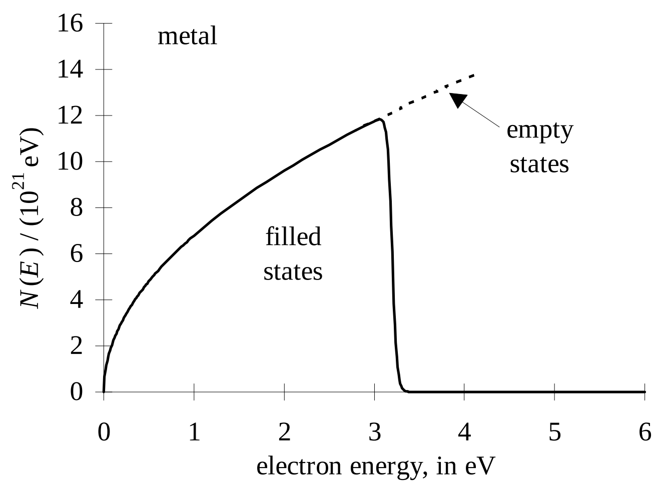
ChemE 2200 - Physical Chemistry II for Engineers
Solutions to Exercises for Calculation Session 4

1. Metals have a continuous band of energy levels at the Fermi level. There are empty energy levels immediately above the filled energy bands, at increments of about 10^{-15} eV. Electrons in metals can absorb photons at microwave energy ($E_{photon} = h\nu = (6.6 \times 10^{-34}$ J·sec$)(10^{11}$ sec$^{-1}) = 6.6 \times 10^{-23}$ J $= 4 \times 10^{-4}$ eV $\approx 10^{-4}$ eV) because there are vacancies at the higher energy levels.



The metal will absorb microwave energy photons indefinitely. Excited electrons will collide with the atoms, transfer their extra kinetic energy to the atoms (which heats the metal) and fall to a lower, unoccupied state. Or, an electron may continue to absorb microwave photons until it is ionized away from the metal object. This will appear as a spark, or electrostatic discharge.

Molecular solids, such as paper, are generally not metallic and thus do not have available levels within reach of a microwave photon's energy. Thus, the microwave photon is not absorbed by paper.

2. The following succinct response is sufficient for full credit.

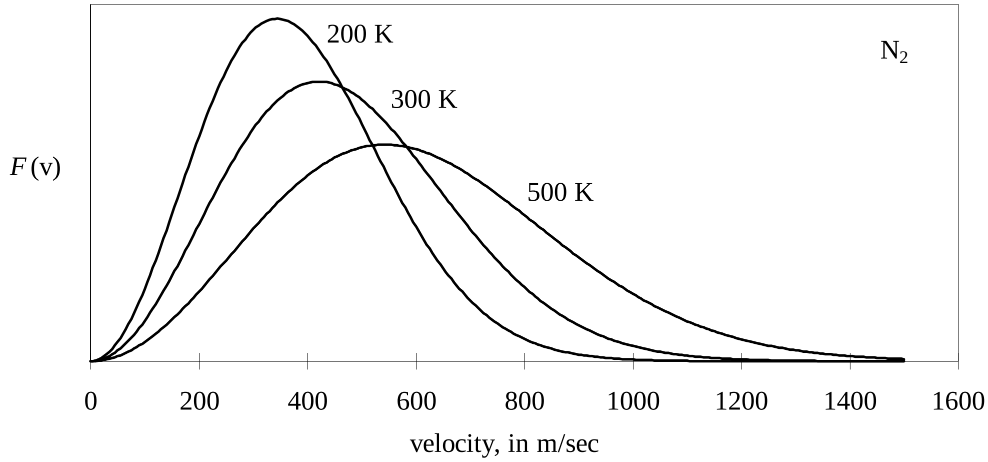
> "Only a small fraction of the electrons are close enough to E_F to absorb E and jump to a higher energy state. In an ideal gas, on the other hand, *all* gas atoms have the ability to absorb energy."

Here is a more detailed explanation.

