/min (11)} \]

Substitute equations 10 and 11 into equation 9.

\[ q_{\text{heater}} = \Delta q_{H_2O,1 \rightarrow 6,12} + \Delta q_{\text{oatmeal,3 \rightarrow 6}} = 39,030 + 8,560 \text{ J/min} = 47.6 \text{ kJ/min} \]

Heat is delivered to the water evaporator + steam heater at 47.6 kJ/min.

(B) Consider the porridge boiler. The steam in stream 9 cools to 100°C from 220°C. The heat released by cooling the steam warms the porridge to 100°C from 85°C and evaporates some of the water in the porridge. Draw an equivalent unit of elementary energy changes.

![Energy Diagram](image)

Calculate \( q_{\text{cooler}} \) the energy released by cooling the steam to 100°C from 220°C. Draw system borders around the fictitious steam cooler, write an energy balance, and use thermodynamics for an expression for the temperature change of the steam.

\[ \text{rate in} = \text{rate out} \]

\[ q_9 = q_{9a} + q_{\text{cooler}} \]

\[ q_{\text{cooler}} = q_{9a} - q_9 = F_{\text{steam,9}} \frac{C_p,\text{steam}}{220 - 100^\circ C} \]

\[ q_{\text{cooler}} = \frac{179.2 \text{ g steam}}{\text{min}} \left( \frac{1.94 \text{ J}}{\text{g steam}(^\circ C)} \right)(220 - 100^\circ C) = 41,718 \text{ J/min} \]
Calculate \( q_{\text{warmer}} \), the energy absorbed by heating the porridge to 100°C from 85°C. Draw system borders around the fictitious porridge warmer, write an energy balance, and use thermodynamics for an expression for the temperature change of the porridge (water and oatmeal).

\[
q_{5} + q_{\text{warmer}} = q_{5a}
\]

\[
q_{\text{warmer}} = q_{5a} - q_{5} = (F_{\text{water},5}c_{p,\text{water}} + F_{\text{oatmeal},5}c_{p,\text{oatmeal}})(100 - 85°C)
\]

\[
q_{\text{warmer}} = \left[ \frac{128.2 \text{ g water}}{\text{min}} \left( \frac{4.17 \text{ J}}{\text{g water}(^\circ \text{C})} \right) + \frac{78.1 \text{ g oatmeal}}{\text{min}} \left( \frac{1.37 \text{ J}}{\text{g oatmeal}(^\circ \text{C})} \right) \right](100 - 85°C)
\]

\[
q_{\text{warmer}} = 9624 \text{ J/min}
\]

Calculate \( q_{\text{evaporator}} \), the energy available to evaporate some of the water in the porridge. Draw system borders around the fictitious energy splitter and substitute the values for \( q_{\text{cooler}} \) and \( q_{\text{warmer}} \) above.

\[
q_{\text{cooler}} = q_{\text{warmer}} + q_{\text{evaporator}}
\]

\[
q_{\text{evaporator}} = q_{\text{cooler}} - q_{\text{warmer}} = 41,718 - 9,624 \text{ J/min} = 32,090 \text{ J/min}
\]

Use \( q_{\text{evaporator}} \) to calculate the flow rate of water through the fictitious water evaporator. Draw system borders around the fictitious water evaporator and write an energy balance.

\[
q_{5a} + q_{\text{evaporator}} = q_{5c}
\]

\[
q_{\text{evaporator}} = q_{5c} - q_{5b} = F_{\text{water},5b} \frac{\Delta H_{\text{vap,water}}}{\Delta H_{\text{vap,water}}}
\]

\[
F_{\text{water},5b} = \frac{q_{\text{evaporator}}}{\Delta H_{\text{vap,water}}} = \frac{32,090 \text{ J/min}}{2280 \text{ J/g water}} = 14.1 \text{ g water/min}
\]

The cooling steam evaporates 14.1 g water/min from the hot porridge. Draw system borders around the fictitious porridge splitter and write a mass balance on water.

\[
F_{\text{water},5a} = F_{\text{water},5b} + F_{\text{water},6}
\]

\[
F_{\text{water},6} = F_{\text{water},5a} - F_{\text{water},5b} = 128.2 - 14.1 \text{ g water/min} = 114.1 \text{ g water/min}
\]

The porridge contains 78.2 g oatmeal. Thus the weight fraction water is 114.1/(114.1 + 78.2) = 0.593.

