

ChemE 2200 - Physical Chemistry II for Engineers

Quiz 7 - March 12, 2025

Name: Solution

Substitute the expressions for $q_{\text{isothermal}}$ and w_{cycle} into the equation for efficiency.

$$\varepsilon = \frac{-w_{\text{cycle}}}{q_{\text{isothermal}}} = \frac{RT_1 \ln \frac{V_2}{V_1} - \frac{3}{2} R(T_1 - T_2)}{RT_1 \ln \frac{V_2}{V_1}} = 1 - \frac{3(T_1 - T_2)}{2T_1 \ln \frac{V_2}{V_1}}$$

We need to express the ratio V_2/V_1 in terms of temperature. Recall for an adiabatic expansion of an ideal monatomic gas,

$$\begin{aligned} \left(\frac{T_1}{T_2}\right)^{3/2} &= \frac{V_2}{V_1} \\ \ln \left(\frac{T_1}{T_2}\right)^{3/2} &= \ln \frac{V_2}{V_1} \\ \frac{3}{2} \ln \left(\frac{T_1}{T_2}\right) &= \ln \frac{V_2}{V_1} \end{aligned}$$

Substitute this expression into the expression for the efficiency.

$$\varepsilon = 1 - \frac{3(T_1 - T_2)}{2T_1 \ln \frac{V_2}{V_1}} = 1 - \frac{3(T_1 - T_2)}{2T_1 \frac{3}{2} \ln \left(\frac{T_1}{T_2}\right)} = 1 - \frac{(T_1 - T_2)}{T_1 \ln \left(\frac{T_1}{T_2}\right)}$$

$$\varepsilon = \frac{RT_1 \ln \frac{V_2}{V_1} - \frac{3}{2} R(T_1 - T_2)}{RT_1 \ln \frac{V_2}{V_1}} \quad +5 \text{ correct plug-in of } \frac{-w_{\text{cycle}}}{q_{\text{isothermal}}}$$

$$= 1 - \frac{3(T_1 - T_2)}{2T_1 \ln \frac{V_2}{V_1}} \quad +2 \text{ correct simplification}$$

$$\left(\frac{T_1}{T_2}\right)^{3/2} = \frac{V_2}{V_1} \quad +2 \text{ correct use of equation for adiabatic expansion of ideal gas}$$

$$\varepsilon = 1 - \frac{(T_1 - T_2)}{T_1 \ln \left(\frac{T_1}{T_2}\right)} \quad +1 \text{ correct simplification}$$