Recall the velocity distribution of gas molecules obtained from the kinetic theory of gases. For those who favor equations, there is equation 27.40 from McQuarrie & Simon (p. 1112).

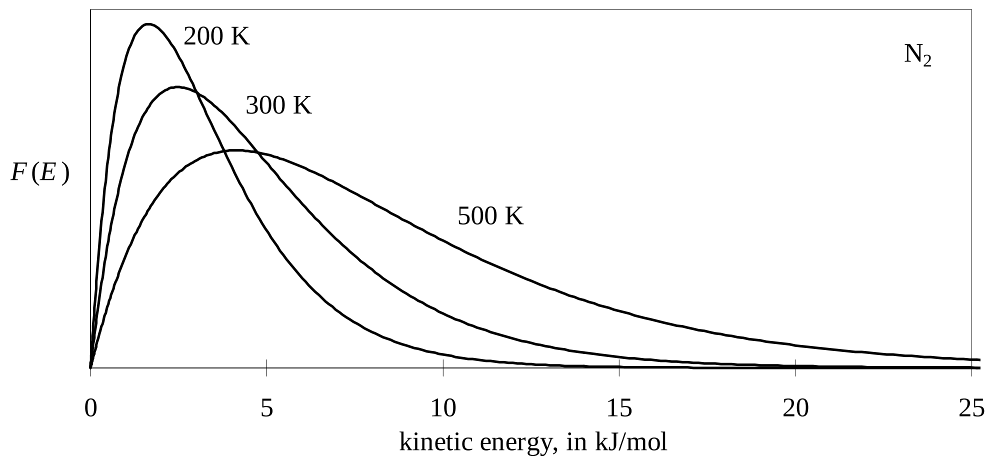
$$F(\mathrm{v}) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \mathrm{v}^2 e^{-mv^2/2kT}$$

For those who favor visual representations, there is figure 27.2 of McQuarrie & Simon (p. 1106). Some representative distributions are plotted below.



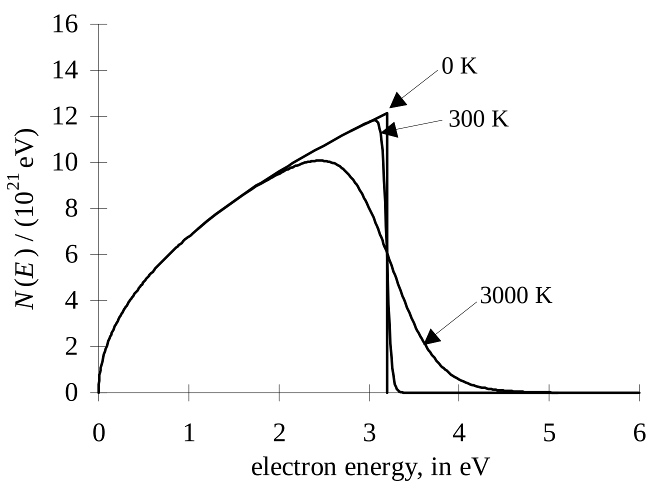
The velocity distribution can be converted to a kinetic energy distribution ($E = \frac{1}{2}m\mathrm{v}^2$),

$$F(E) \; = \; 8\pi m^{1/2}\left(\frac{1}{2\pi kT}\right)^{3/2} E\mathrm{e}^{-E/kT}$$



When the temperature increases, the entire energy distribution shifts to higher energy. The energy of every gas molecule increases. Energy must be given to every gas molecule to increase the temperature.

Consider the distribution of electron energies predicted by the free electron model (figure 7 from my monograph, *Electrons in Solids*.)



When the temperature increases, only the electrons with energies near the Fermi level may increase in energy. Most of the electrons cannot change energy. Thus to increase the temperature of the free electron gas, one needs only to add energy to a small fraction of the electrons. Near 300 K, this small fraction is about 1/30 of the electrons.

3. To dope a crystal of Ge one should use elements from the same period, so the substitutions are approximately the same size and do not distort the crystal lattice. To add electrons to the conduction band, one would add small amounts of an element with one more electron than Ge; use As. To create holes in the valence band, use an element with one fewer electron than Ge; use Ga.