The hot porridge product is 59.3 wt% water.

2. We must calculate the Return on Investment (ROI) to choose the better option. Start with option 1. Calculate the capital costs for option 1.

option I capital costs = reactor + separator I + separator II

= 143,000 $ + 971,000 $ + 776,000 $

= 1,980,000 $

Calculate the operating costs for option 1.

option I operating costs = reactant G + reactants F + I + reactor + separator I + separator II + depreciation

= 407,000 $/year + 98,000 $/year + 47,000 $/year + 147,000 $/year + 101,000 $/year + 1,890,000 $/10 years

= 989,000 $/year
Calculate the revenue for option 1.

\[
\text{option 1 revenue} = \text{product } P + \text{ by-product } B \\
= 1,524,000 \text{s/year} + 109,000 \text{s/year} \\
= 1,633,000 \text{s/year}
\]

Calculate the profit for option 1.

\[
\text{option 1 profit} = \text{revenue} - \text{operating costs} \\
= 1,633,000 \text{s/year} - 989,000 \text{s/year} = 644,000 \text{s/year}
\]

Calculate the ROI for option 1.

\[
\text{option 1 ROI} = \frac{\text{profit}}{\text{capital costs}} = \frac{644,000 \text{s/year}}{1,890,000 \text{s}} = 0.34 \text{/year}
\]

Repeat the calculations for option 2. Calculate the capital costs for option 2.

\[
\text{option 2 capital costs} = \text{reactor} + \text{separator I} \\
= 143,000 \times \left(\frac{7.41}{6.22}\right)^{0.6} + 971,000 \times \left(\frac{4.47}{3.22}\right)^{0.6} \\
= 158,800 \text{s} + 1,182,200 \text{s} = 1,341,000 \text{s}
\]

Calculate the operating costs for option 2.

\[
\text{option 2 operating costs} = \text{reactant } G + \text{ reactants } F + I + \text{reactor} + \text{separator I} + \\
\text{disposal} + \text{depreciation} \\
= 407,000 + 98,000 + 47,000 \times \left(\frac{7.41}{6.22}\right) + 147,000 \times \left(\frac{4.47}{3.22}\right) + \\
23,000 + 1,341,000 \text{s/10 years} \\
= 922,000 \text{ s/year}
\]

Calculate the revenue for option 2.

\[
\text{option 2 revenue} = \text{product } P = 1,524,000 \times \left(\frac{0.94}{1.00}\right) \text{s/year} = 1,432,560 \text{s/year}
\]

Calculate the profit for option 2.

\[
\text{option 2 profit} = \text{revenue} - \text{operating costs} \\
= 1,432,560 \text{s/year} - 922,000 \text{s/year} = 510,400 \text{s/year}
\]

Calculate the ROI for option 2.

\[
\text{option 2 ROI} = \frac{\text{profit}}{\text{capital costs}} = \frac{510,400 \text{s/year}}{1,341,000 \text{s}} = 0.38 \text{/year}
\]

Option 2 has the higher ROI and is therefore the better option.

Grading Rubric:

- Capital Costs: 2 pts for each option
- Operating Costs: 2 pts for each option
- Revenue: 2 pts for each option
- Profit: 2 pts for each option
- Used ROI to select better option: 4 pts.
3. Consider an energy balance on the overall system. We need to calculate the energy change from stream 1 to the total of streams 4 and 7. Draw a lever with 173 kg/min at the right end corresponding to stream 4 (steam at 270ºC) and 55 kg/min at the left end corresponding to stream 7 (a mixture of 20 wt% steam and 80 wt% water at 100ºC.) Locate the position of stream 7, which lies in the liquid+vapor line segment. The length of the liquid+vapor line segment is 117.5 mm. Stream 7 is \( \frac{20}{20+80} = \frac{1}{5} \) of the distance from “water at 100ºC.” \( 117.5 \div 5 = 23.5 \) mm. Draw the lever (shown below).