4. Assume the ISO uses a semiconductor detector and that the semiconductor was selected because its band gap is slightly less than the lowest energy photon to be detected. Much of the electronic noise from a detector is caused by electrons thermally excited across the band gap into the conduction band. For a given semiconductor detector, the Fermi-Dirac distribution indicates that the number of thermal electrons (noise) depends exponentially on temperature. The ISO detector, which operates at 10 K, will have much fewer thermal electrons than an Earth-bound detector at 300 K.

5. To detect a photon, one should use a material with a band gap slightly smaller than the energy of the photon.

(A) Infrared photon, $E = hc\tilde{v} = (6.626\times10^{-34}\ \text{J}\cdot\text{sec})(3\times10^{10}\ \text{cm/sec})(1000\ \text{cm}^{-1})\dfrac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}} = 0.12\ \text{eV}$.

Use HgSe.

(B) Infrared photon, $E = hv = (6.626\times10^{-34}\ \text{J}\cdot\text{sec})(1\times10^{14}\ \text{sec}^{-1})\dfrac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}} = 0.41\ \text{eV}$. Use InAs.

(C) Visible photon, $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{sec})(3\times10^{10}\ \text{cm/sec})}{5000\times10^{-8}\ \text{cm}}\dfrac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}} = 2.5\ \text{eV}$. Use ZnTe.

6. There are two concepts needed to analyze these detectors.

i. A semiconductor can detect a photon if the photon can excite an electron across the band gap, from the valence band to the conduction band. Thus the semiconductor's band gap must be less than the energy of the photon to be detected.

ii. After the electron is excited into the conduction band, a current will flow if there is an electric potential across the semiconductor. The current is then detected by an ammeter.

Thus the type A semiconductor (E_{gap} = 1.1 eV) will absorb both photon 1 (1.7 eV) and photon 2 (1.2 eV). The type B semiconductor (E_{gap} = 1.6 eV) will absorb only photon 1; photon 2 will pass through the type B semiconductor.

Detector W (type A semiconductor) will detect both photon 1 and photon 2.

Detector X (type B semiconductor) will detect only photon 1. Photon 2 has insufficient energy to excite an electron into the conduction band.

Detector Y has a sheet of type A semiconductor above the detector circuit. Both photon 1 and photon 2 will be absorbed by the cover and thus neither will reach the type B detector connected to the electrical potential and ammeter.

Detector Z has a sheet of type B semiconductor above the detector circuit. Photon 1 will be absorbed by the cover and photon 2 will pass through and be absorbed by the type A detector connected to the electrical potential and ammeter.

Summary:

(A) Detects only type-1 photons? Detector X

(B) Detects only type-2 photons? Detector Z

(C) Detects both types of photons? Detector W

7.(A) To detect a photon at 1.0 eV, we need a semiconductor with a band gap as large as possible but less than 1.0 eV. This is solid B.

(B) A metal is used to attach the semiconductor detector to the battery. We need a material with no band gap at the Fermi level (at the highest occupied electron level). Solids D, E, and F are metals. It is best to have a material with a Fermi level comparable to the conduction electrons from the semiconductor, about 3.9 eV. This is solid F.

(C) The only other material with a band gap less than 1.0 eV is solid A.

(D) Because solid A has a small band gap, it will have a larger number of electrons thermally excited across the band gap. These thermal electrons are indistinguishable from electrons excited across the band gap by photon absorption. Solid A will have more electronic noise than solid B.

Also, solid A will detect lower-energy photons as well as the 1.0 eV photons. Solid A will detect photons as low as 0.5 eV. These low-energy photons will produce conduction electrons. The signal will be indistinguishable from a conduction electron caused by a 1.0 eV photon.

8. The principal concept is that P-doped silicon has electrons above the band gap and B-doped silicon has vacancies below the band gap. Each type of doping transforms a pure silicon semiconductor into a conductor. In P-doped silicon the conduction electrons are in energy states above the band gap. In B-doped silicon the conduction electrons are in energy states below the band gap.