Locate the fulcrum of the lever. The length of the lever is 110 mm. Calculate the distance from the fulcrum to the steam at 270ºC.

\[
\text{distance from fulcrum to stream 4} = \frac{\text{flow rate of stream 7}}{\text{total flow rate of streams 4 and 7}} \times (\text{length of lever})
\]

\[
= \frac{55}{173+55} \times 110 \text{ mm} = 26.5 \text{ mm}
\]

Add the lever fulcrum and measure the distance to stream 1 (water at 20ºC): 124.5 mm. Convert mm to kJ/kg. The distance from 0 to 3000 kJ/kg is 155 mm.

\[
124.5 \text{ mm} \times \frac{3000 \text{ kJ/kg}}{155 \text{ mm}} = 2410 \text{ kJ/kg}
\]

The total flow of stream 1 is 173 + 55 = 228 kg/min. Thus the total heat flow is 2410 kJ/kg \times 228 kg/min = 549,400 kJ/min.

Only about 2 significant figures can be justified. The highest claimed on an exam was 7 significant figures, which corresponds to an accuracy of \( \pm 0.00001 \) mm. The pencil lead in most mechanical pencils is 0.5 mm diameter.

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Another method is to calculate the flow rate of water through the boiler/steam heater. Draw a graphical mass balance on the combiner. From above, the fulcrum (stream 7) is 23.5 mm to the right from “water at 100ºC.” The lever ends are at stream 6 (water at 20ºC) and stream 5 (steam at 270ºC). Balance 55 kg/min on the lever.

The distance from the left end to the fulcrum is 40.5 mm. The distance from the right end to the fulcrum is 110.5 mm. The ratio of the flow rates is inversely proportional to the lever arm lengths.

\[
\frac{55 - F_{T,5}}{F_{T,5}} = \frac{110 \text{ mm}}{41 \text{ mm}}
\]

\[
41(55 - F_{T,5}) = 110F_{T,5}
\]

\[
151F_{T,5} = 2255
\]

\[
F_{T,5} = 14.9 \text{ kg/min}
\]

From a mass balance on the splitter, \(F_{T,3} = 173 - 15 = 188 \text{ kg/min}\). Measure the energy difference between stream 2 and stream 3: 151 mm. As measured above, the distance from 0 to 3000 kJ/kg is 155 mm.

\[
151 \text{ mm} \times \frac{3000 \text{ kJ/kg}}{155 \text{ mm}} = 2920 \text{ kJ/kg}
\]

Thus the total heat flow is 2920 kJ/kg \(\times 188 \text{ kg/min} = 549,000 \text{ kJ/min}\), which agrees with the graphical method above.
4. It is useful to begin by identifying landmarks on the ternary diagram. Label the points that correspond to pure $P$, pure $B$, and pure solvent. Label the starting point and label the two goals: product with $\geq 94\%$ $P$ and waste below the line at $P/B \leq 2/3$.

Begin the process by mixing solvent with the $P+B$ mixture. How much solvent to add? I chose a tie line such that the $P$-poor end of the tie line was on the line $P/B = 2/3$ (30 mol\% $P$, 45 mol\% $B$, and 25 mol\% solvent) and thus this stream is discarded as waste. The stream corresponding to the $P$-enriched end of the tie line is distilled to remove the solvent. The process to this point is shown below and the graphical model is shown on the ternary diagram. Note - it was not necessary to write compositions on the stream in the process flowsheet if the streams were indicated on the ternary diagram.

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We have increased the concentration of $P$ in stream 5 to 87 mol%, but we are short of the goal of 94 mol% $P$. We must again mix with solvent and distill. Chose a tie line such that the unmixing line produces 94 mol% $P$. This is shown in the process flowsheet and ternary diagram below.
To increase the flow rate of product we must not discard stream 7, which has a $P/B$ ratio of 2; stream 7 contains higher relative concentration of $P$ than the feed stream. We can harvest product from stream 7 in a manner the same as we harvested product from stream 1: mix with solvent, obtain a $P$-enriched stream, and repeat. We need not add any units because a mixing line for solvent + stream 7 intersects the tie line for the first liquid-vapor separator. Mix solvent with stream 7 to obtain stream 10 and send to the first liquid-vapor separator. The final process and graphical model are shown below.
Goal: 94% P, 6% B

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Niu Niu Pontius ('21) proposed a creative, superior design that uses one less distillation column. She recycled the vapor stream from the flash drum that produces the liquid waste to the flash drum that produces the produce. This process and the graphical model is shown below.