(A) The positive applied voltage injects conduction electrons into the P-doped silicon. The electrons have energies above the band gap. When the electrons reach the B-doped silicon, they continue to travel left to right because there are empty states at the same energy.

(B) The negative applied voltage injects conduction electrons into the B-doped silicon. The electrons have energies below the band gap. When the electrons reach the P-doped silicon, they are forbidden to enter because the energy states below the band gap are completely occupied.

(C) The conduction electrons can enter the P-doped silicon if they are promoted to energies above the band gap. Because silicon has a band gap of 1.12 eV, the breakdown voltage is about 1.12 eV. The breakdown voltage depends only on the band gap; it is independent of the Fermi level.

The Team Competition Champions for Calculation Session 4:

Team ‘Moda & Bepeh’

Bepeh Amama, Moda Kurma

Score = 5 out of 5 and the 2nd submitted!

There are two concepts needed to analyze these detectors.

i. Electrons in the valance band cannot flow in an electric current. Electrons in the conduction band may flow in an electrical current. A semiconductor can detect a photon if the photon can excite an electron across the band gap, from the valence band to the conduction band. Thus the semiconductor’s band gap must be less than the energy of the photon to be detected.

ii. After the electron is excited into the conduction band, a current will flow if there is an electric potential across the semiconductor; the semiconductor is wired to a battery. The current is then detected by an ammeter.

Thus the type A semiconductor (E_{gap} = 1.0 eV) will absorb photon 1 (1.1 eV), photon 2 (1.7 eV), and photon 3 (2.2 eV). The type B semiconductor (E_{gap} = 1.6 eV) will absorb only photon 2 and photon 3; photon 1 will pass through the type B semiconductor. The type C semiconductor (E_{gap} = 2.1 eV) will absorb only photon 3; photon 1 and photon 2 will pass through the type C semiconductor.

Detector R (type A semiconductor) will detect photon 1, photon 2, and photon 3.

Detector S (type B semiconductor) will detect photon 2 and photon 3.

Detector T (type C semiconductor) will detect only photon 3.

Detector U: The semiconductor A above the detector absorbs photon 1, photon 2, and photon 3. Thus all three types of photons are filtered from reaching the detector semiconductor. Detector U detects no photons.

Detector V: The semiconductor B above the detector absorbs photon 2 and photon 3. Photon 1 passes through the filter and is absorbed by semiconductor A which is connected to the detection circuit. Detector V detects only photon 1.

Detector W: The semiconductor B above the detector absorbs photon 2 and photon 3. Photon 1 passes through the filter and but photon 1 is not absorbed by semiconductor C. Detector W detects no photons.

Detector X: The semiconductor C above the detector absorbs only photon 3. Photon 1 and photon 2 pass through the filter but only photon 2 is absorbed by semiconductor B which is connected to the detection circuit. Detector X detects only type 2 photons.

Detector Y: The semiconductor C absorbs only photon 3 and thus filters photon 3 from reaching semiconductor A. Photon 1 and photon 2 are absorbed by the semiconductor A which is connected to the detection circuit. Detector Y detects both type 1 photons and type 2 photons.

Detector Z: The semiconductor C absorbs only photon 3. Semiconductor C is connected to the circuit so detector Y detects at least photon 3. Photon 2 is absorbed by the semiconductor B. Photon 1 passes through the semiconductor B filter and is absorbed by semiconductor A. Detector Z detects both type 1 photons and type 3 photons.

Summary

(A) Detects all three types of photons? Detector R

(B) Detects only type-1 photons? Detector V

(C) Detects only type-2 photons? Detector X

(D) Detects only type-3 photons? Detector T

(E) Detects type-1 and type 3 photons? Detector Z

Grading Rubric: 4 points for each question; 2 points for the correct detector and 2 points for a valid explanation. “Type A semiconductor detects all three photons because it absorbs all three photons” with no preamble is not a sufficient explanation.